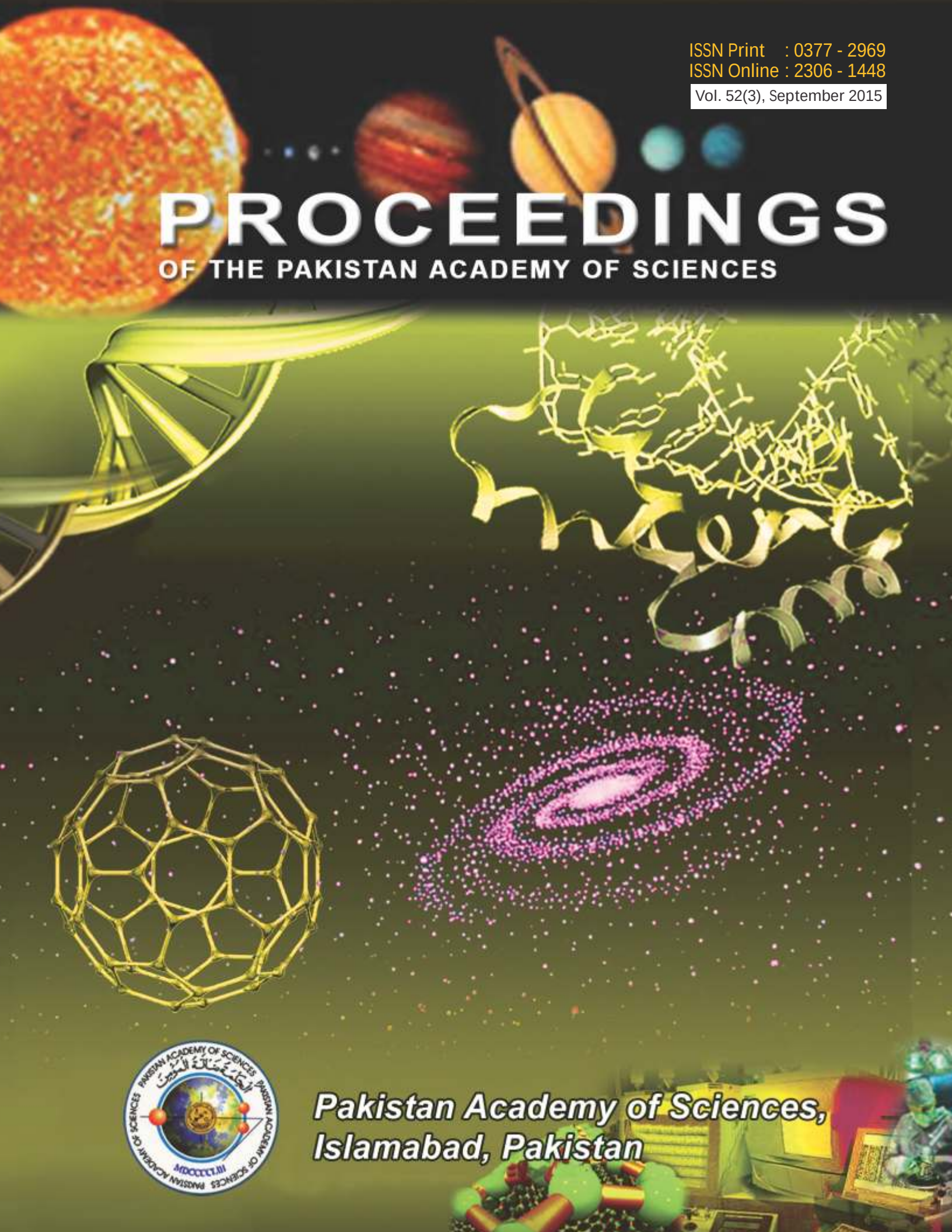
ISSN Print : 0377 - 2969
ISSN Online : 2306 - 1448
Vol. 52(3), September 2015

PROCEEDINGS

OF THE PAKISTAN ACADEMY OF SCIENCES



Pakistan Academy of Sciences,
Islamabad, Pakistan

PAKISTAN ACADEMY OF SCIENCES

Founded 1953

Proceedings of the Pakistan Academy of Sciences, published since 1964, is quarterly journal of the Academy. It publishes original research papers and reviews in basic and applied sciences. All papers are peer reviewed. Authors are not required to be Fellows or Members of the Academy, or citizens of Pakistan.

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HEC Recognized, Category X; PM&DC Recognized

Published by **Pakistan Academy of Sciences**, 3 Constitution Avenue, G-5/2, Islamabad, Pakistan
Tel: 92-5 1-9207140 & 9215478; **Fax:** 92-51-9206770; **Website: www.paspk.org**

Printed at **PanGraphics (Pvt) Ltd.,** No. 1, I & T Centre, G-7/l, Islamabad, Pakistan
Tel: 92-51-2202272, 2202449 Fax: 92-51-2202450 E-mail: pangraph@isb.comsats.net.pk

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Proceedings of the Pakistan Academy of Sciences 52 (3): 171–186 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Gesture Recognition through Android Interface for the Blinds

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Abstract: Blind people cannot actively participate in the society; due to lacking of using capability of latest technology. There is a need of an interface that may help the blind people in communicating with normal people by using latest technology. Human Computer Interaction (HCI) is a hot topic in research areas. This research provided such an interface which is helpful for blind people. A few devices have been designed for the blinds but their accessibility is limited due to specific design of the device. Therefore a standard interface is required that may be acceptable/ adjustable on any device. Mostly Braille language is used for the blinds for alphanumeric text input. There is a standard pattern used for each alphabet or digit. Android is most widely used platform for smart phones in these days. Research's focus is on the usage of Android for blind people by using braille language. Therefore, after researching about the blinds; braille and android suggested an interface based on gesture recognition. It will be operated by fingers movements like drawing the digit pattern on the screen and voice source's involvement is not there.

Keywords: Blind people, android, braille language, touch mobiles, gestures

1. INTRODUCTION

The Blind people are part of this world and due to their limited capacity of using latest technology just like normal humans; they cannot actively participate in the society. There is a need of an interface that may help the blind people in communicating with normal people by using latest technology. Several blinds have shown their endowment in many fields. Just because of communication gap, blind people are unable to become the active members of our society and a lot of talent is being lost. The treatment of completely blinds is impossible and we are inspired to develop an android based interface for blinds and its use will make their lives easy like normal human beings. A few devices have been designed for blinds but their accessibility is limited due to their specific designs. So a standard interface is required that may be acceptable/adjustable on any device. Few years back, Google introduced its mobile operating system (OS) namely 'Android' which has become number one choice for smart phones around the world. Android has overtaken windows and Symbian mobiles in terms of number of users and research agencies have confirmed this report [1, 3]. It's a Linux based Operating System and it is Open source. Reason of this popularity is that it is free of cost and its help is easily available. There are many applications in android available to disabled people. In these days 'Human Computer Interaction' (HCI) is a hot topic in research areas and it motivates me to research about the blind people's needs. Mostly Braille language is used for the blinds for alphanumeric text input. There is a standard pattern used for each alphabet or digit. In this research focus is on the usage of Android for blinds. The voice source is used for all applications used for call making and message creating for blind people. Still there is need of such an application that can be used without voice or some other resource. After conducting research on blinds, Braille and Android suggested an interface that will be operated physically by finger movement like drawing the

Received, May 2014; Accepted, June 2015
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digit's pattern on the screen. In it, there is no voice source involvement. Our proposed interface is proficient in taking information from screen. It will detect the image from screen and will extract the associated digit or alphabet with that image. It will extract information by using predefined patterns of comparing the image made by blind person by touching the screen and listening the voice generated by mobile that what digits or character he has entered in the mobile. After correctly entering the input, blind person will be able to make a call or sending a SMS on any number. This system will pass through different experiments and it will help to reduce the communication gap between blind people and society.

2. MATERIALS AND METHODS

The eye movement of a person can be used for communication. It can be very helpful for people who can't speak due to some hard stuff or deafness. By technical perspective, the phenomenon is that the cameras of our mobiles will detect the movements of eyes and a corresponding algorithm will take the eyes movement as an input and will match it with the action performed against it. Suppose if the eye moves up, it maximizes the screen and minimizes it as the eye moves down [1]. The more related application regarding this eye movement could be something like a message typed by eyes.

Facial expressions play an important role in expressing one's intentions and thinking. At the same time theyare an authentic and useful source to understand the feelings and intentions of a person who cannot express oneself verbally. The above mentioned application will be implemented in a way that only once we will record the expressions of a xyz person and will label these expressions as sad, smile, angry or what so ever [2, 4]. Blind people can be assisted in shopping. This is not a standalone/one-way system; the contribution of shopping malls is required in such a way that they should record the information against every product by tagging a barcode number [4, 9]. The blind user can retrieve the information by scanning the barcode through mobile camera and the application will voice that scanned information. GPS (Global Positioning System) is a worldwide service used to find location on earth [8, 11]. It has many applications like finding location of a specific place, nearest shopping center, bus stop, the shortest path, or to find the location of specific person having the GPS in his mobile by just sending a SMS (Short Message Service) or making a call. The human tongue is not only used for taste and talk but also for some other purposes. As our tongue freely moves in our mouth, it is useful for performing specific tasks [5]. Suppose a person can't move any part of body, a wireless sensor operating by tongue can perform this task. The sensor having buttons on it will be operating by tongue for moving the wheel chair. [11, 10]. The wireless sensor and its button's interaction with tongue are shown in Fig. 1.

Fig. 2 shows how different components of the smart home system are interconnected. User interacts with the system through user interface which communicates with a central control panel, there should be a component with functional Bluetooth and a transceiver for receiving and transmitting data from the user interface. For the sake of ease and accessibility; a system is designed that controls the wheelchair by using touch screen and voice functionality [9]. To implement/install this system, supporting components are installed on the wheel chair. User will simply touch the screen of mobile and it will perform different tasks like controlling the wheel chair. Disable people suffering from tetraplegia needs an efficient human machine interface centered on a camera for controlling an exoskeleton orthosis for upward limb movements [6]. It is optimized that how far an intelligent camera interface can be used for monitoring and controlling the interface for exoskeletonorthosis. A system is proposed that controls the association of mouse by capturing imagery of head movement by using a marker that is placed on the user's head.

Moreover, a special interface can be used for disabled people for interaction with a personal computer or a gaming interface etc. [3]. People

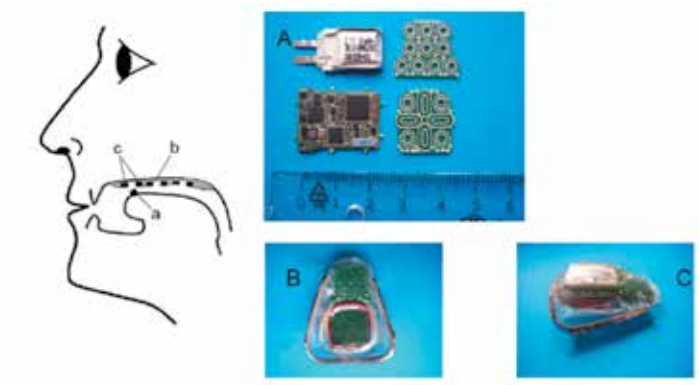

Fig. 1. Wireless sensor and its buttons interaction with tongue [5].

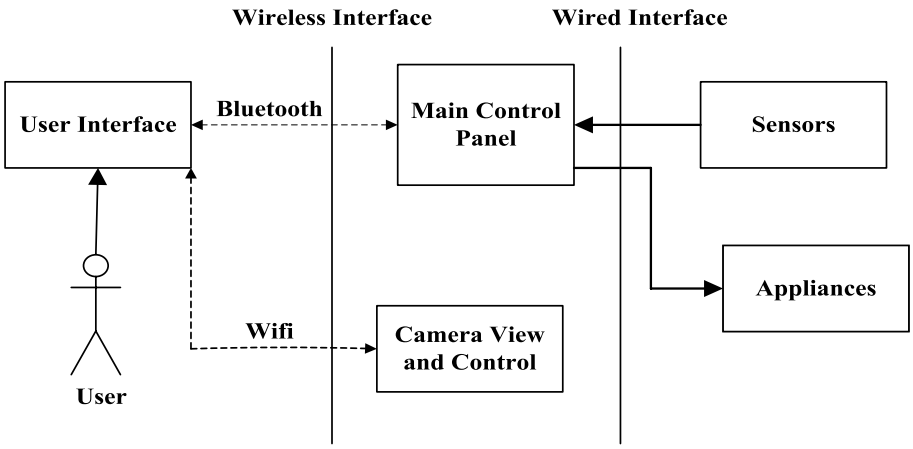

Fig. 2. Android based smart home system for disabled people [7].

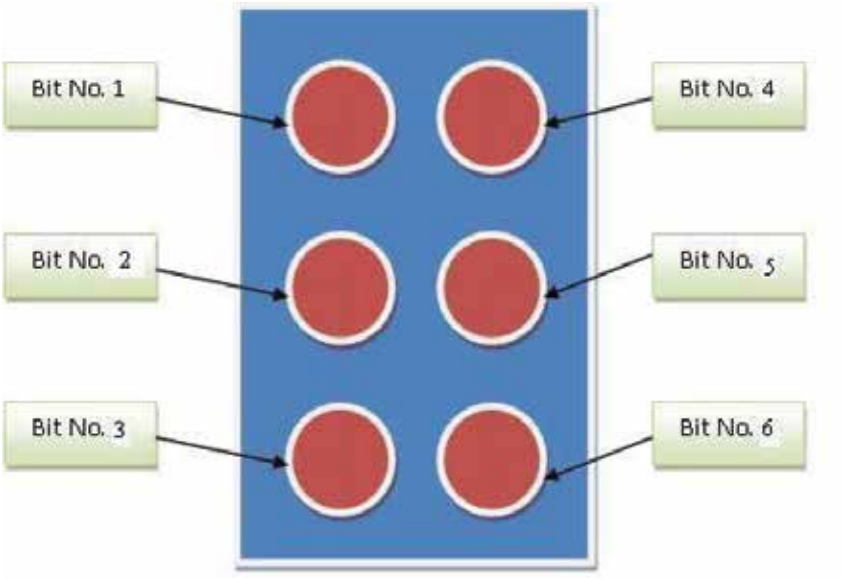

Fig. 3. Representation of braille cell [19].

having the disability of fingers are also assisted in android in such a way that they can use the system by just moving a small move of a single finger. Finger movements are monitored and perform actions accordingly, so that the disable user can use the computer normally [12].In the field of pain research, android provides a tool by the name of Pain Droid [13]. This is used to assist in pain assessment especially for the doctors and also for a lay man. A smart home system supports disabled persons in their domestic activities. A smart home system is developed such that it is wirelessly controlled by Bluetooth and Wi-Fi technology [7]. This application is adaptable to mobile phones and PDAs by using the Android OS. Basic purpose of this application is to control the switches of electrical devices through Bluetooth from a maximum distance of 25 meter from the main controller. Complete pictorial description of the system is presented in Fig. 2.

The blinds are considered sacred with supernatural powers in some nations, whereas in others blindness is considered as penalty for discourteous decent or public behavior. Negative perception about blindness results in the social barring and refusal of blinds. They have limited occasions for learning and service. This decreases self-confidence and creates a sense of insignificance in them. Estimated number of blind people in different provinces of Pakistan is given in the Table 1.

As a kind of disability; blindness results in joblessness, loss of revenue, large scale paucity, and low standard of life and non-affordance of health cautious facilities [18]. A lot of disable persons due to blindness are deprived of their financial efficiency and eminence of life. Visually impaired jobless persons are facing larger complexity of recognition in limited group of people. Moreover, carelessness from Govt. and community delays in delivering a healthy atmosphere for persons affected by blindness and prevents them to become the creative members of society. Both in rural and urban areas the no of males with disability is larger than females. This happens due to the more occurrence of child death among female children caused by social bias, deep rooted gender selfishness and liking for the male child within the households.

Table 1. Blind People distribution in different provinces of Pakistan [18].

Province	Estimated Number of Blind People
Punjab	769000
Baluchistan	52000
KPK	114000
Sindh	200000
Total	1140000

2.1 Braille Transcription

Braille is a tangible scripting system that is utilized by the blind and the visually impaired and it is utilized for signs, books, silo buttons and currency. Operators of Braille are able to read the screen of computers and other electronic provisions by refreshing Braille spectacles. Blinds are able to write down the braille with innovative stylus and slate or write it on a braille writer just like the movable braille note-taker. Blinds can also do it on a PC. Braille is a structure of encoding of printing in stamped dot shapes utilized for writing and reading by blinds. Every character of Braille has a cell of fixed size. It contains 2 columns of dots that are marked from top to the bottom 1, 2, 3 and 4, 5, 6. Representation of braille cell is shown in Fig. 3.

Spot images are used to represent the Braille characters; as an example we are listing the representations of the first ten Braille characters (a-j) of the alphabet. When they are headed through the marks for numeric entry then they also represent the ten numeric digits (1-9, 0). Representation of alphabets (a-j) and numeric digits (1-9, 0) shown in braille cell is given in Fig. 4. Overall there are 64 Braille characters with usage of space character. According to Library of Congress standard size of Braille cell is given. The reality that only sixty four Braille typescripts are accessible, it is obvious that

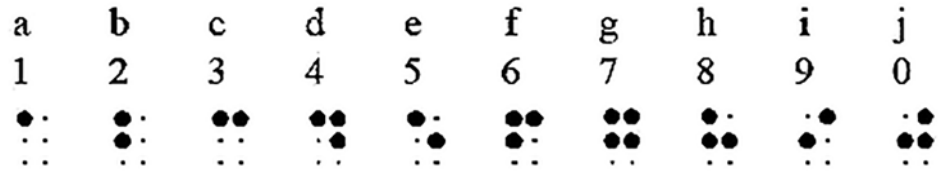

Fig. 4. How Numeric digit represented in braille [19].

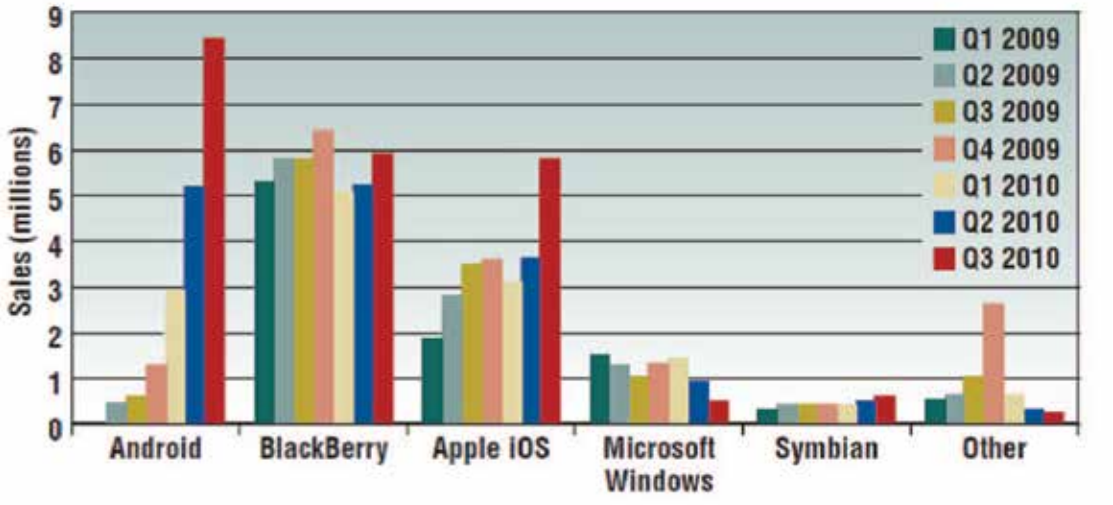

Fig. 5. Android mobiles sale in different time quarter [17].

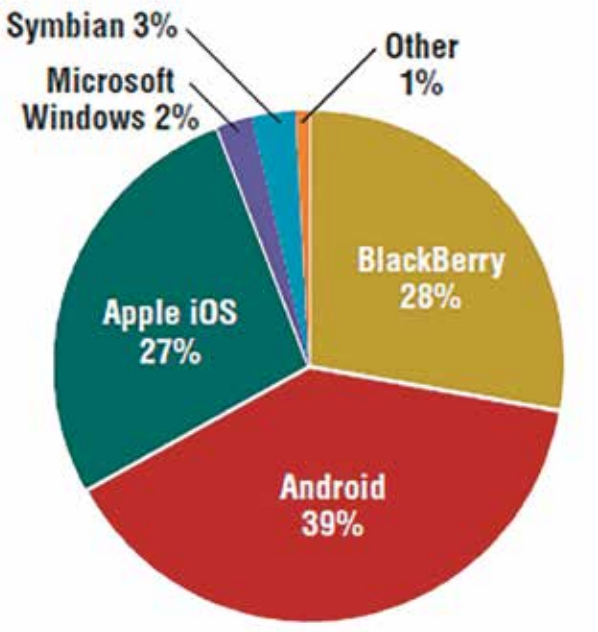

Fig. 6. Android market share in 2010 [17].

particular encoding guidelines have to be settled for diverse apps.

2.2 Android Platform Support

In March 2013, Android's share in the worldwide smart phone market, led by Samsung products, was sixty four percent. At the end of May 2013, forty eight billion applications were downloaded and installed from Play store of Google [14]. In July 2013, there were 11,868 models of Android devices, and 8 operating system types were at the same time employed. On September 3, 2013, total no of active android devices were 1 billion. As a result, instead of usage in tablets and mobile phones, android has further application for digital cameras, television games, consoles and other types of electronic devices. Android's Open nature has ensured huge society of enthusiasts and developers to utilize open source code as a base for public focused developments that affix latest capabilities for complicated consumers. Android mobile's sale is shown in Fig. 5 that is rising day by day as compared to other Mobiles.

So we choose Android platform for our application for Blind's Aid. Features like lenient licensing and open-source code let the software to be revised free of cost and spread by wireless carriers, manufacturer and support developers. Approximately there were 700,000 applications offered by Android in October 2012. Twenty five billion applications were downloaded from the Google Play and Android key Application Store. According to Developer's Analysis in May 2013, it came to know that Android is used by seventy one percent of the mobile developer population as the most admired platform. Linux kernel of Android's has an additional change in architecture by Google exterior to normal Linux kernel improvement series [15]. In 2010, Android turned into market head by 39 percent market shares as given in Fig. 6.

By using Android Software Development Kit (SDK), applications are developed in Java language. SDK comprises of a set of development tools, including software libraries, a debugger, example code, certification and tutorials. Although Linux-based development phase sparked the people attention, but there were further hesitations about Android in front of strong opposition from familiar companies in the smart phone marketplace; for example Microsoft and Nokia and opponent Linux mobiles OS[16].

3. RESULTS AND DISCUSSION

People with special needs are the part of our society. They cannot participate efficiently in our community due to lacking of communication abilities. The Blind people, due to lacking of using latest technology like normal people cannot actively participate in the society. Particularly when talked about blind people, their needs are different from deaf and dumb people. They need a standard interface according to braille language that is mostly used for blinds. Android based Smart phones and touch screen devices are mostly used in these days. A few devices have been designed for blinds but they limit their accessibility to specific designs of such devices. To achieve accessibility to all android based phones, a standard interface is required that may be acceptable/adjustable on any device. Mostly Braille language is used for blinds for alphanumeric text input. There is a standard pattern used for each alphabet or digit in braille. There are bundles of applications available on android regarding disabled people. The voice source is used for input in all applications related to call making and messaging for blind people. There are some drawbacks of such applications such as:

What will be if you are on such a place where noise is disturbing your voice?

What will be if you are going to write a message through voice and voice conversion to words is not accurate due to language problem?

The second thing which we are going to address in this research is the gesture recognition for blinds. After studying different gesture recognition algorithms and techniques, we come to know that they did not fulfill the needs of blinds as they need

quick response for their input; so we propose a new technique for the recognition of gestures for blind people. Due to problems relevant to voice input as described above; a single-touch entry system is presented for touch screen devices. By studying different aspects of blind people's needs, Braille language and Android platform we designed an interface based on a specific gesture recognition technique that will help the blind persons to utilize latest technology with ease. With the application named "Application for Blinds Aid (ABA)", blind people will be able to enter input just like they were writing Braille by using the conventional Six-dot code in a matrix form. Braille System is very easy yet influential, in this system any character, can be developed by grouping of 6 or less than 6 dots. This system takes benefit of the information to permit user to enter transcript ordering into a solo display made up of six marks representing the Braille matrix.

An interface is designed for blind people in Android. A blind person can simply make calls and send messages using this system without voice input. User can move mobile in predefined direction and the application get started and enter into the call or message mode accordingly. The application will use the earth gravity to detect the user movement. User dial phone number by drawing digits on the screen. User can draw the digit in braille language on screen by moving his finger accordingly. Our application would be well-organized and efficient to take the information from screen. System will detect the image from screen and then extract the associated digit or alphabet with that image. Finally it will extract info from input by utilizing well defined pattern comparisons with the image drawn by blind person on the touch screen. The application process that drawn image and analyze the digits. After dialing the phone number or typing the message the user will make a call or send message. In addition, the voice output and mobile vibrations can also be implemented in such a way that voice output can be used to hear dialed number and vibration can be used to sense the drawn digit. Mechanism of image extraction from the screen is shown in the Fig. 7.

There are various techniques used to categorize and distinguish gestures as discussed earlier; however, all these techniques are not completely compatible with our problem associated with gesture recognition for blind people. A few researches have also provided the methods to achieve this goal. Although researchers have a number of constraints and limitations yet major difficulty of the real time or immediate gesture recognition process is their trust on a few suppositions to do work appropriately and these suppositions limit user's freedom of utilization. For example, providing few proficient consequences in the laboratory's atmosphere may not give the suitable consequences exterior to the laboratory. Therefore, an innovative technique for gesture recognition is developed called as *Gesture Recognition Technique for Blinds (GRTB)*. The steps followed in this technique are given below:

1. User inputs the braille combination for digit; the entered pattern is then matched with sample space's quintile values and then concentrate on standard deviation of the entered pattern with quintile value of the sample space.
2. Next step is to setup a threshold value (that is the greatest deviation value a gesture is able to contain) and based upon threshold value we locate the lowest variation of the sign with the sample space.
3. If lowest or smallest deviation is lesser than the threshold value then the sign is classified for that digit and take into account as that gesture has least standard deviation from the threshold value.
4. When a latest gesture arrives which is not registered and it does not match with the sample space then an audible error message will be generated.

Our recently developed system appears as the adaptation of above procedure, this system is nowadays broadly accepted in touch mobile devices. In this system, the screen of a touch mobile serves as the depiction of Braille cell that contains 6 big dot targets which represents all the dot positions.

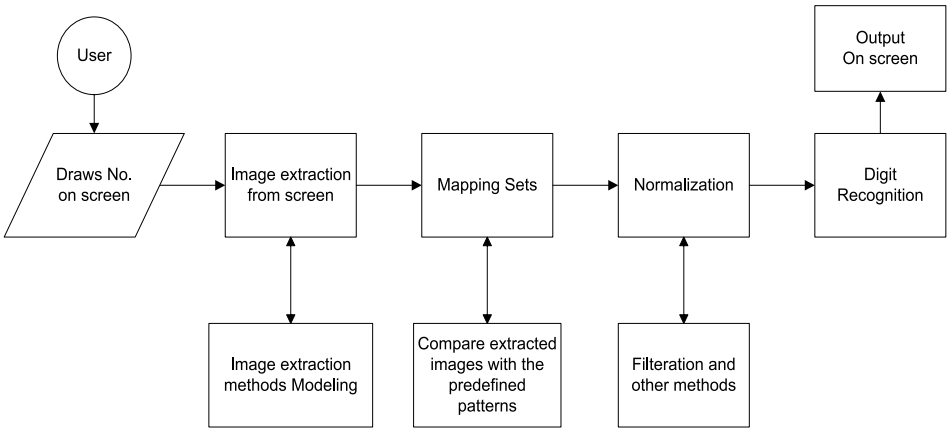

Fig. 7. Extraction of digits from screen.

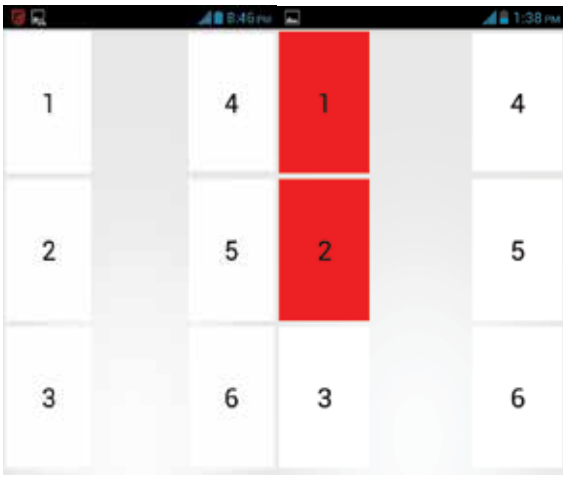

Fig. 8. Representation of the six target zones and how user makes entry.

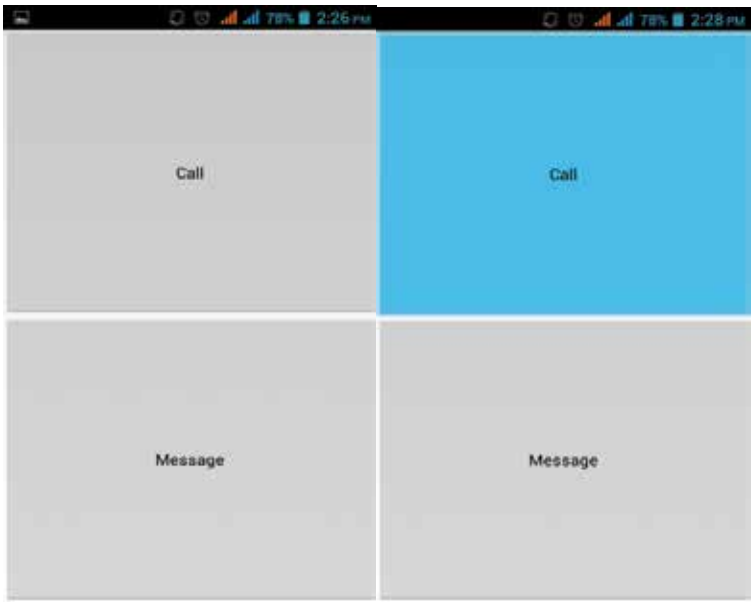

Fig. 9. Interface for selecting call or message mode.

To permit a simple and easy search, these dot targets are set large and are mapped to corner and borders of mobile screen. As these targets are also represented according to the well-recognized and predictable Braille cell, therefore targets becomes easy to locate. Six target zones visual depiction and how user makes entry in show in the Fig. 8.

To maintain the system straightforward and simple, multi touch practices are ignored. Every communication by this system is prepared through solo touch input. Every time a user press or drag his/her finger to the new target then the succeeding dot number will vibrate on selection, but the target will not instantly elected. A little clock is incorporated to avoid unconscious selections from the user and a double tap in center will ensure that user has completed the selection of braille pattern. In the Braille cell, to spot a dot the user simply hasto touch a target and there will be a vibration prompt for confirmation. If he repeats this process on a dot which was previously selected then it would eradicate its selection. Once the essential spots for Braille character will be marked in either category that user wishes then it will be accepted by a double tap in the center of screen. In the developed application "Space" key is obtained by empty Braille cell entry. When user attempt to allow a wrong arrangement of buttons then Braille matrix will be clean and a fault sound like "No Such combination exists" will be announced.

Swipe on left side will clear the entered Braille cell if partially dots are marked in the past or it will delete previously entered character if there is no selection made for any cell. This technique seeks to present a fewer tense initial approach by means of the touch mobiles by dipping amount on the screen target. Through minimizing amount of faults and enable the users to be successful, we would coagulate their assurance and will let them go beyond. Moreover, we can plan to gain the advantages of capabilities of those people, who use Braille on the usual base, however it also permit those persons who did not learn or conserve Braille practice throughout simple everyday connections.

Extracted features are utilized to categorize gestures and to make conclusion on the basis of data group, after applying the feature extraction and segmentation techniques. Gesture recognition is a stage where data investigated from the pictorial figure of sign is documented or categorized in a specific gesture. This stage includes pattern recognition techniques and methods [20]. Classification technique can be divided in two portions, Machine learning methods and Rule based methods. First method is aptitude based or machine learning present disciplined and vigorous mapping among the feature set of highest dimension and gestures. Because of this reason the machine find out the gesture's model from the provided training set rather than by means of brain just like human being. Second method is the rule based approach that follows a few rules that are fixed or determined by particular researchers manually [21]. Gestures extracted from pictorial images are acceptable if the extracted features fulfill these statutes and if the statutes are fulfilled then gesture is documented as predefined.

Cutler et al. suggests a system that utilizes object identification approach based on rules and also suggests 6 rules to differentiate 6 gestures only. Still, dissimilar boundaries are there and it's too hard to utilize. It's not capable for real time apps. It's extremely hard to retain in mind rules and then follow them and describing the rules on runtime, it will increase the calculation expenses and performance problem. We use our newly developed gesture recognition technique for the extraction of features and recognition for specific pattern drawn by blind people on the mobile screen. We have compared the principles of gestural interface for different application for blinds with our newly developed Gesture Recognition Technique for Blinds (GRTB) and different parameters have been considered in context of this comparison, by considering their shortfalls, we can improve them in our new Gesture Recognition Technique. In Table 2, a detailed comparison against different features is given.

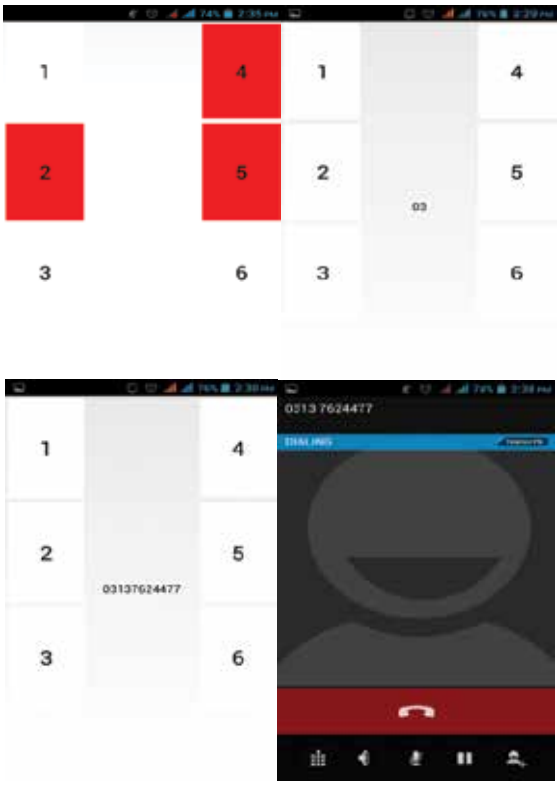

Fig. 10. Making a call on a mobile number.

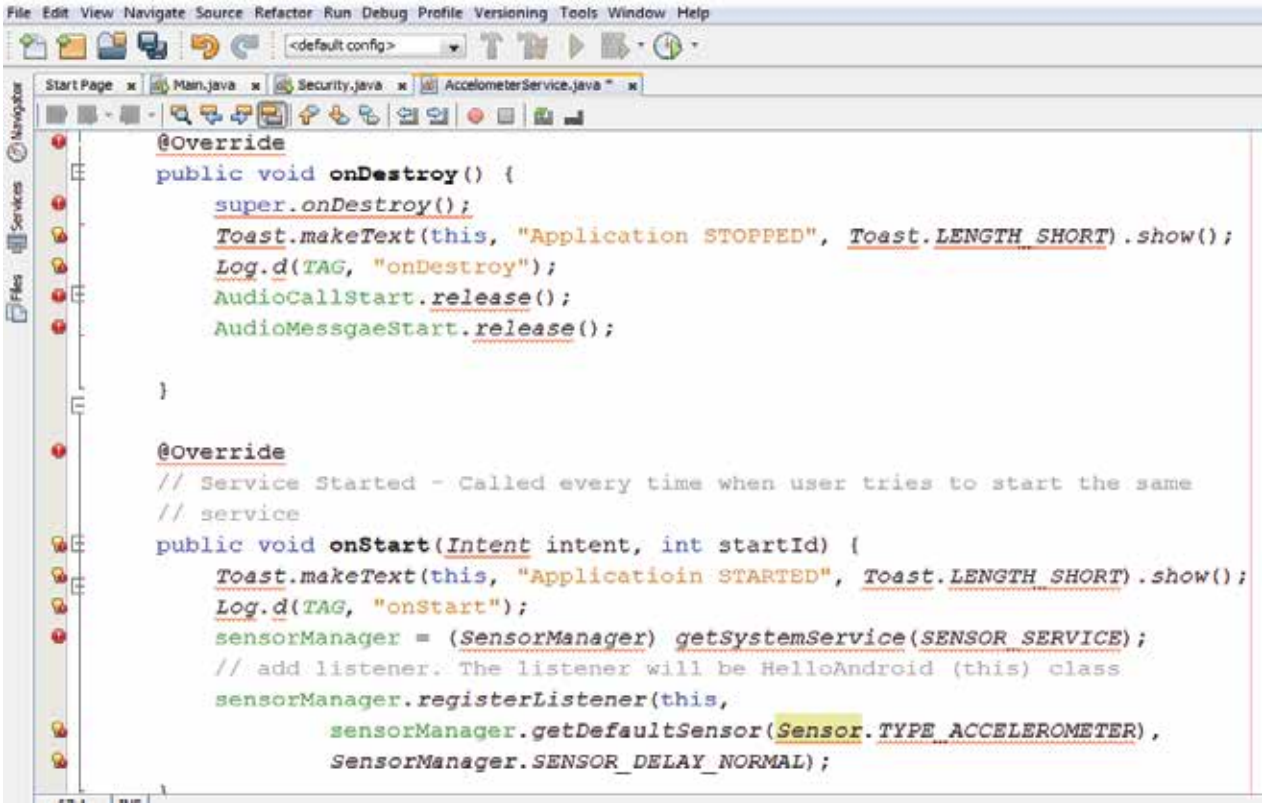

Fig. 11. Accelometer service.

Table 2. Comparison of gestural interface principles for different applications.

Applications/ Features	VoiceOver	No-Look Notes	NavTouch	BrialleTap	ABA
Learnable	Difficult	Not easy to learn	Easy to learn	Highly learnable	Easy to learn
Memorable	Difficult	Difficult	Hard to memorize	Highly memorable	Highly memorable
Responsive	Highly responsive	Highly responsive	Highly responsive	Highly responsive	Highly responsive
Meaningful	Gestures are meaningful	Highly meaningful	Not very meaningful	Highly meaningful	Gestures are meaningful
Clever	Not so clever	Not so clever	Not clever	Not clever	Not so clever
Playful	Highly playful	Error prone	Hard to make an error	Errors may occur	Highly playful

We have implemented the call and SMS mechanism for the proposed interface, detailed design and implementation for both of them is given in section 7.1 and section 7.2.

3.1 Design and Implementation of Call Mechanism

when we start the application from icon and handover the mobile device to the blind person to make a call or send message then he will shake the mobile with a reasonable velocity and application will be turned into call or message mode upon blind person's selection for call or message. There will be a voice generated for the blind person to tap on the upper half for call and lower half of the screen for messaging. It's shown in the Fig. 9.

Upon selection of call mode, an interface for entering mobile number will be appeared and a voice will be generated for blind person to enter the number for making a call. Blind person will enter the braille pattern for the corresponding digit by selecting the holes and then double tap in the center of the screen to make an entry. Upon selecting hole, its space will be vibrated and blind person will be ensured that he has selected a hole. When he has selected a hole then on double tapping, sound for the corresponding digit will be played and blind people will be ensured that he has entered the correct digit. If a wrong digit is entered then blind user will perform left swiping on the bottom of the screen, the entered number will be removed and a voice will be generated that number is removed. In this way blind user will enter the complete mobile number and then perform right swiping at the bottom to make a call. Call process is shown in the Fig. 10.

We have used Eclipse as an Integrated Development Environment (IDE) for the development of this application. Eclipse is most suitable IDE and it consists of extensible plug-in system for customizing an environment according to your own needs. There is a base work space that is mostly written in Java language. So, we also used Java and Eclipse Software Development Kit (SDK) that consists of Java development tools. Instead of copy pasting complete source code here, we have taken the snapshot for major modules of the application. In the Fig. 11 the code for Accelometer service is shown, that enables the mobile device to detect motion of the device for starting the application. As there are two major parts of this application named "Call" and "Message" therefore, code for the call activity is maintained in a separate ".java" file and it is given in the Fig. 12. Selection for braille pattern is also given in the Fig. 12.

3.2 Design and Implementation of SMS Mechanism

On selecting Message button form the lower corner of the screen, application will be turned into the SMS mode. Like call mechanism, an interface for entering mobile number to send sms will be appeared in front of the blind person. There will be a voice for blind user to enter number for sending text message. Interface is shown in the Fig. 13.

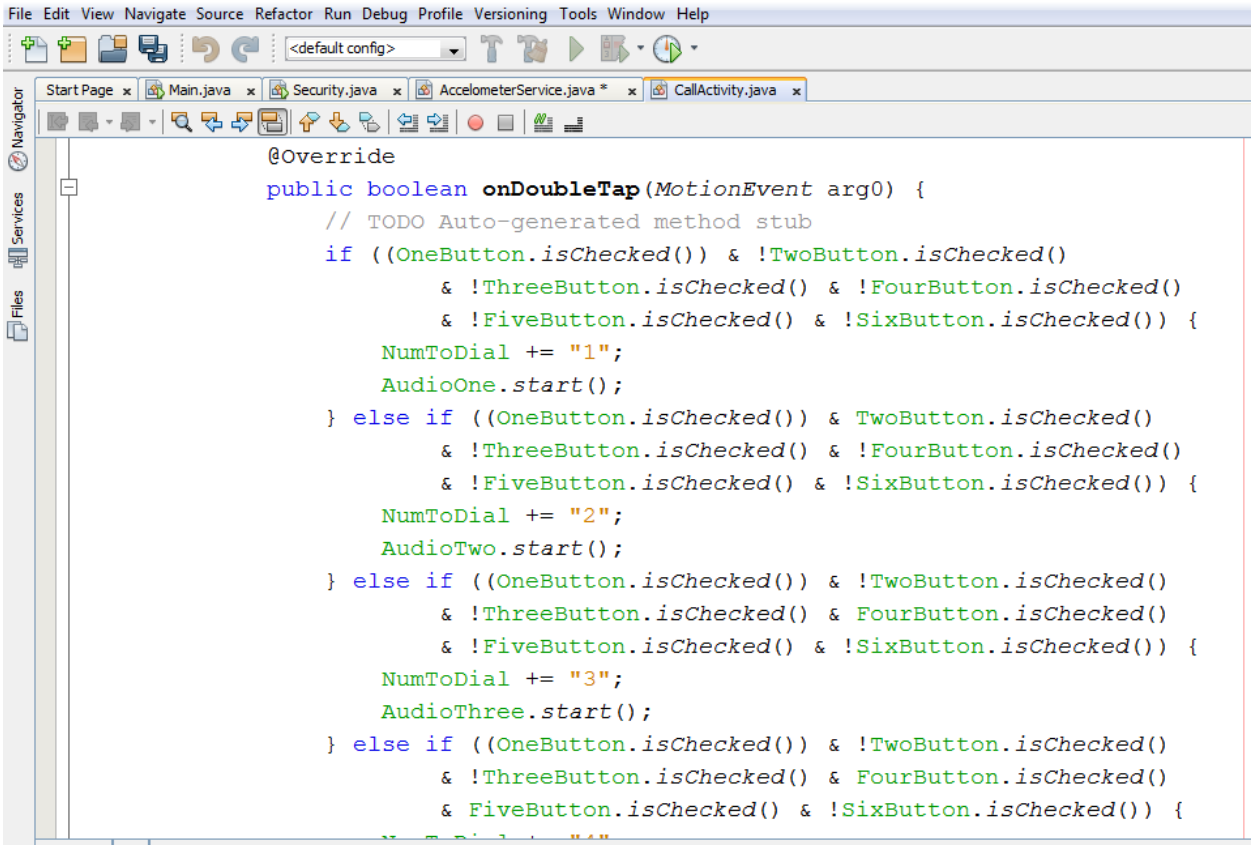

Fig. 12. Call activity.

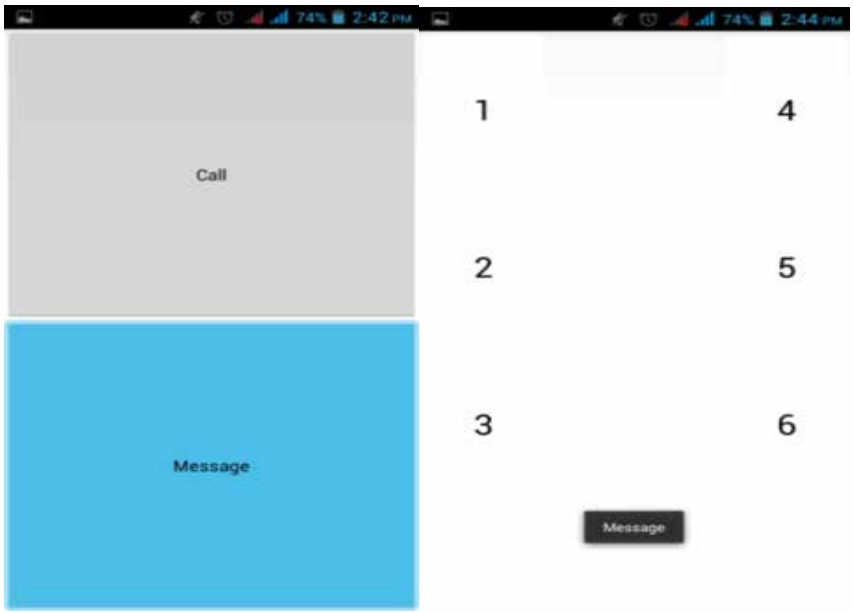

Fig. 13. Selecting message mode.

Fig. 14. Message sending in braille.

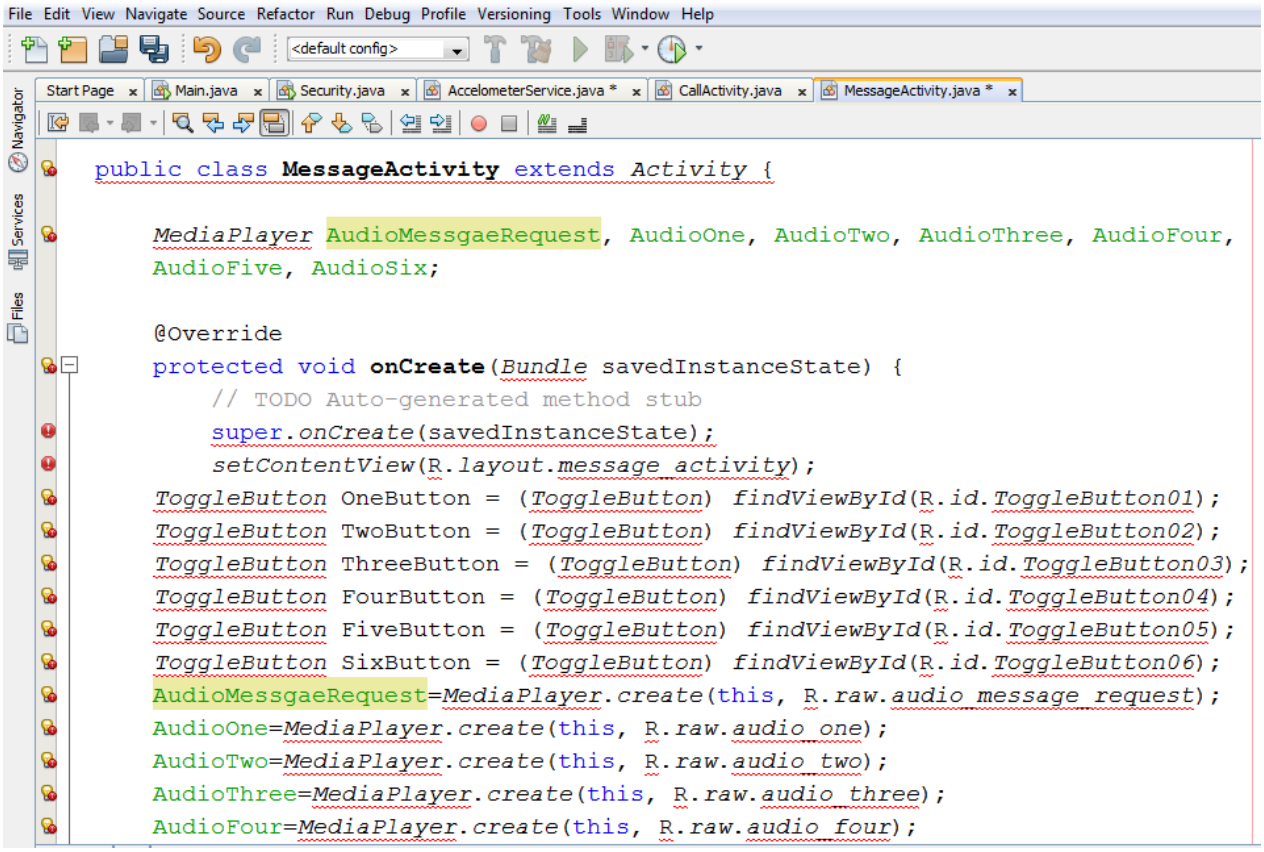

Fig. 15. Message activity.

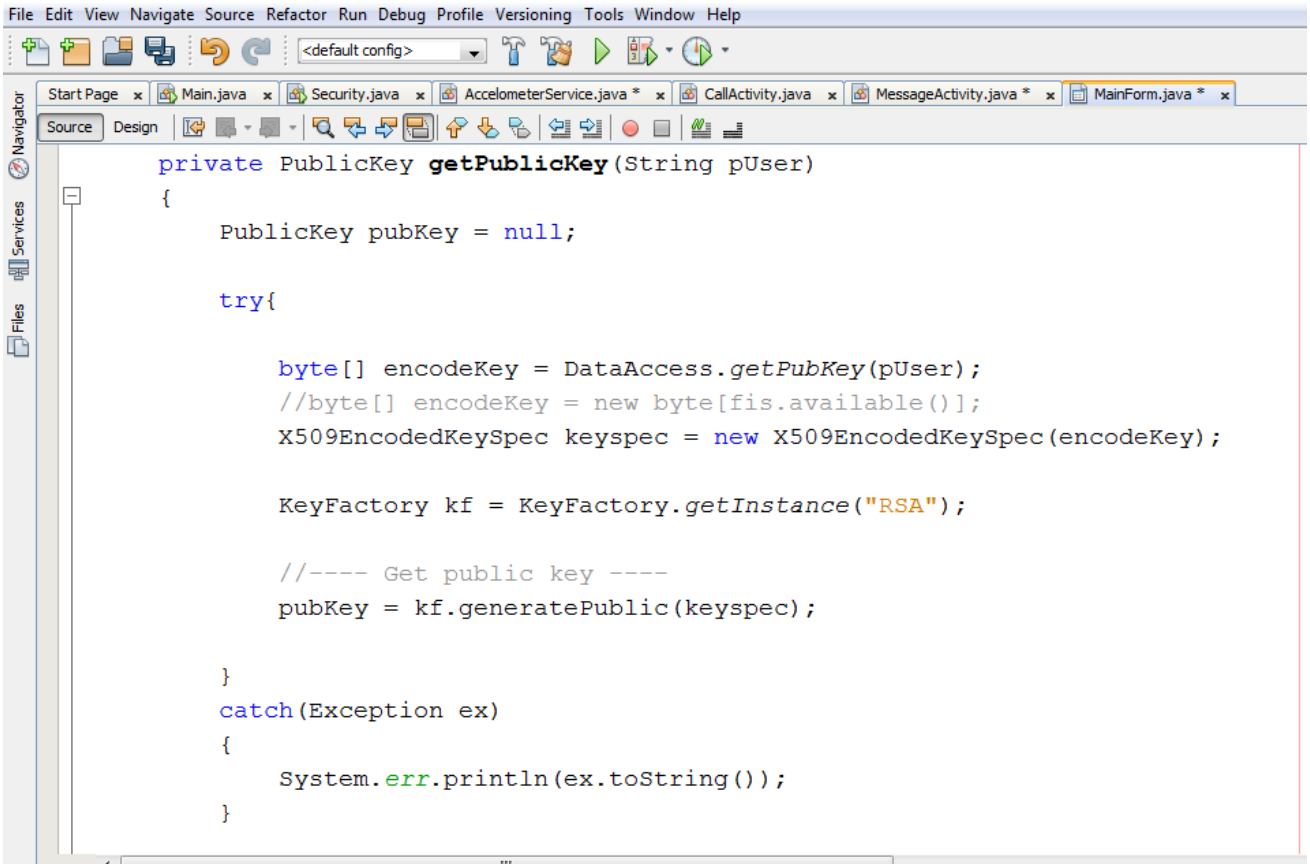

Fig. 16. Main menu.

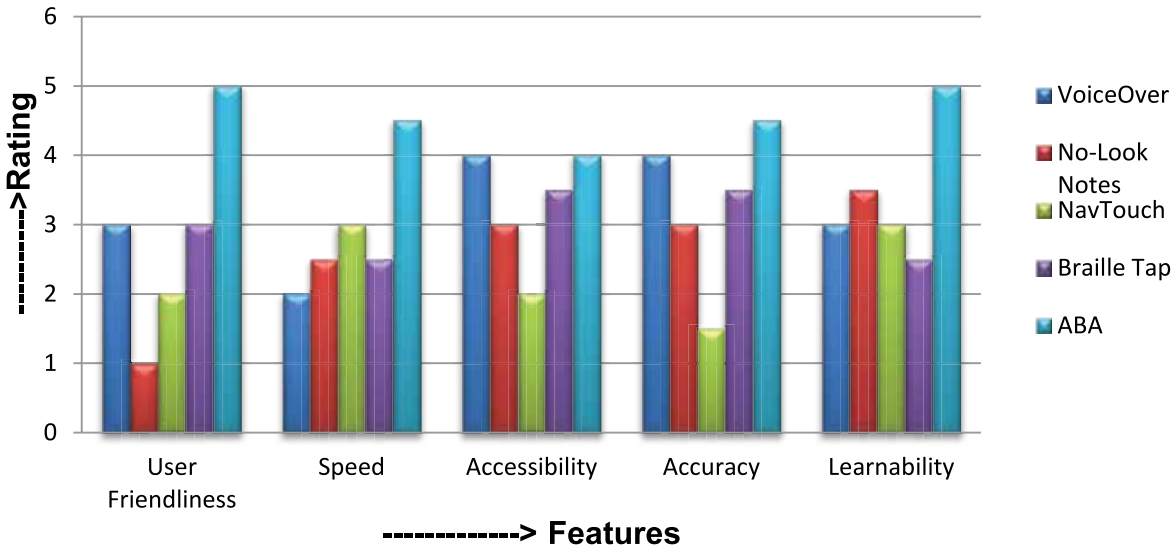

Fig. 17. Comparison of ABA with other applications.

Table 3. Comparison of ABA design with other applications.

Application Design	Easy to use	Speed	Accessibility	Accuracy	Learnability
VoiceOver	3	2	4	4	1
No-Look Notes	1	3	3	2	4
NavTouch	2	4	2	1	3
Braille Tap	4	2	3	1	2
ABA	5	4	4	5	3

After selecting the message mode, user will enter the mobile number where he/she wants to send the text message. This process is similar to entering the number for making call but here after entering the mobile number, user will perform right swiping and application will ask the user to type text message by using the same pattern as used before. Voice will be generated at each step for the guidance of blind user. When user will completely enter the text message then he/she will perform right swiping again to send the message. A sound will be generated for the confirmation of the sent message. Message sending is show in the Fig. 14. Similar to call activity, code for Message activity is given in the Fig. 15.

Code for main menu is given in the Fig. 16, in the "Main Menu.java" all the events are handled regarding this application. We have compared the ABA design with other relevant applications for blinds and noted down the user's view in a tabular form for different parameters. Blind users who have already used other blind specific apps were asked to use ABA application and to give their ratings to this application out of 5 (5 is the highest rating). Average of ratings by different users is given in the Table 3 and Fig. 17. Table 3 shows that our developed application is up to blind user's satisfaction and he/she rates this application in a better way than others. We analyzed shortcoming of other related applications for blinds and developed ABA to overcome these shortcomings, as a result we got better response from the blind users.

4. CONCLUSION AND FUTURE WORK

Gesture recognition based on patterns has been explored since last decade, but it is not qualified up till now. Therefore it requires many researches for strength and competence. After improvement of such kind of method, human being will experience relaxation by utilizing HCI softwares and associated hardware for the valuable purpose. In this research work a new technique for gesture recognition is developed which is discussed earlier. The developed Gesture Recognition Technique for Blinds (GRTB) is efficiently working for the recognition of the braille patterns and it helps to fill the gap between blind people and latest technology. It is better and easy as compare to Voice over and other braille apps. With this application blind user can easily use the latest smart phones for their use like here we implemented the functionality of Call and SMS. Scope of this research work is limited due to shortage of time and resources. Same interface can be used in other touch screen devices like tablets and PCs for text documents writing. Based upon this idea, a blind user can do typing on a computer for writing an email or composing a text document by using the same patterns mapping which is developed for typing sms. Also in mobiles, besides call &sms functionality, other features can be added. For instance, to start a specific application we associate a number with that application and upon selecting this number corresponding application will start running. This research area can be further explored and implemented in other ways mentioned above, to make the interaction of blinds people with the real world more convenient.

5. REFERENCES

1. Vazquez, L.J.G., M.A. Minor & A.J.H. Sossa. Low cost human computer interface voluntary eye

movement as communication system for disabled people with limited movements. In: *Pan American Health Care Exchanges (PAHCE)*, p. 165-170 (2011).

2. Hofree, G., P. Ruvolo, C. Reinert, M.S. Bartlett & P. Winkielman. Why are you smiling? In a strategic context, people's affective responses reflect the meaning of android's facial expressions. In: *IEEE International Conference on Development and Learning and Epigenetic Robotics (ICDL)*, p.1-2 (2012).
3. Higa, H., T. Dojo, K. Mihara, H. Uehara & Vijayan K. Asari. A video-based user interface for people with disabilities of the fingers. In: *International Association of Computer Science and Information Technology - Spring Conference*, p. 459-463 (2009).
4. Parmar, K., B. Mehta & R. Sawant. Facial-feature based human-computer interface for disabled people. In: *IEEE International Conference on Communication, Information & Computing Technology (ICCICT)*, p. 1-5 (2012).
5. Lotte, N.S., A. Struijk, E.R. Lontis, B. Bentsen, H.V. Christensen, H.A. Caltenco & M.E. Lund. Fully integrated wireless inductive tongue computer interface for disabled people. In: *Annual International Conference of the IEEE on Engineering in Medicine and Biology Society*, p. 547-550 (2009).
6. Baklouti, M., S. Couvet & E. Monacelli. Intelligent camera interface (ICI): A challenging HMI for disabled people. In: *IEEE First International Conference on Advances in Computer-Human Interaction*, p. 21-25 (2008).
7. Ramlee, R.A., D.H.Z. Tang & M.M. Ismail. Smart home system for disabled people via wireless Bluetooth. In: *IEEE International Conference on System Engineering and Technology*, p. 1-4 (2012).
8. Rifat, M.R., S. Moutushy, S.I. Ahmed & H.S. Ferdous. Location based information system using open street map .In: *2011 IEEE Student Conference on Research and Development,* p. 397–402 (2011).
9. López-de-Ipiña, D., T. Lorido & U. López. Indoor navigation and product recognition for blind people assisted shopping. III *International Workshop on Ambient Assisted Living – IWAAL*, p. 33-40 (2011).
10. Baron, R., M. Mateo's, A.L. Barrios, I. Muriel & G. Villarrubia. Alternative human-machine interface system for powered wheelchairs. In: *IEEE 1st International Conference onSerious Games and Applications for Health (SeGAH)*, p. 1-5(2011).
11. An, A., Hae-Duck J. Jeong, J. Lim & W.S. Hyun. Design and implementation of location-based sns smartphone application for the disabled population. In: *Sixth International Conference on Innovative Mobile and Internet Services in Ubiquitous Computing (IMIS)*, p. 365- 370 (2012).
12. Murata, Y., N. Sato, T. Takayama & S. Onodera. A gesture-based remote control for finger disabled people. In: *IEEE 1st Global Conference on Consumer Electronics (GCCE)*, p. 406– 410 (2012).
13. Spyridonis, F., J.Hansen, Tor-Morten Grønli & G. Ghinea. Pain Droid: an android-based virtual reality application for pain assessment. *Multimedia Tools and Applications,*Online publication: (2013), DOI: 10.1007/s 11042-013-1358-3·
14. Nauman, M., S. Khan & X. Zhang. Apex: extending Android permission model and enforcement with user-defined runtime constraints. In: *ASIACCS '10 Proceedings of the 5th ACM Symposium on Information, Computer and Communications Security*, p. 328-332 (2010).
15. Lessard, J. & G.C. Kessler. Android forensics: Simplifying cell phone examinations. *Small Scale Digital Device Forensics Journal* 4(1) (September 2010), www.garykessler.net/library/SSDDFJ_V4_1_Lessard_Kessler.pdf
16. Kilgo, P.M. Android OS: A robust, free, open-source operating system for mobile devices. Available: http://students.mint.ua.edu/~pmkilgo/etc/android-os.pdf: (Oct. 2013).
17. Butler, M. Android: Changing the mobile landscape. *Pervasive Computing*. *IEEEXplore* 10(1) (2011). http://ieeexplore.ieee.org/xpl/tocresult.jsp?reload=true&isnumber=5676140
18. Awan, Z.H., P.S. Mahar & M. Saleh Memon. Blindness and poverty. *Pakistan Journal of Ophthalmology* 27(3) (2011). www.pjo.com.pk/27/3/zahid%20Hussain%20Awan.pdf
19. Kumar, D., H.S. Muktawat & S. Islam. E-mail client having articulation and braille transcription of e-mails for the blinds. *International Conference Special Issue of IJCCT* 2(2, 3, 4) (2010).
20. Saffer, D. Designing gestural interfaces. *Canada: O'Reilly Media*, p. 1-24 (2008).
21. Kane, S.K., J.O. Wobbrock & R.E. Ladner. Usable gestures for blind people: Understanding preference and performance. *Proceedings CHI'11, ACM*: p. 413-422 (2011).

Proceedings of the Pakistan Academy of Sciences 52 (3): 187–200 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

An Ontology-based Approach for Handling the Issues in Requirement Engineering

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Abstract: Requirement engineering (RE) has a vital role in the whole software development life cycle. It is a proven fact that stakeholders involved in the requirements elicitation and specification process may have different backgrounds and source; this may result in inconsistencies, ambiguities in requirements due to lack of domain knowledge, conflicting and contradictory views, communication and coordination issues. A few examples in the implementation of software systems, reciprocally the resulting system did not fulfill stakeholder expectations. To deal with these problems, it is necessary that RE activities ought to be integrated and improved by higher domain, application and instance level knowledge of requirements, which must be able to facilitate understanding about the context and develop shared understanding, amongst stakeholders. This study aimed at evaluating the framework of knowledge management to address mentioned issues within the RE process.

Keywords: Requirement engineering; RE challenges; ontology; knowledge management; formalization; RE process improvement.

1. INTRODUCTION

Requirements engineering (RE) is the first phase of software engineering and considered the most crucial and essential part of the entire software development life cycle. According to "Object Watch" report published in 2010, the software failure cost has been reached to about $ 6 trillion or $500 billion per month and this trend continued further; some research has concluded that systems failure in approximately ninety percent (90 %) of large software products traced back is due to poor requirements elicitation and specification. In short, the poor requirement engineering activities are the major reason of project failure [1, 2, 3]. The RE domain holds certain challenges, especially in elicitation and analysis, which are lack of communication and coordination, incomplete and contradictory knowledge of requirements, conflicting views of stakeholders a few examples [4]. One of the major reasons for project failure is the incomplete, insufficient understanding and management of requirements. The designer of information system begins designing system too early before understanding the customer need. The cost of correcting the error after delivery stage has been higher than the cost of correction during analysis phase [5]. Software and Requirement Engineers usually are not domain experts, therefore it is necessary for them to learn the problem domain because different understanding may results to incomplete and an ambiguous specification. Therefore, all participants have shared understanding about the problem [6]. RE process must elicit and understand the background of requirement knowledge, established common terminology among the diverse stakeholders and develop shared understanding among the stakeholders. Thus, software engineers redesign

Received, December 2014; Accepted, August 2015
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and iterate specification due to lack of information and interpretation [7].

Business knowledge is considered to be useful for Requirement Analyst to thoroughly collect business stakeholders' expectations of the system as initial requirements. It suggests that understanding and modeling the interaction between crucial users can help identifying unknown users, and also assists users to organize their thoughts and ideas purposefully [8]. Communication and coordination are the primary challenges facing global software development (GSD) industry, because there are different stakeholders with different background and sources involved in elicitation and specification process. The deficiency in communication and coordination can harm the relationship and trust, in these situations, knowledge management can be useful to handle these issues in the GSD and knowledge sharing is necessary, keeping the people update and improve shared understanding among members of the team [9]. Requirements can be viewed as statements that capture stakeholder demands, whose understanding requires domain knowledge to help bridge between a stakeholder's design on what a system needs to do and what is practically implementable in that system. The domain knowledge, which also contains rules and assumptions about the system's operating environment, offers us a practical means to identify inconsistencies and overlaps in requirements that may arise from the competing objectives and/or different stakeholders' preferences. Some types of requirement inconsistencies may not be detectable in the absence of such domain knowledge [10]. The weaknesses in the RE processes are misalignment or lack of RE knowledge with actual business process knowledge, misunderstanding, lack of coordination among the stakeholders, communication which are probable risk to bring the inadequate solution [4, 11]. Data information and knowledge are the essential building blocks of information, we exchange these concepts, however, there exists certain distinction among them, data is the assortment of facts in unstructured form and stored in un-organized way. Whereas to make this data meaningful, we process this data, examples are computed, summarized information, most of the information based application relies on this block. While knowledge is something different, it has a capability to link different information and present the meaningful knowledge which is clear in semantics, have a well-defined relationship, understandable by humans and as well as machines.

The comprehensive study shows that there is no practical approach for knowledge management to handle the shortcoming of RE, moreover, most of the literature contributes to develop the theoretical foundation, therefore lacks in providing end to end solution or applied approach. Our research objective was proposing and evaluating the framework of knowledge management for handling the issues in RE.

2. RELATED WORK

Gasevic et al [12] presented the literature review on the use of ontology in different phases of the software engineering life cycle, such as documenting, modeling of domain background, testing, artifacts, interaction / collaboration. They discussed on how ontology helps in solving issues face during the SDLC. Their study provides a good basis to work on the applied ontologies in the areas mentioned in their work. Reyes-Ortiz et al [13] presented knowledge representation in a medical diagnostics domain; their developed ontology cover three major components which are symptoms, diseases and their risk factors. Their paper described detailed usage of ontology, however, they did not described implementation detailed of their proposed model as well as evaluation has not been discussed in their presented work. Kayed et al [14] described the importance of ontology in the RE process for e-government applications. The key objective of their research work was to develop common and important concepts of e-government domain by using different tools, i.e., text-to-onto and developed their own tool KAON's which further refined the concepts. They emphasized that if concepts in a given domain of interest have

sufficient and rich semantics, they will develop a shared understanding of the requirements of various e-government applications [14]. Nahar et al [4] presented step-wise refinement model for requirement elicitation and discussed the various issues in elicitation. Their elicitation model contains user request, domain analysis, feasibility study, stakeholder analysis, elicitation techniques and prototyping. They used the traditional approach for elicitation and demonstrated with the case study in Hospital domain. Castaneda et al [7]. discussed the usage of ontology in the requirement engineering life cycle. They elaborated its use in the development of requirement ontology, specification document, and application domain ontology. They presented the literature review of the benefit of applying ontologies in areas which is good for building the theoretical foundation. Reddy et al [15] described benefits of ontologies in developing the common understanding, reuse of domain knowledge, explicit management of the domain assumption. Ahsan et al [16] presented an approach of domain modeling and applied it in an area of agriculture by taking crop as case study; they argued that their proposed work is beneficial for farmers as well as agriculturist to understanding the semantics as well as reducing the inputs for searching. Barforush and Rahnama [17] presented the literature review and performed a comparison of different ontology learning tools to acquire the knowledge from semi structured and unstructured data. In case of ontology construction from the text they discussed various tool like Bole, OLE, Onto-cmap and Text2Onto. In the semi-structured data they discussed the tools, i.e. AEON, RelExt, OntoGen and GALEIN.

The different literature reviews and related study suggest that there is need to develop and demonstrate the model with an applied approach to handle the mentioned issues and challenges with the help of ontology.

3. PROPOSED METHOD & APPROACH

We have proposed a comprehensive approach for dealing with the various mentioned issues affecting the RE process by framework of knowledge management. We have used design science research methodology (DSRM) [18] to demonstrate the proposed model by applying in hospital registration and admission module and evaluated by the professionals to see if the proposed framework and its demonstration can help to develop shared consensus among the stakeholders and beneficial in understanding the problem domain so that developed system may minimize the risk of failure and expectations of stakeholders. We have divided our model into the following phases:

- Requirements Knowledge Acquisition;
- Requirements Knowledge Formalization;
- Persistent Storage;
- Knowledge Distribution; and
- Knowledge Integration.

4. DEMONSTRATION OF PROPOSED FRAMEWORK

Our objective is to develop and evaluate the framework of knowledge management to handle issues faced during the RE process. Fig.13 shows the framework of requirements knowledge management.

4.1 Requirements Knowledge Acquisition

The most important activity after obtaining set of high level requirements is the identification and selection of sources for knowledge elicitation to understand the current business process as well as to find the gaps and problems in current business model. There are different approaches which are used to acquire the domain knowledge from learning objects and non-learning objects. In the case of learning objects, types of input could be structured data, semi-structured data and unstructured data. In case of non-learning objects, human experiences, thoughts are the examples and direct and indirect approaches used to obtain this sort of knowledge [17]. In order to acquire knowledge of the domain we have used both techniques semi-automated as well manual. If the domain is of type of Learning then a lot of material is available in various

locations such as internet, Wikipedia and other encyclopedias, to extract such types of concepts and relationships that exist among them in the given domain of interest, learning techniques and tools can be used for obtaining initial set of concepts and then refining these concepts with the help of domain experts. In case of an organization's business process, domain knowledge cannot be obtained from the learning objects because such knowledge is a combination of tacit and explicit which is an organization's intellectual property and hidden in the company manuals, confidential documents, workers experience and in their brain. This type of knowledge can only be obtained from the business stakeholders and documents through direct and indirect approaches and finally knowledge engineer with the help of stakeholders analyze about which type of knowledge should be the part of their knowledge base for implementation of proposed solution or business process re-engineering. The conceptual model is the output of the knowledge acquisition process in the shape of concepts and relationship among them along with their taxonomies. Some of the identified concepts in the given domain of interests are shown in Table 1.

Table 1. Top level concepts and relations.

Domain	Relation	Range
Patients	has Full Name	Literal
Patients	has Registraion No	Registration
Patients	has Identity	Literal
Patients	belongs To	Regions
Registration	paid Amount	Literal
Registration	Belongs To	Patient
Registration	Authorized By	Roles
Registration	Expires On	Literal
Registration	Received Documents	Artifacts
Registration	has Entitlement	Entitlements
Registration	has Panel Associated	Panels
Admission	requires Some	Registration
Admission	has Assigned	Doctor
Admission	has Diagnosed	Disease
Admission	has Alloted	Ward

Since we are going to formalize the Patient registration and admission process, Table 1 illustrates the conceptual model of some of identified classes and relationships among them, which we have extracted during the process of knowledge acquisition. Patient concept describes that each instance of the patient concept has its full name and other bio data, registration number allotted during the process of registration. Patient may or may not associate with the panel. Similarly admission process requires that patient must be registered and have valid registration process attached with him and for each admitted patient Role of doctor must be attached with him who will visit him and admission fee has been paid as initial entry amount. Each admission process is based on some disease diagnosed in the patient etc.

4.2 Requirements Knowledge Formalization

Knowledge acquisition is the first process which knowledge engineer performs by applying different knowledge acquisition techniques. In order to transform the conceptual model into formal model we have followed the process introduced by Noy [19], i.e.:

- Outline the Scope of the system
- Consider reusing formerly established ontologies.
- Extract the key terms in the ontology.
- Defining classes and its taxonomy.
- State the properties of classes, slots.
- Express the facets of the slots.
- Create the instances.

According to the first step, the scope of the system is to manage the application level knowledge of the patient registration and admission process. As we intend to model the part of application domain of the hospital business process, we could not be able to find the existing ontologies in the given domain of interests due to the fact that process is mapped to specific business needs owned by stakeholders and it is the property of the organization.

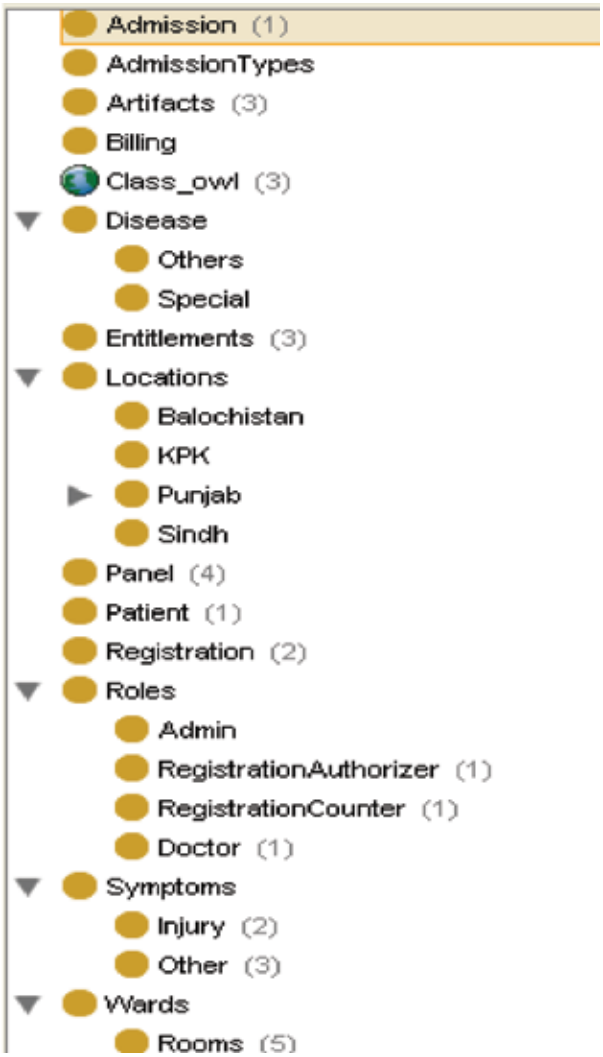

Fig. 1. Knowledge formalization.

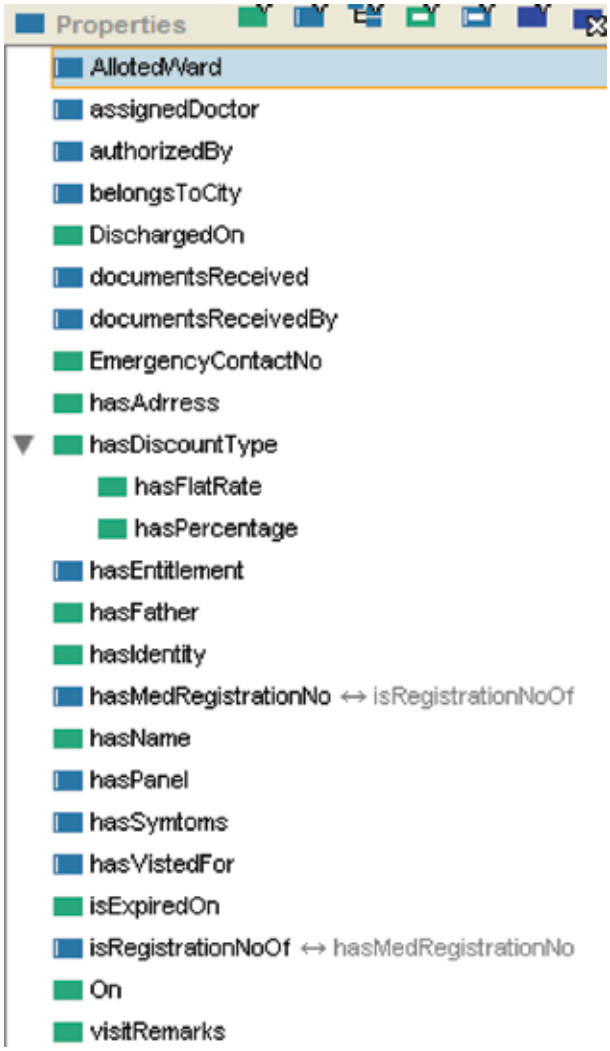

Fig. 2. Object and data properties.

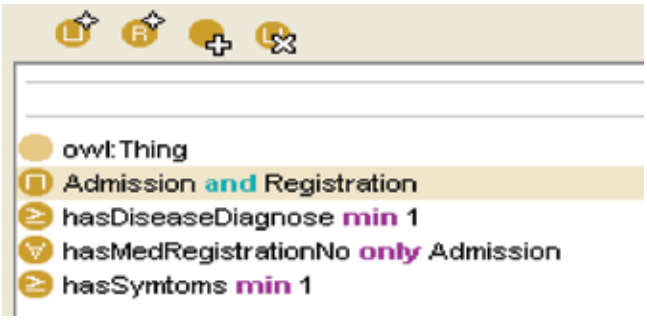

Fig. 3. Constraints.

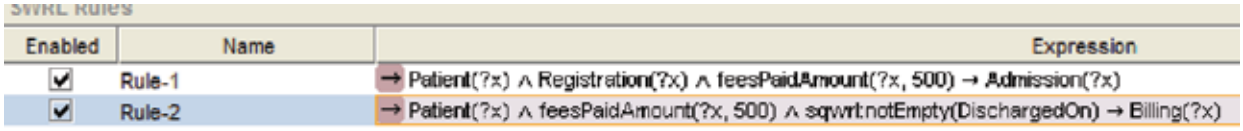

Fig. 4. Object and data properties.

Extraction of important terms is the process of identifying the part of knowledge which is needed to model, an organization's knowledge may be stored in the bundle of manuals, but our intention is to manage the application level knowledge which is consumed by the knowledge based system. So the all stakeholders have shared consensus about the process which needs to be implemented further.

Defining class and class hierarchy is the step which is performed by the knowledge engineer. The protégé ontology editor was chosen for this reason, because it provides strong reasoning and inference capabilities. We have adopted the top-down approach for building the concept taxonomy. Fig.1 illustrates the taxonomy of concepts in the domain of interest.

When the classes and its taxonomy is defined, then the next step is to identify and define the properties among the concepts. A property relates the domain concept with the range. There are two nature of properties which protégé supports to define: 1) An object property which holds the relationship among instances of concepts; 2) while the data property assign the Literal, i.e., string, date and numbers to an instance of concepts. There different types of properties, i.e. functional property for defining one to one relationship an example max one registration is attached with each patient and vice versa is inverse functional, similarly transitive property defined as the patient is linked with registration and registration is associated with admission, so the patient is associated with admission, similarly symmetric property same property name used for functional and inverse functional. Fig. 2 shows a list of properties associated with patient, registration and admission process.

Next step is to define the constraints on the properties, constraint limits the values. While Reasoners help us to ensure that the knowledge, fulfill all the constraints defined. Some of the restrictions are applied to the Admission concept.

Some rules and constraints depend on the conditions like if then else clause. Protégé offers SWRL semantic web rule engine to write more powerful rules and implicit knowledge. A typical example in this scenario is a patient can only get admission if he is having valid registration number and paid initial fees for admission. Similarly if the patient is associated with the panel then discount rate will be applied based on the discount rate defined in the agreement.

Final step is to instantiate the classes to see the actual behavior of the objects. This step is done by Knowledge Engineer which transforms the tacit knowledge and unstructured explicit knowledge in the knowledge base. Fig. 5 shows the instance of the registration and its associated properties while Fig.6 shows the instances of admission class and its associated properties.

A knowledge base is the combination of a T-Box and A-Box and often written as K = (T, A) where T is a set of axioms and A is set of facts. When all the instantiation is complete, it is necessary to check the consistency and completeness of the developed knowledge model if it conform all the constraints and rules. There are certain Reasoners available in protégé i.e. Pellet, Racer which check the consistency and completeness.

4.3 Knowledge Persistence Storage

There is a need for persistent storage of the ontology knowledge model for scalability and better performance and security features where knowledge base may accessible through the user interface. There are certain storage models discussed in the literature.

One way to store semantic data in the form of simple relational data structure approach and then perform simple SQL queries over the knowledge base. There are three main approaches deals with SQL based approach. 1) Triple store consists of three columns (SUB, PROP, OBJ), all data stores in a single Table. 2) N-array where the table is created on the basis of subject and their properties. Each subject and its associated properties are stored in a single Table. 3) Binary Tables where table is based on the properties, so the number of tables is directly proportional to the number of distinct relationships.

Table 2. Experts rating based on "Agreed/Partially Agreed/Not Agreed".

Category	Parameters	Agreed (0-3)	Partially Agreed (4-6)	Not Agreed (7-10)
Framework	Knowledge Management	8	1	1
Framework	Integration Support	9	1	0
Framework	Ease of use	7	2	1
RE Process	Develop Shared Consensus	8	2	0
RE Process	Customer Satisfaction	7	3	0
RE Process	Conflicts/Ambiguities Removed	7	3	0
RE Process	Inconsistencies Removed	7	3	0
RE Process	Improve Communication	9	1	0
RE Process	Improvement in Elicitation and Validation	7	2	1

The described techniques use the pure relational model to store the knowledge and do not have much inference, semantic and reasoning capabilities and it's also nearly impossible to define explicitly constraints and facet on relationships. So a need is raised for providers which can have ability to store semantic data with inference and reasoning support also have capability to embed SWRL rules [20].

A recent improvement in the ontology organization is the storage of semantic data in a shape of the URI so that native SPARQL queries may run over it and it may able to provide semantic capabilities and inference rule support that are not supported in the typical relational model. Virtuoso, Sesame, Oracle, 4store, Allegro Graph, Fuseki are the some of the semantic storage providers [21]. Oracle Database 11g Enterprise Edition, provides built-in provision for RDF/OWL /RDFS/ SKOS principles, this semantic based data storage provider facilitates developers and application programmer to take advantage of a scalable, open, integrated, secure and proficient platform for OWL and RDF-enabled applications. These semantic features of Database facilitate saving, accessing, loading, and DML access to OWL/ RDF data and ontologies, inference using RDFS, OWL and SKOS semantics and user-defined rules. Oracle has introduced the new column type named as SDO_RDF_TRIPLE_S to store the RDF data. Every RDF data model consists a set of triples as subject, object and predicate which are structured as an OWL/RDF graph of direct labeled edges. The edges are called relationship or links that joins a subject node with an associated object node and is labeled as a predicate. The normalized, compressed and partitioned storage architecture manages the complexity arising from repeated usage of typically long URIs and literal values associated with the subjects, objects and predicates across triples. This provides space-efficient storage that requires 75% less disk hardware than uncompressed semantic data, and scalable and perform ant loading, querying, and inference. In addition to that it provides user-defined rules, Fine grained security, indexing of documents, scalability and bulk load operations to import and export OWL/RDF data [22].

4.4 Knowledge Integration

The pattern of knowledge is more complex than data and information, as the knowledge should have the ability to link with other source of knowledge stored, distributed on heterogeneous servers so it must have a homogenous structure, that's the feature of ontology driven knowledge model. Since the develop knowledge model is exportable into RDF, XML, OWL format, it complies W3C recommendations and provide maximum interoperability. The developed knowledge base is in the shape of Triple so SPARQL queries are used to apply to the knowledge base stored as

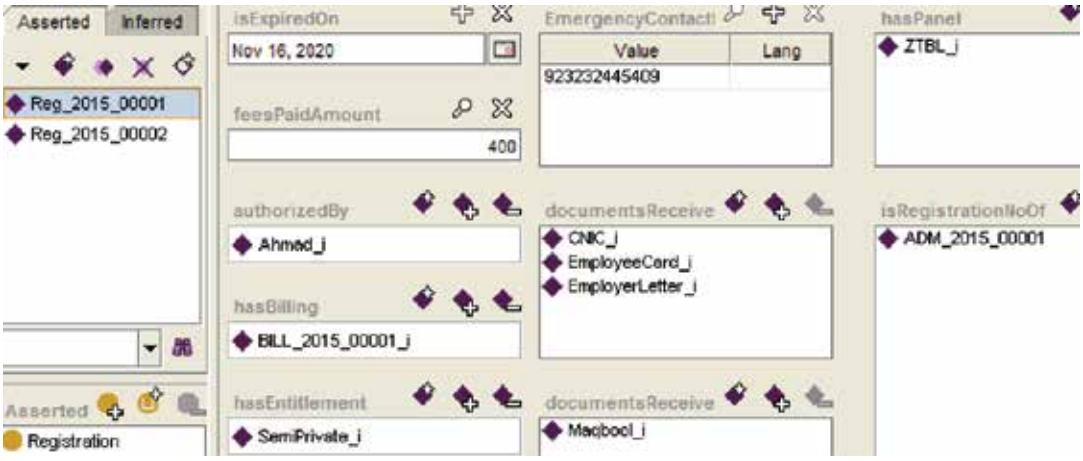

Fig. 5. Instances of registration class.

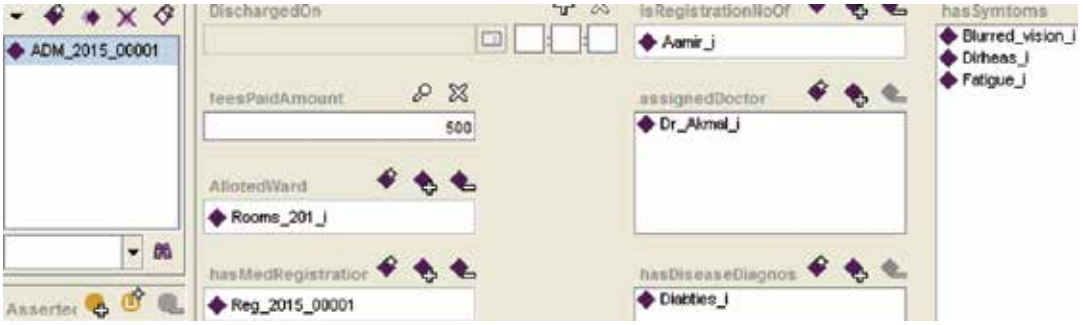

Fig. 6. Instances of admission class.

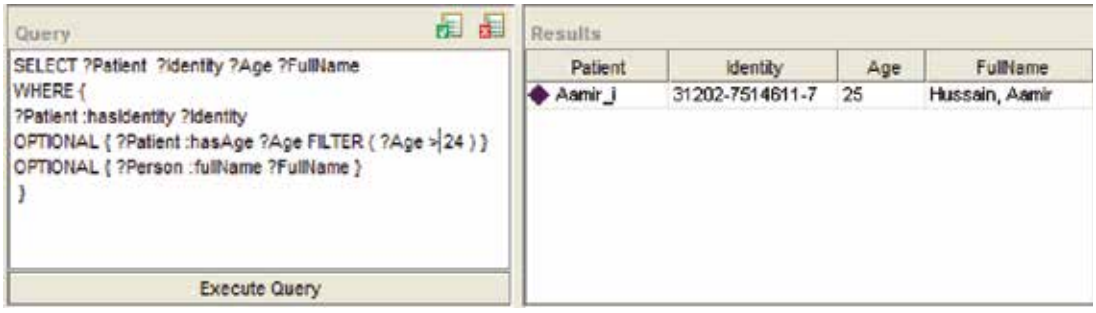

Fig. 7. SPARQL query showing patient.

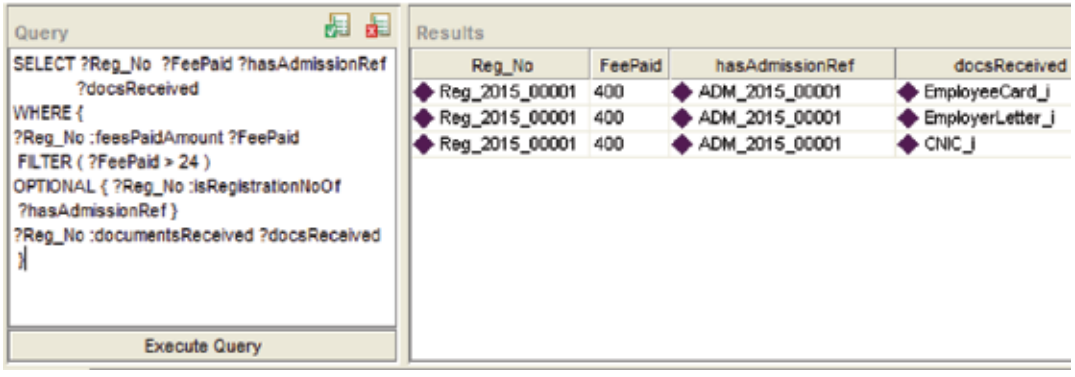

Fig. 8. SPARQL query showing registration.

Triple. RDF works and supports with the data to be distributed or decentralized. RDF graph and models can be combined easily, and RDF serialization can easily be made possible and easily exchanged over the HTTP. Applications can be loosely linked to various RDF enabled data sources over the Web. The SPARQL query matches with the pattern and return the result set in the XML, N3, RDF, OWL format.

A SPARQL query encloses a set of triple patterns, so-called a basic graph pattern. A triple form is similar to an RDF triple (subject, object and predicate), but any component can be a variable. We say that a basic graph pattern matches a sub-graph of the RDF data, when RDF terms of that sub-graph may be substituted for the variables; the result of the matching is an RDF graph equivalent to the sub-graph. The query tries to match the triples of the graph pattern against the RDF data model. Matching means find a set of bindings such that the substitution of variables for values creates a triple that is in the set of triples making up the RDF graph. Each matching binding of the graph pattern variables to the model nodes becomes a query solution, and the values of the variables named in the SELECT clause become part of the query results. Fig. 7 and Fig. 8 shows sample SARQL queries applied on the knowledge-base for the patient and registration process.

4.5 Knowledge Distribution

Knowledge distribution is the process of distributing the knowledge to the right person at the right time. Since the knowledge engineer is the person who develops the knowledge base, in order to distribute this knowledge, there is need to develop the interface through which all stakeholders may use the knowledge base easily. Since the developed knowledge base may not only be used by stakeholder for developing shared understanding and refinement in the current business process flow, but after completion of the requirement process this refined knowledge base will be used in the development, verification and validation team to check the software fulfill all the rules and flows defined in the knowledge base by the knowledge engineer with the help of stakeholders. We have develop the interface on the knowledge-base in ASP. NET with dotNetRdf [21] an open source library. The knowledge base initializes by the loading graph Graph graph = new Graph () statement. Fig. 9 represents the graphical user interface of instance level knowledge of registration and its relationship with other instances and literals.

Fig. 9 represents the instance level knowledge of the admission and its relationship with other instances and literals.

We have used an open source java based NLP parser to parse the text and then match with the developed ontology. We have used it for the part of speech tagging. The POS tagger namespace is used to parse a given sentence using NLP techniques and assign the parts of speech to the words and display into the graphical format in a tree shape. When the input string is given to this library it initially tokenize each word in the sentence using "English Maximum Entropy Tokenizer" and return pipe separated token in lower case and then part of speech tagger assign part of speech to each word and return an array of tokens. Fig.11 displays parse context using NLP parser, we can see that it has recognized malaria as NNP (proper noun singular) similarly VBD (verb past tense). Lithium library is used to display in graphical format.

5. EVALUATION & DISCUSSION

We have evaluated our framework by the group of professional working in a software company 'X' provides healthcare solutions located in Islamabad, Pakistan. Their development team consists of five developers two Requirement Engineers one Business Analyst and two Quality Assurance Engineers. We have demonstrated our proposed framework, developed knowledge-base and its user interface which covers the process of registration, admission and billing to their software development team. We have asked them if such knowledge-base on given domain of interest has been developed by using our proposed framework of knowledge management,

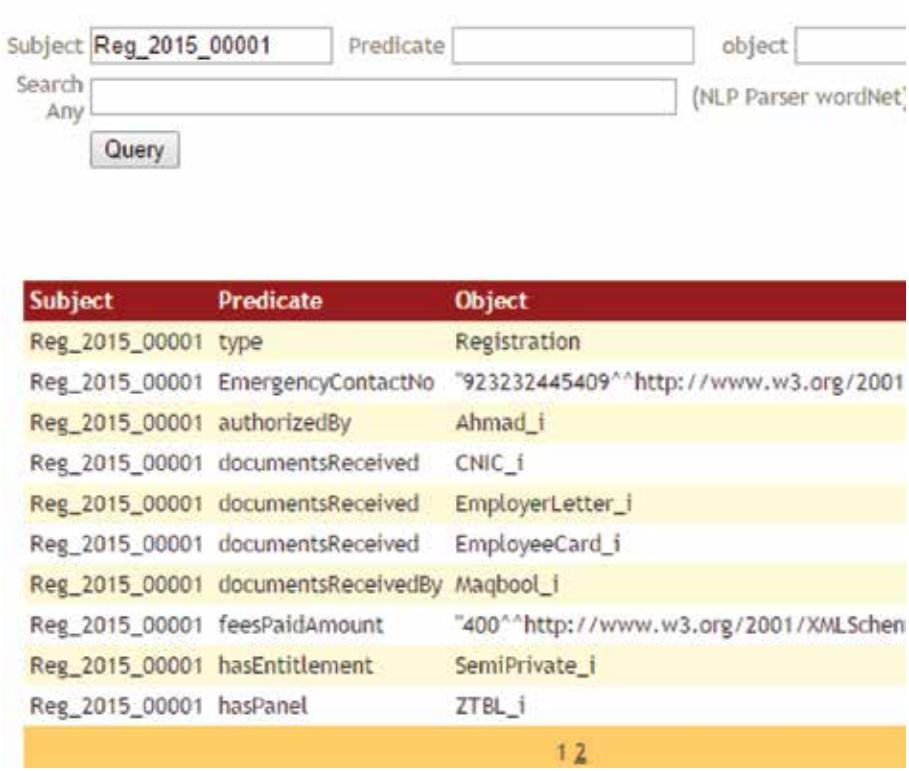

Fig. 9. GUI shows instance level knowledge of regsiterd patient.

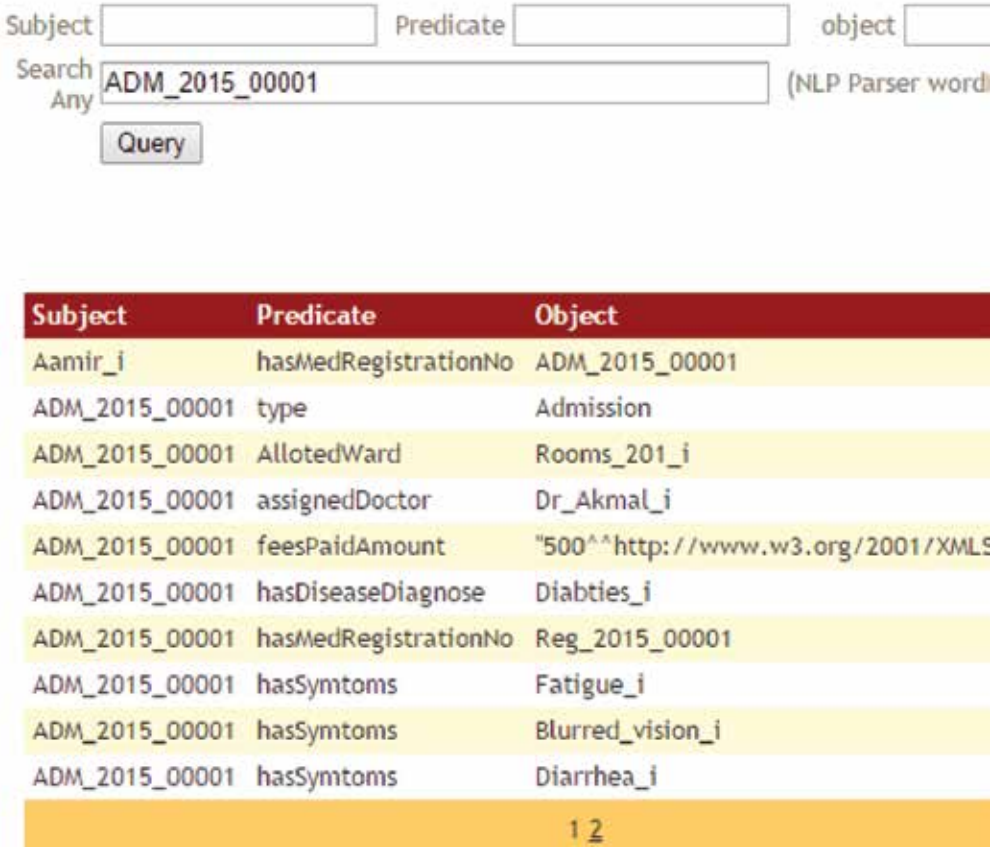

Fig. 10. GUI shows instance level knowledge for admitted patient.

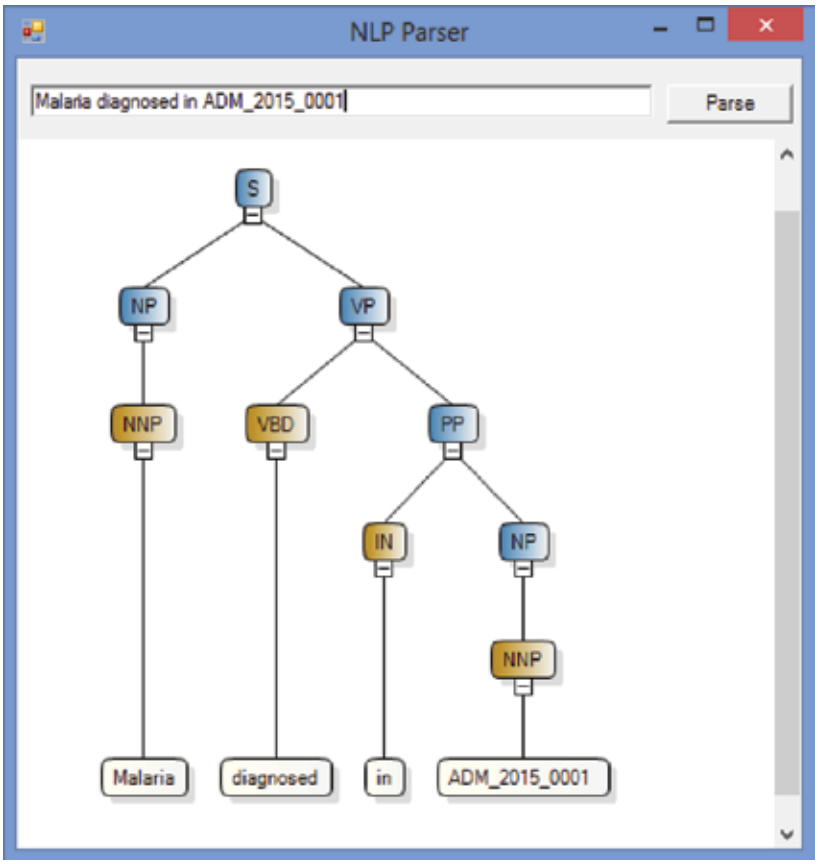

Fig. 11. NLP parser.

Fig. 12. Graphical representation of experts rating.

would it help in understanding the business domain as well as instance level knowledge along with whether, it would helpful in improving the process of Requirements Engineering by removing the ambiguities and conflicts among the stakeholders, understanding the context and knowledge about the domain. We have proposed the parameters which are based on the knowledge management model as well as parameters impacting on the requirement engineering process improvement. They have applied the rating ranging from 0 to 10. We have tagged rating like 0-3 as Not Agreed, 4-6 as Partially Agreed and 7-10 as Agreed.

We have summarized the result based on agreed, partially agreed and not agreed by applying the mean on all the resultant values of each parameter which is shown in Table 2.

Fig.12. presents the graphical representation of the summarized result of each parameter based on Agreed, Not Agreed and Partially Agreed.

Based on the feedback from the experts, they were mutually agreed that the proposed approach of knowledge management is better to manage the diverse knowledge in a homogenous structure, subject, object and predicate and is capable to manage the knowledge of any domain of interest hence due to homogenous structure it can easily integrate to any other source of knowledge. The developed interface on the knowledge-base is easy to use for searching, however, the initial cost to develop the knowledge model require some technical knowledge and require a knowledge engineer. The developed knowledge base help in shared understanding, removing the conflicts about the context requirement and domain as well as when all customers agreed on the same set of concepts the resultant system will require minimum changes. The improvement in communication has been observed among the stakeholders with different background, geographically distributed as well as among the software agents because it provides semantics and maximum interoperability and conform w3c recommendations. The knowledge based on the domain of interests will not only beneficial for all stakeholders, but the developers and verification

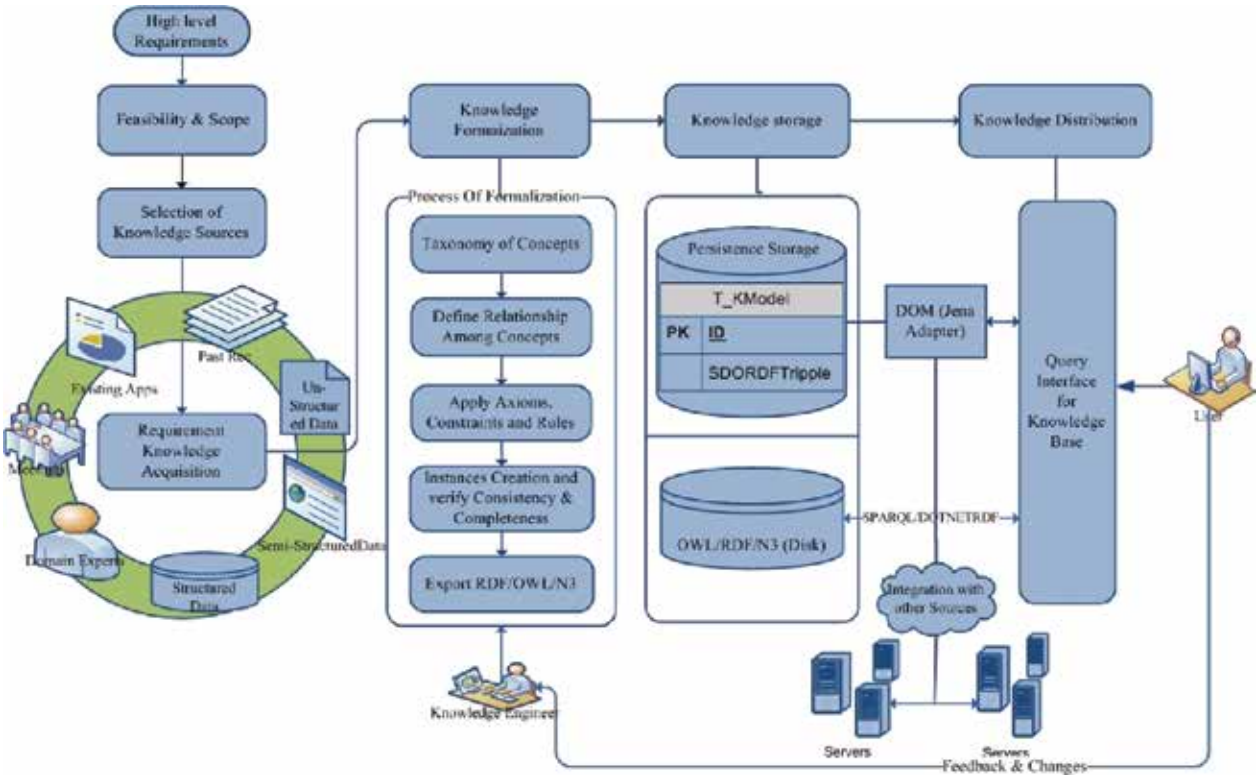

Fig. 13. Requirements knowledge management framework.

team may also use the knowledge base for managing their application level knowledge to check & validate the specification of the software. Hence the whole Requirement engineering life cycle would be improved.

6. CONCLUSIONS

This paper presents the framework of knowledge management. Our proposed framework of knowledge management comprises of five phases: 1) knowledge Acquisition; 2) knowledge Formalization; 3) Persistence Storage; 4) Knowledge Integration; and 5) Knowledge Distribution. The proposed ontology based knowledge management framework demonstration in a Health care domain and its evaluation from the experts shows that it will not only help stakeholders including Analysts, Requirements Engineers to understand the context about the problem domain with instance or behavioral knowledge, along with it will also help to remove the ambiguities and conflicts among business users and technical users by developing a shared understanding of concepts, removing ambiguities and conflicts, improve the communication especially for the stakeholders located geographically. Our next focus is to further evaluate the proposed approach of knowledge management by applying it in the domain of banking industry to see its effectiveness.

7. ACKNOWLEDGEMENTS

We would like to thank Zarai Taraqiati Bank Limited (ZTBL), Islamabad, Pakistan for the help and support regarding this study.

8. REFERENCES

1. Zhang, X. *An Interactive Approach of Ontology-based Requirement Elicitation for Software Customization.* Electronic Theses and Dissertations (University of Windsor): 347, (2011). http://scholar.uwindsor.ca/etd/347.
2. Isserman, S. *Challenges of Requirements Elicitation*. http://www.umsl.edu/~sir3b/Sean_Isserman_Requirements_Elicitation_Home.html.
3. Sethia, N.K. & A.S. Pillai. The effects of requirements elicitation issues on software project performance: An empirical analysis. *Requirements Engineering*: *Foundation for Software Quality, Lecture Notes in Computer Science* 8396: 285-300 (2014).
4. Nahar, N., P.K. Wora, & S. Kumaresh. Managing requirement elicitation issues using step-wise refinement model. *International Journal of Advanced Studies in Computer Science and Engineering* 2: 27-33 (2013).
5. Farfeleder, S., T. Moser, A. Krall, T. Stalhane, I. Omoronyia & H. Zojer. Ontology-driven guidance for requirements elicitation. *Lecture Notes in Computer Science (ESWC)*. 6644, 212–226 (2011).
6. Happel, H., & S. Seedorf. Applications of ontologies in software engineering. *5th International Semantic Web Conference (ISWC 2006)* Cite Seer x, 1–14 (2001). http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.89.5733
7. Castañeda, V., L. Ballejos, M.L. Caliusco & M.R. Galli. The Use of Ontologies in Requirements Engineering. *Global Journal of Researches in Engineering specification*. 10, 6, 2–8 (2010).
8. Jason, G. *An Ontology-based Knowledge Support System for Requirements Analysis,* Massey University, Manawatū, New Zealand (2013). http://hdl.handle.net/10179/4881
9. Humayun, M., C. Gang. An empirical study on improving shared understanding of requirements in GSD. *International Journal of Software Engineering and Its Applications*. 7, 1, 79–92 (2013).
10. Huan, T., B.Q. Vo, M. Lumpe & J. Grundy. KBRE: A framework for knowledge-based requirements engineering. *Software Quality Journal (c)*. *Springer*, March, 35 (2013).
11. Haron, A., S. Sahibuddin. The strength and weakness of requirement engineering (RE) process. *2nd International Conference on Computer Technology and Development*. *Icctd*, 56–59 (2010).
12. Gasevic, D., N. Kaviani & M. Milanovi. Ontologies and software engineering. p. 1–22 (2008). *DOI: 10.1007/978-3-540-92673-3_27*
13. Reyes-Ortiz, J.A., A., L. Jiménez, J. Cater, C., A. Meléndez, P. Márquez, B. M.& García, Ontology-based knowledge representation for supporting medical decisions. *Research in Computing Science*. 68, 127–136 (2013).
14. Kayed, A., M. Nizar, & M. Alfayoumi. Ontology concepts for requirements engineering process in E-Government applications. 2010 *Fifth International Conference on Internet and Web Applications and Services*. p. 396–400 (2010).
15. Reddy, P.N. & K.P. Kumar. An efficient software engineering ontology tool for knowledge sharing. *IJCSI International Journal of Computer Science Issues*. 7, 4, 19–27 (2010).

16. Ahsan, M., Y.H. Motla & M. Asim. Knowledge modeling for e-agriculture using ontology. *8th IEEE International Conference on Open Source Systems and Technologies 18-20 December (ICOSST-2014),* University of Engineering & Technology (KICS), Lahore. 112-122 IEEE Explore (2014). *doi: 10.1109/ICOSST.2014.7029330*
17. Barforush, A.A. & A.L.I. Rahnama. Ontology learning: revisited. *Journal of Web Engineering*, Vol. 11, No. 4 (2012) 269-289 © Rinton Press. 11, 4, 269–289 (2012).
18. Peffers, K., T. Tuure, M. Rothenberger & S. Chatterjee. A design science research methodology for information systems research. *Journal of Management Information Systems* 24(3): 45–78 (2008).
19. Noy, N. & D. McGuinness. Ontology development 101: A guide to creating your first ontology. Development. 32, 1–25 (2001).
20. Levandoski, J.J., M.F. Mokbel, RDF Data-Centric Storage. 2009 *IEEE International Conference on Web Services*. 1, 911–918 (2009).
21. DotNetRDF Semantic Web, RDF and SPARQL Library for C#/.Net, https://bitbucket.org/dotnetrdf/dotnetrdf/wiki/UserGuide/Querying with SPARQL.
22. Corporation, O. ORACLE DATABASE SEMANTIC, http://docs.oracle.com/cd/E18283_01/appdev.112/e11828/sem_jena.htm, (2013)

Proceedings of the Pakistan Academy of Sciences 52 (3): 201–214 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

QoS Assurance in Higher Mobility Mobile Ad Hoc Networks using Multipath Admission Control Protocol

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Abstract: The cheap and easy availability of wireless devices boosted the MANETs supported applications. Due to these emerging applications MANETs are not only deployed in military sector but in every walk of life. Now QoS assurance to these applications is an essential part rather than additional feature in MANETs. The routing protocols provide only data route rather than assuring any kind of guaranteed QoS to applications. The routing protocols must be equipped with additional features such as traffic scheduling, QoS awareness, admission control, and traffic priority to assure guaranteed QoS.In this article we will present our designed Flow aware admission control protocol that work with Dynamic Source Routing (DSR) to assure guaranteed QoS provisioning. The admission control assures high throughput in highly mobile scenario and volatile topology of MANETs by sustaining partially disjoint multiple paths between source and destination. The protocol injects the data traffic to the network on the basis of availability of resources. The protocol calculates the available bandwidth using channel idle time ratio (CITR) and takes the decision of accepting or rejecting the new data traffic to the network. The protocol repairs the route locally and reduces the network load and results in high performance. The protocol is compared with the state of the art admission control protocols using network simulator-2.

Keywords: Admission control, multimedia applications, MANETs, QoS, multipath

1. INTRODUCTION

Mobile ad hoc networks (MANETs) are combination of mobile nodes, works as an end node as well as a router, communicate over a wireless channel [1]. The nodes can join or leave the network on their own will, which results in a dynamic and unpredictable topology. MANETs have no infrastructure or centralized control. Due to these characteristics of MANETs, it became very popular in military as well as every walk of life. The widely acceptance of MANETs compel the research community to support different kinds of applications over MANETs. The applications range from normal text data to video. These applications have different Quality of Service (QoS) requirements. Some of the applications just need throughput guarantee while others required guaranteed throughput, boundedend-to-end delay and jitter [1].QoS assurance in MANETs is challenging and research oriented task [2].

Research studies show that Routing Protocols only provide routes for the data without any kind of QoS assurance to the applications. Many researchers have made efforts to provide guaranteed QoS to the applications and majority of these QoS-aware routing protocols find the route on the basis of capacity availability, traffic congestions and node stability. These routing protocols provide a

Received, March 2015; Accepted, April 2015
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uni-path between source and destination. In case of route failures, the routing protocols have to search for another route, which causes extra delay and degraded throughput. The link failures occurs either due to congestions or nodes mobility.

The designed Flow aware admission control protocol with multipath tackles both the reasons of longer delay as well as degraded throughput. The protocol will inject the traffic load to the network according to the available capacity and will maintain multiple paths between source and destination to cope with the route failure to high mobility of nodes. The protocol works with the Dynamic Source Routing (DSR) protocol [3]. The paper presents the design, characteristics and performance of Flow Aware Admission Control-Multipath (FAAC-Multipath) protocols and comparison with the other state of the art protocols such as Contention Aware Admission Control protocol (CACP) [4], Multipath admission control protocol for MANETS (MACMAN) [5] and Flow Aware Admission Control (FAAC) protocol [6]. The protocols are evaluated in different high mobility scenarios. The remaining sections of the paper presents: the background study, designing of FAAC-Multipath protocol, comparison with other well-known admission control protocols and conclusion of the paper.

2. BACKGROUND WORK

Most of the reactive routing protocols support only best effort services without any kind of QoS guarantees. DSR and AODV are most common and well accepted protocols in this category. AODV-BR [7]is the extended version of AODV that uses intermediate route repair locally. The optimized version of AODV-BR [8] maintains the backup paths not only with the help of route reply but as well as due to the data packet as well. The data packets following the routes also help the protocol to find the data route for data sessions. The intermediate route repair or intermediate backup path did not assure the route availability at any cost, so it is not a sufficient technique for backup route availability. The application needs the guaranteed backup path so it can switch the data at the same time of primary route failure. The author [9] uses limited flood technique to solve the route failure problem, but this is not optimal solution in unpredictable and volatile topology networks like MANETs. The work presented in [8] uses multiple constraints such as delay, reliability and capacity for route selection but did not explain how routes can be updated and can remove the stale information. While in [10] the author proposed to use pre-emptive techniques to cope with route failure but this is also not feasible in MANETs like network where the topology is totally unpredictable and where route failures occurs not only due to traffic congestion but due to nodes movement as well.

Multipath routing technique can solve the problem upto some limit of route failure due to changing mobility but unable to solve the problem of route failure due to congestion or more traffic injection to the network than the its capacity. The congestion problem can be solved using admission control protocols. Multipath protocols show higher performance than the uni-path protocols in higher mobility scenarios [11, 12]. These protocols provide best effort services. Some of the QoS-aware routing protocols based on contention free MAC [13, 14]. As there is no centralized control in MANETs, so it is not feasible to use contention free MAC. The hidden node can cause collision of data packets and exposed node reduces the efficiency of channel utilization [15]. Therefore, we will consider the protocols that uses contention aware MAC for capacity calculation.

It is fairly an open research issue in research community to increase the battery life of portable or mobile devices such as PDA, smart phone etc. Although research achievement has begun to solve the problem of limited battery life, it is still a fact that portable or mobile devices have less power supply as compared to wired networks devices [16]. Therefore, the design of protocol should minimize the overhead, because it will drain energy of the device proportionally [17].

MP-DSR [18] discovers the routes on the basis of route reliability but does not assure guaranteed throughput. The protocols select the best available reliable routes but the requirement satisfying routes. Adaptive Dispersity QoS Routing (ADQR) Protocol [19] divides the traffic among the multiple routes which gives the problem of re-arranging the data traffic at the receiver. The protocol does not provide any solution to that problem. Interference aware QoS Multipath Routing (IMPR) [20] protocol proposes to find the routes on the basis of link stability and available bandwidth but did not provide any information about the mechanism of combination of these two metrics.

Scalable Multipath on Demand Routing in Mobile Ad Hoc Networks (SMORT) [21] tackles the route failure problem by providing multiple routes to intermediate nodes only. In route stability-based multipath QoS routing (SMQR) [22] protocol calculate the route capacity only considering the individual node's capacity not neighbours capacity. In wireless communication, one node transmission affects the available capacity of the neighbours' nodes as well. So only to consider the nodes capacity by itself will severely affect the performance. It will inject more traffic than the available capacity and will result in degraded throughput and high packet loss. MACMAN provides multiple routes for each data session but these multiple routes are fully disjoint. It is very difficult to find and maintain such routes in MANETs. It introduces a lot of overheads in the network while finding such routes. The admission control mechanism of MACMAN is a combination of CACP and Perceptive Admission Control (PAC) protocols [23].

3. FAAC-MULTIPATH PROTOCOL

Flow Aware Admission Control-Multipath (FAAC-Multipath) protocol incorporates both routing and admission control aspects of operation. Its purpose is to provide end-to-end guaranteed throughput services to application data sessions that have a strict constraint on the minimum level of throughput they require. FAAC-multipath protocol establishes and maintains multiple paths between source and destination on demand. The protocol assures guaranteed throughput to the application in high mobility scenarios. The protocol includes features to discover routes that nominally have adequate capacity to support admission of data sessions, as well as to admit only those new sessions that would not have a derogatory effect on the throughput of the previously-admitted sessions and finally to uphold the level of throughput that it has promised to sessions by way of admitting them. The design and implementation of the protocol is presented in this paper and performance of the protocol is compared with other well-known admission control protocols. FAAC-multipath partially utilizes the function of DSR protocol. The protocol finds and maintains partially disjoint routes between source and destination. Both the routes i.e. primary and secondary are established on capacity estimation of the said routes, means both routes must fulfil the capacity requirement of the application. The following sections give a full description of its operation as well as the design choices made. The protocol working mechanism is a combination of application layer and network layer. We have explained the behaviour and characteristics of each layer involved in our protocol.

3.1 Application Layer Model

Application layer is the 5th layer in TCP/IP suite and is basically responsible for different services. Different applications run on application layer. We have developed an application that generates constant bit rate data and the application agent defines the notion of a session. A new data session is specified by the following fields: data session ID, start time (s) of data session, minimum required throughput (bps), and data packet size (bytes). The session ID is allocated by the application agent. The throughput requirement defines how many bits, and therefore packets, are generated per second, as well as the desired end-to-end throughput. Traffic is modelled by constant bit rate sources, since this adequately demonstrates the ability of FAAC-Multipath to handle various traffic loads and to make admission decisions.

When a new session is generated by a user, a blocking timer is set to expire in 10s and a session request (SReq) message is passed to the network at the source node. The source application agent will block the session if it does not receive the session reply (SRep) in 10s. The blocking timer is set to 10s, so that the application agent can generate two SReq for each data session before blocking the data session. The SReq is passed down to the User Datagram Protocol (UDP) agent. The UDP agent encapsulates the SReq in a UDP packet, giving it a unique sequential packet ID. The SReq is carried as the application data and passed down to the routing agent, which takes over the handling of SReq.

3.2 Network Layer Model

Network layer is the 3rd layer in TCP/IP suite and routing protocol runs on this layer. As FAAC-Multipath protocol is partially coupled with DSR protocol, therefore it is implemented on network layer. Application data sessions that are requesting service from and admission to the network are assumed to specify their desired traffic characteristics to the protocol. The characteristic of the data session is modelled in the form of Session Request (SReq) packet. The SReq is passed down to the network layer to model the arrival of a session admission request at a traffic source node. The routing agent will find the route in route cache or will initiate the Route Request (RtRq). When route is found then the protocol will test the route nodes resources according to Session Request (SReq). The Novelty of the designed protocol is the method of propagating Session Request (SReq), resource checking and to find the route for throughput sensitive data session. Figure 1 shows the position of FAAC-Multipath protocol in TCP/IP suite. The protocol works on network layer and as well MAC layer, because MAC layer calculate the remaining resources for the protocol to take admission decision.

Application Layer	Session Request
Transport Layer	UDP/TCP
Network Layer	FAAC Protocol
Data Link Layer	Link Layer
Physical Layer	Physical Layer

Fig. 1. FAAC protocol in view of TCP/IP suite.

3.3 Protocol Implementations

FAAC-Multipath protocol is implemented in two phases:

- In first phase, the protocol searches the route from source to destination in route cache. If the route is available in the route cache, then the protocol checks the resources for that route in second phase of the protocol implementation. If there is no source to destination route in route cache, then the routing agent generates the route request (RtRq) and finds the routes between intended source and destination.
- In second phase of admission control, local and neighbour resources are tested before forwarding the SReq to other nodes. As in the second stage, the full route is known to the source, so protocol checks the resources with the full knowledge of contention count (C_{count}).

3.4 Route Discovery

In this process, the protocol finds the route from source to destination on the basis of application's requirements. The application agent specifies the date session requirements in a control packet called Session Request (SReq). The Network layer receives these requirements from application layer and encapsulates these SReq requirements in Route Request (RtRq) packet and store the information in the cache of source node of the data session.

The source node checks its route cache for the route to destination. If route is available then it starts the capacity testing of the route else initiate the

route discovery. The source node first conduct its own capacity estimation and after this initiate route request in case of sufficient capacity availability. The source forward the RtRq and each receiving node do the capacity estimation locally.

Every intermediate node only forwards the RtRq if it can support the new data session without affecting the quality of already admitted data sessions. The source as well as all the intermediate nodes reserves the resources for the specified data in RtRq. At this stage, the protocol partially admits the data session means not fully. The RtRq which has encapsulated the SReq propagates in this manner and reaches the destination finally if it is possible. The destination node may have received more than one RtRqs for the same data session. The destination node sends Route Reply (RRep) to the source node. On a way back of RRep to the source node, each intermediate node checks its two hops neighbours capacity using a control packet called admission request. If the two hops neighbours of the intermediate node can satisfy the request of the new data session, then it forwards the RRep to the next node on a route. In this method the primary route is established between source and destination. The secondary or backup route process is explained in a paper in a later section.

3.5 Capacity Testing

The required capacity of a node (C_{req}) can be estimated by using the following equation. The session single hop requirement is calculated as:

$$C_{req} = b_{req} * W_{req} \quad (1)$$

Both types of capacity are measured in bits per second. Where b_{req} is the required capacity by the session and W_{req} is the weighting factor means the overheads of different layers to be included with the data capacity as show in following equation 2.

$$W_{req} = \frac{(T_{DIFS} + 3T_{SIFS} + T_{RTS} + T_{CTS} + T_{DATA} + T_{ACK} + T_{backoff} + T_{MAChdr} + T_{IPhdr} + T_{UDPhdr} + T_{SRhdr} + T_{QoShdr})}{T_{DATA}} \quad (2)$$

Here T_{DIFS} and T_{SIFS} are the times taken by distributed coordinated function (DCF) inter-frame space (DIFS) and short inter-frame space (SIFS) employed by the direct sequence spread spectrum (DSSS) physical layer (PHY) specification in IEEE 802.11 standard [24], T_{RTS}, T_{CTS}, T_{DATA} and T_{ACK} are the times taken to transmit request-to-send (RTS),clear-to-send (CTS), Data and ACK frames (along with the physical layer preambles) respectively, $T_{backoff}$ represents the time for which a node backs off before each packet transmission and T_{MAChdr}, T_{IPhdr}, T_{SRhdr}, $T_{UDPhdr,}$ T_{QoShdr} are the times taken to transmit the fixed size MAC, IP, source route, UDP and QoS-specification (SReq contents) headers on each data frame. So for any node to forward the SReq should satisfy the following equation.

$$(T_{idle} - T_{resv})\beta > C_{req} * C_{count}$$
$$where \ \text{resv} \in 1,2,3,4,,,,, \quad (3)$$

Where T_{idle} is the fraction of channel idle time, T_{resv} is the session reserved fraction of the channel time, which is not yet being used, but which has been reserved by previously processed session request (SReq), and β is the node transmission rate, which specifies the raw channel capacity in bps. FAAC-Multipath protocol requires that the 802.11 MAC protocol monitors the status of the channel reported by the virtual and physical carrier-sensing mechanisms. The basic unit of time in the 802.11 MAC specifications is the time slot, the duration of which is between 9μs and 20μs depending on the type of PHY assumed. In our model, the MAC protocol simply checks the channel status once per time slot, since this is a computationally cheap operation, and records the number of slots for which it is deemed idle. This number is aggregated for one second before being reported to the higher layer protocol. This avoids responding to momentary fluctuations in the CITR.

Fig. 2 represents the testing of local and neighbours' node resources. Small circle represent mobile nodes, middle and large circles represent the transmission and carrier sensing range of node 'B' respectively. Node 'S' is the source and node 'D' is the destination of the data session. Solid

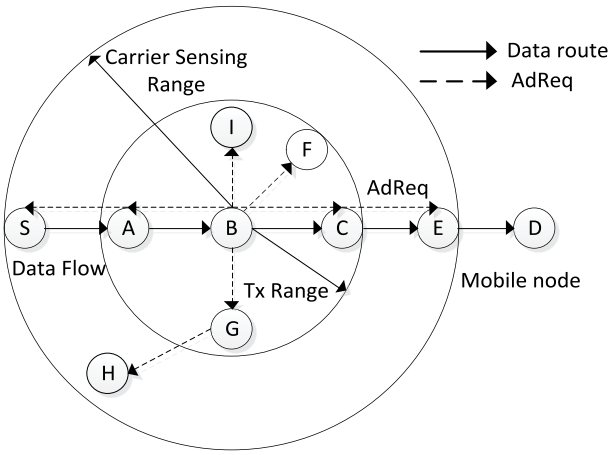

Fig. 2. Capacity test at local and neighbour nodes.

arrows represent the intended data route from 'S' to 'D' and dotted arrows represent the transmission of Admission Request (AdReq) control packets from node 'B' to its two hops neighbours to check their capacity.

3.5.1 Processing of Session Request (SReq)

The receiving node of SReq tests its local resources according to equation (3). If it can satisfy the requirement of the new session then it tests the resources of its two hops neighbour by transmitting admission request (AdReq) packet. If the SReq node did not receive the Admission Denial (AD) packet within the specified time, then it considers that its neighbours can accommodate the new data session. The node forwards the SReq to other node on the intended route of the data and reserves the required resources of the data session. Each node will continue the process of checking local and neighbours' resources and forwarding the SReq till destination node. After receiving SReq by destination, it generates Session Reply (SRep) and transmits back to source of the data session on same route followed by SReq. FAAC-Multipath protocol checks the node resources during the session request phase with full knowledge of contention count (C_{count}). Contention count of the node can be calculated by the following formula [4].

$$C_{count} = (CSN \cap R) \setminus D \quad (4)$$

Here Contention Count (C_{count}) is the combination of Carrier Sensing Neighbours (CSN) and tentative route (R) of the data traffic excluding the destination node (D). The destination node does not transmit the data further therefore, it is not considered in Ccount. The following algorithm explains the processing of SReq by each individual node.

```
# Received SReq
If (Bavail>= Breq) then
        -Broadcast AdReq
Note:   If (AD) then
                -Drop SReq
                -Inform Source Node
        Else
  If (time>=SReqtime) then
                -Reserve resources
                -Propagate SReq
          Else
                        -Goto Note:
                End if
        End if
Else
        -Drop SReq
        -Inform Source Node
End if
```

3.5.2 Processing of Admission Request (AdReq)

The receiving node of AdReq also tests their local capacity using equation (3). If it can satisfy the requirements, then it stores the session and route information, otherwise will send the admission denied (AD) packet to the AdReq source node. The following algorithm explains the processing of AdReq by each node. AdReq time to live (TTL) represents the number of nodes to which AdReq packet has to be forward.

3.6 Selection of Backup/Secondary Route

The protocol establishes and maintains a backup route for all data sessions. These routes must be partially disjoint. For the backup routes, the protocol checks for route in a source node cache, if it is available then the protocol starts the testing

and partially disjoint-ness of the backup route to the primary route. The backup route request (BRReq) carries the primary route information and checks the disjoint-ness with the primary route. At any stage, when both the routes are found sharing the maximum 50% of nodes in common, the secondary route is rejected.

In backup route selection, every node starting from source to destination, tests its local as well as two hops neighbour's capacity in a similar way to primary route. But capacity test of local and neighbours are conducted during BRReq forwarding, not at a time back up route reply (BRRep) as in primary route selection. In backup capacity estimation, contention difference is used instead of contention count, because contention count may underestimate the capacity of the backup route. The contention difference can be easily calculated using the following formula.

$$CD = \langle |C_{count}| - |CSN \cap R_{curr} \backslash \{D\}| \rangle \quad (5)$$

CD is contention Difference, C_{count} is contention count, CSN represent Carrier sensing range of the node, whose capacity is being estimated, R_{curr} represent the current data route and D represent the destination of the data session. Figure 3shows the explanation of Contention Difference.

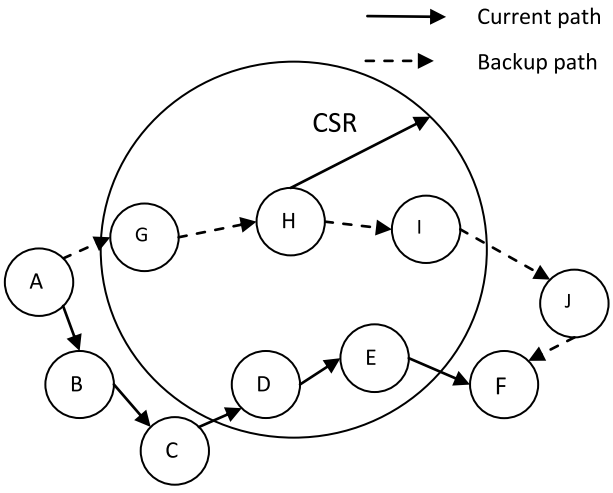

Fig. 3. Calculation of contention difference.

Small circle are mobile nodes and large circle represents the node's CSR. The primary route for the data session is represented by solid line between source and destination while backup route is represented by dotted line. The contention count of node H is 3, while CD of the node H is 1. Two of the neighbour nodes of node H is already in primary route, so it will not be included in capacity estimation, i.e., CD, because the data traffic will divert to secondary route only when primary route fails. In a similar fashion, the CD of all nodes can be find easily.

3.7 Reliability of Backup Route

In Mobile Ad hoc Networks, it is very difficult to maintain fully disjoint routes particularly in high mobility scenarios. So instead of fully disjoint routes, FAAC-Multipath protocol will establish partially disjoint routes. It means that both the routes can share the nodes 50%. This disjoint-ness improves the reliability of the backup route and reduces the chances of both routes failure simultaneously. The following equation finds the reliability of the backup route.

$$B_{avail} - B_{resr} = CD.B_{req} \quad (6)$$

B_{avail} is the available bandwidth of the node, B_{resr} is the reserved bandwidth for some session but not yet utilizing, CD is contention difference and B_{req} is the required bandwidth of the data session.

3.8 Switching Mechanism

The switching mechanism is one of the most important aspects of FAAC-Multipath protocol. It actually avoids the route failure, avoids the collision and tries to uphold the guaranteed throughput in low as well as high mobility scenarios. The fast switching mechanism benefited the protocol to avoid the session pausing mechanism, which increases end-to-end delay and results in collision and route failure. Switching mechanism is implemented in the following three different scenarios:

- The protocol switches the data session from primary to secondary route, when the primary route is not satisfying the requirements of the data session. The failure to satisfy the requirement can be due to node mobility or collision. When

a route node of other data session moves into the interference range of the earlier stated data session's route nodes, it affects the throughput of the session and also increases PLR which in turn increases end-to-end delay.

- The FAAC-MM protocol switches the data session from primary to secondary route when primary route fails either due to mobility or due to failure of excessive re-transmission at the MAC layer. When a route nodes move out of the transmission range of the data sending node then failure detecting node informs the source node and the source node switches the data session from primary route to secondary route.
- The protocol also switches the data session from primary to secondary route when it finds that the secondary route offers higher throughput. The protocol admits the data session when it finds a route from source to destination that satisfies the session requirements. As the protocol does not wait for secondary route discovery to initiate the data session, so when source node become aware that secondary route is offering higher throughput, then the protocol switches the data session from primary to secondary route. One thing must be noted in this scenario that the primary route is still satisfying the requirements of the data session. It upholds the guaranteed throughput and bounded end to end delay.

4. SIMULATION ENVIRONMENT

FAAC-Multipath protocol is tested using extensive simulation with other well-known admission control protocols under different simulation environment and network traffic load. The simulation results show the comparison of the performance of the protocols. The paper presents the comparison of FAAC-Multipath protocol, CACP, MACMAN and FAAC. FAAC and CACP maintain uni-path between source and destination. CACP is a well-known and leading admission control protocol for MANETs. MACMAN maintains multiple paths between source and destination. MACMAN utilizes the functionality of CACP and Passive Admission Control (PAC) protocol. The simulation results show the performance under different node mobility and also the capability of the protocols admission control techniques.

4.1 Simulation Setup

The simulation of the protocols carried out using well accepted network simulator NS-2. The data files of the simulations are further processed by text based programming language, AWK. Table 1 show the simulation parameters, which are used during simulation of the protocols. The number of nodes, simulated area size and the average transmission range were selected using the guidelines in [25] for rigorously evaluating a multi-hop routing protocol.

Table 1. Simulation parameters.

S. No.	Parameters	Values
1	Total mobile nodes	100
2	Total traffic sources	50
3	Per source sessions	20
5	Data packet Size	512 bytes
6	Mobility model	Random WayPoint
7	Routing protocol	DSR
8	Node speed	2,4, 8, 16, 32
9	Tx rage	250m
10	CSR	500m
11	Channel bandwidth	2Mbps
11	Simulation area	1500m * 1500m
12	Pause time	801sec
13	Simulation time	800 sec
14	Results averaged over	10 runs

4.2 Mobility Model

Number of mobility models is available to check the performance of the protocols in MANETs. These mobility models are used to generate node position and movements' pattern of the nodes during simulation. Literature survey shows 64% of the researcher's research papers used Random Waypoint Mobility (RWP) model to test the

protocols in MANETs [26]. RWP model excellently exhibit the movement pattern of the mobile nodes, but the initial velocity of nodes must not be zero. Zero initial velocity of mobile nodes creates concentration of the nodes in the middle of the simulation area. We have used RWP mobility model to test our protocol performance and the designed protocol also work with any other mobility model as well.

4.3 Communication Model

IEEE 802.11b, Distributed Coordination Function (DCF) is used in our simulation as a communication model [24]. DCF uses CSMA/CA technique for channel contention among mobile nodes. Channel capacity is shared among mobile nodes within their Carrier Sensing Range (CSR).

4.4 Performance Evaluation Metrics

Different metrics can be used to evaluate the performance of the protocols. The careful selection of metrics helps in fair analysis of the protocols. The protocols performance and efficiency are evaluated using traffic admission and completion of session with routing load, etc.

5. SIMULATION RESULTS ANALYSIS

This section of the paper presents the analysis of the simulation results on basis of performance evaluation metrics. Each protocol is analysed deeply according to their performance and results.

5.1 Session Admission Ratio

Figure 4 shows the Session Admission Ratio (SAR) of the studied protocols at different node speed. Node speed affects the performance of the protocols due to frequent topology changes. Node movement causes collision and frequent route failures. As the node speed increases, the SAR of the protocols decreases because the protocol generates more control overheads to find or recover the data route.

The CACP protocol admits more sessions than FAAC protocol because CACP does not consider

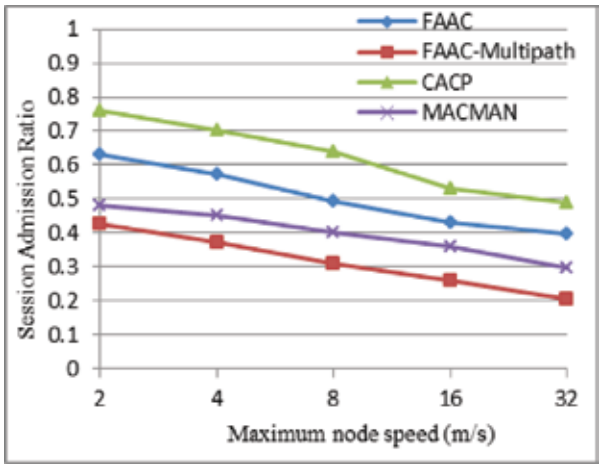

Fig. 4. Session admission ratio.

the effect of new data session on the existing data session in the network. The CACP protocol drops session and then uses this free capacity for the admission of other new sessions. Data session admission ratio in FAAC decreases as the node maximum speed increases because the provisioning of guaranteed throughput in such mobile scenario becomes difficult. The main task of FAAC protocol is to assure the guaranteed throughput to the admitted session and complete the session that have been admitted.

SAR of FAAC-Multipath is low and it decreases from 42.6% to 20.5% when speed increases from 2 to 32 m/s. Higher speed of nodes causes frequent route failures, more re-routing, local route repair, increases Packet Loss Ratio(PLR) and average end-to-end delay that results in consumption of network capacity and decrease the SAR. SAR of MACMAN is higher than FAAC-Multipath because FAAC-Multipath test the resources very thoroughly during the admission control and consider the effect on previously admitted sessions, because the main objective is to complete the data session not only to admit the data session.

5.2 Session Completion Ratio

Figure 5 represents the Session Completion Ratio (SCR) of the studied protocols and their behaviour at different node speed. Higher node speed decreases the SCR of the protocols because it changes topology frequently and results in collision

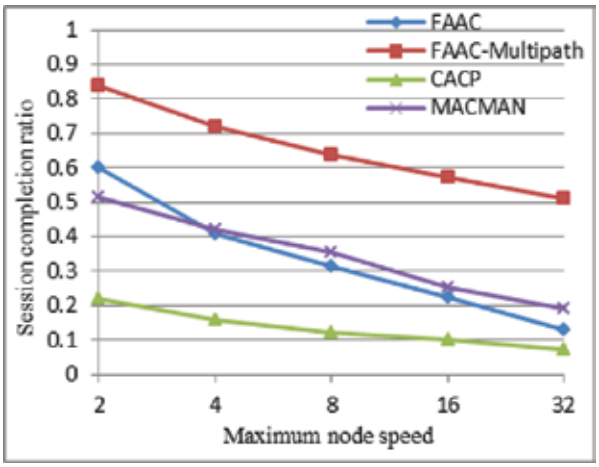

Fig. 5. Session completion ratio.

at MAC layer. The excessive re-transmission at the MAC layer causes the route failure, which either results in switching the data session to another route or initiate new route discovery. The switching mechanism or initiating the route discovery increases the overheads and results in degraded throughput. The session drops if its requirements are not fulfilled. It is clear from the figure that the data session completion ratio of FAAC is higher than CACP protocols. The completion ratio of FAAC protocol varies from 60.3 % to 12.8% by increasing speed from 2 to 32m/s while the completion ratio of CACP decreases from 21.9% to 7.4%, respectively. CACP admits more data sessions and then drops the sessions due to failure of providing the guaranteed throughput to data sessions.

SCR of MACMAN protocol is higher than FAAC protocol at higher node speed because higher speed cause frequent route failure and MACMAN takes an advantage of back up route availability. The SCR of the MACMAN is decreases from 51.4% to 19.2% when node speed rises from 2 to 32m/s. FAAC-Multipath performs better at different node speed among all the studied protocols. It's fast re-routing mechanism and local route repair mechanism helps to maintain the agreed throughput to the data session. Moreover the thoroughly controlled admission of data session also helps to achieve high SCR. Its SCR decreases from 83.7% to 51.1%, when node speed changes from 2 to 32m/s.

5.3 Packet Loss Ratio

Node speed has a great effect on the Packet Loss Ratio of the studied protocols. Figure 6 shows the Packet Loss Ratio of the four studied protocols i.e., CACP, FAAC, FAAC-Multipath and MACMAN. Nodes mobility make the data route stale and also causes route failure, which results in data packet loss. CACP protocol is severely affected by higher node speed, which increases the collision and as a result the protocol drops the data packets. FAAC protocol PLR is lower than CACP due to thorough admission control and efficient utilization of resources. FAAC finds alternate routes for the data session faster than CACP protocol, which results in low PLR. However, the PLR of FAAC protocol is higher than MACMAN and FAAC-Multipath protocols because it initiates route discovery or start the testing of available routes in source cache for the session. The PLR has great impact on session completion ratio of the protocols.

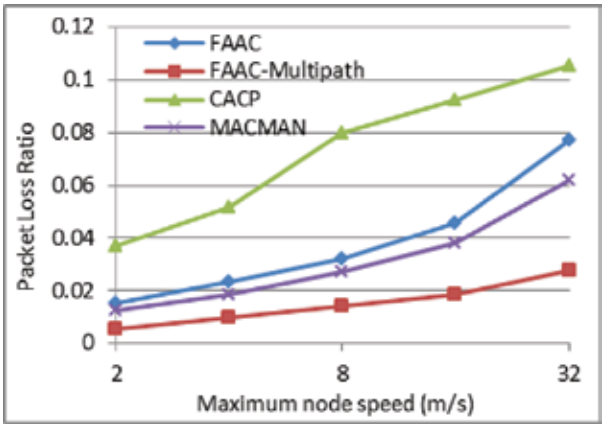

Fig. 6. Packet loss ratio.

PLR of MACMAN is higher than FAAC-Multipath at different node speed and its PLR are increasing as the node speed increases. The increase in node speed changes the topology very frequently and the node movement may affect the capacity of each other and as a result either decreases the session throughput or fails the data route. The node movement not only causes collision at the MAC layer, but also causes the buffer overflow. The MACMAN protocol session pausing mechanism although reduces the PLR that is due to collision,

but on the other hand, session pausing mechanism increases the average end-to-end delay which in turn increases the PLR that is due to the buffer overflow. Its PLR increases from 1.2% to 6.2% when node maximum speed changes from 2 to 32m/s.

FAAC-Multipath protocol has lowest PLR due to thorough admission control, fast re-routing and local route repair of routes. The reserved capacity plays a vital role in frequent topology changes and switches the data session from primary to secondary route. Its PLR increases from 0.57% to 2.7% when node speed changes from 2 and 32m/s.

5.4 Average End-to-End Delay

The average end-to-end delay is the second most important metric for evaluation of network layer protocols. It shows the efficiency of the protocols to deal with congestion, mobility, PLR and utilization of available capacity. Excessive dropping of packets either due to route failure or due to collision increase the average end to end delay of the data packets. Figure 7 shows the effect of nodes mobility over different protocols. Higher node speed causes frequent route failures and protocols initiate route discovery frequently that introduces more overheads to the network. Higher speed increases the interference that results in high PLR and longer end-to-end delays. FAAC protocol has smaller average end-to-end delay than MACMAN protocol at lower node speed because lower node speed causes less number of route failures. At higher node speed, route failure occurs more frequently and MACMAN protocol takes an advantage of backup routes and attains smaller average end-to-end delay.

MACMAN protocol has a longer average end-to-end delay than FAAC-Multipath protocol, due to its session pausing mechanism and slow re-route mechanism. MACMAN protocol pauses the session, when its achieved throughput is less than the requested. Session pausing mechanism of the MACMAN protocol results in longer average end-to-end delay which in turn also increases the PLR. MACMAN protocol maintains fully disjoint routes, which is very difficult to achieve in such frequent changing topology. FAAC-Multipath uses fast re-routing strategy instead of session pausing mechanism. The fast re-routing mechanism avoids the collision and keeps the average end-to-end delay minimum, which results in higher SCR and lower PLR among the studied protocols. SAR and reserved capacity also contribute to maintain minimum average end-to-end delay at different node speed.

5.5 Aggregate Throughput

The Aggregate throughput is related to the successful transmission of data packets. Route failure, PLR and average end-to-end delay affects the aggregate throughput of the network. Figure 8 shows the aggregate throughput of the FAAC, CACP, FAAC-Multipath and MACMAN protocols. Aggregate throughput of the protocols reduces with the rising node speed. MACMAN protocol achieves

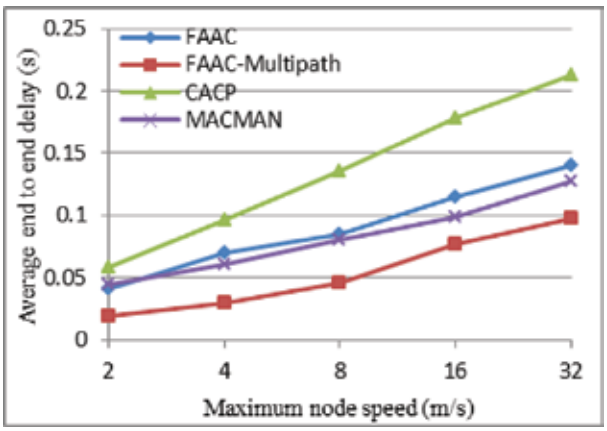

Fig.7. Average end-to-end Delay.

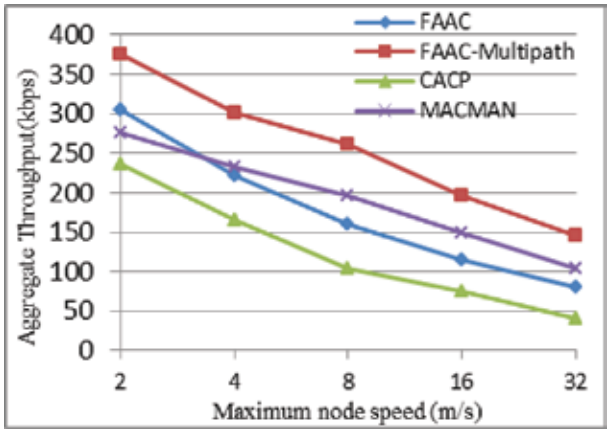

Fig. 8. Aggregate throughput.

higher aggregate throughput than FAAC protocol at higher node speed. The simulation results show that multi-path protocols works better in fast changing topology environment.

MACMAN protocol aggregate throughput is mainly affected by increasing average end-to-end delay with increase in node speed. Although MACMAN protocol uses backup route to achieve higher aggregate throughput, its session pausing mechanism increases the average end-to-end delay, which in turn decreases the throughput. MACMAN protocol attains lower aggregated throughput than FAAC-Multipath protocol.

The main objective of FAAC-Multipath protocol is to assure and uphold the throughput of each session which has been guaranteed at the time of session admission. The thorough admission control, tested backup route, fast re-routing and the absence of session pausing mechanism upholds the highest aggregate throughput of the protocol. It maintains minimum average end-to-end delay among all the studied protocol, which also contribute to the highest aggregate throughput. The SCR of the Figure 4-25 also confirms the result shown in Figure 4-28. Although the aggregate throughput of the FAAC-Multipath decreases with the increase in node maximum speed but still it maintains the guaranteed throughput of a higher ratio of the admitted session into the network.

5.6 Useful Aggregate Throughput

This metric shows only the average aggregate throughput of the completed sessions in Figure 9. The aggregate throughput of sessions, which drops in the middle, may not be useful to the application. Node mobility or speed create challenging environment for the protocols to uphold the guaranteed throughput till session completion. It shows the protocols' behaviour dealing with frequent route failure and unpredictable topology. Useful aggregate throughput is calculated by multiplying the aggregate throughput with the session completion ratio of the protocol. Due to higher aggregate throughput and session

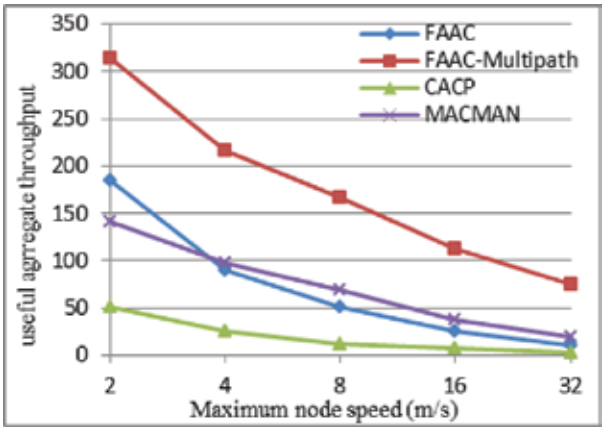

Fig. 9. Useful aggregate throughput.

completion ratio of FAAC protocol, the useful throughput of FAAC is higher than the CACP protocols. The useful aggregate throughput of all the protocols degrades as the node speed increases because higher node speed causes frequent route failure and increases PLR. The figure shows that FAAC-Multipath has maintained its highest useful aggregate throughput due to its highest SCR and aggregated throughput.

5.7 Normalized Routing Load

Normalized Routing Load of the stated protocol increases with the increase in node speed as represented in Figure 10. Here, mobile speed is the main cause of route failure and this failure occurs very frequently. Due to frequent changes in topology, single path AC protocols do not achieve

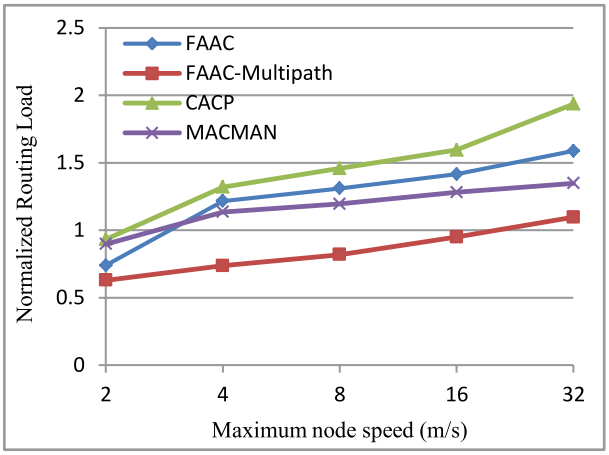

Fig. 10. Normalized routing load.

higher aggregate throughput because most of the session drop in the middle of session duration. CACP and FAAC initiates and test the capacity of route for each route failure. In high mobile scenario, MACMAN achieves higher aggregate throughput and SCR than FAAC protocol, which compensates the higher overhead of multiple routes and maintains lower NRL than FAAC protocol. FAAC-Multipath has lowest NRL among the studied protocols. The partial disjoint multiple routes and fast re-routing mechanism helps to assure aggregate throughput throughout the session duration that results in higher SCR. Higher SCR and aggregate throughput helps the protocol to compensate the routing overheads and maintain lower NRL.

6. CONCLUSIONS

Flow Aware Admission Control (FAAC)-Multipath protocol is designed with the characteristics of tested multipath and local route repair functionality. Both these characteristics and functionality enhances the throughput and session completion ratio enormously of the protocol. The simulation results establish the fact that Session Completion Ratio has improved by 60% and Throughput by 10%. FAAC-Multipath is compared with the state of the art Admission Control Protocols, which are single as well as multipath capabilities.

7. REFERENCES

1. Asif, M., Z. Sun & H. Cruickshank. Admission control protocols in mobile ad hoc networks provisioning QoS. In: *Proceedings of 7th International Conference on Frontiers of Information Technology,* Abbottabad, Pakistan, p. 1-4 (2009).
2. Stefano, B., M. Conti, S. Giordano & I. Stojmenovic. *Mobile ad Hoc Networking: The Cutting Edge Directions,* 2nd ed., Wiley-IEEE (2013).
3. Johson, D., Y. Hu & D. Maltz. *The Dynamic Source Routing Protocol (DSR) for Mobile Ad Hoc Networks*. IETF MANET Working Group Experimental RFC 4782 (2007).
4. Yang, Y. & R. Kravets. Contention-aware admission control for ad hoc networks. *IEEE Transaction on Mobile Computing* 4(4): 363-377 (2005).
5. Lindgren, A. & M. Elizabeth. Multipath admission control protocol for MANETs. *ACM SIGMOBILE Mobile Computing and Communications Review* 8(4): 68-71 (2004).
6. Asif, M., Z. Sun, H. Cruickshank & N. Ahmad. QoS provisioning in contention aware MANETs using Flow Aware Admission Control protocol. In: *Proceedings of IADIS International Conference on Telecommunications, Networks and Systems*, Rome, Italy, p. 99-106 (2011).
7. Lee, S., & M. Gelra. AODV-BR: Backup Routing in Ad Hoc Networks. In: *Proceedings of IEEE Conference on Wireless Communications and Networking Conference*, Chicago, USA, p. 1311-1316 (2000).
8. Wu, S.L., S. Y. Ni., J.P. Sheu & Y.C. Tseng. Route Maintenance in a Wireless Mobile Ad Hoc Network. *Telecommunications Systems*18(1-3): 61–84 (2001).
9. Castenada, R., S.R. Das & M.K. Maria. Query localization techniques for on-demand routing protocols for mobile ad hoc networks. *Wireless Networks* 8(2-3): 137-151 (2002).
10. Goff, T., N. Abu-Ghazaleh., D. Phatak & R. Kahvecioglu. Pre-emptive routing in ad hoc networks. *Journal of Parallel and Distributed Computing* 63(2):123-140 (2003).
11. Marina, K.M & S.R. Das. Ad hoc on-demand multipath distance vector routing. *Wireless Communications and Mobile Computing* 6: 969-988 (2006).
12. Li, X. & L. Cuthbert. Multipath QoS routing of supporting Diffserv in Mobile Ad hoc Networks. In: *Proceedings of IEEE Sixth International Conference on Software Engineering, Artificial Intelligence, Networking and Parallel/Distributed Computing,* Towson, Maryland, USA, p. 308-313 (2005).
13. Lin, C.R. On-Demand QoS Routing in Multihop Mobile Networks. In: *Proceedings of IEEE Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies*, Anchorage, Alaska, USA, p. 1735-1744 (2001).
14. Liao, W.H., S.L. Wang, J.P. Sheu & Y.C. Tseng. A Multi-Path QoS Routing Protocol in a Wireless Mobile Ad Hoc Network. *Telecommunications Systems* 19(3-4): 329-347 (2002).
15. Yu, W., Z. Mi, D. Niu & R. Xu. Algorithm of exposed terminals concurrent transmission based on reverse path in MANET. In: *Proceedings of 2nd IEEE International Conference on Mechanic Automation and Control Engineering,* Hohhot, China, p. 3819-3822 (2011).
16. Wannawilai, P. & C. Sathitwiriyawong. AOMDV with Sufficient Bandwidth Aware. In: *Proceedings of 10thIEEE International Conference on Computer and Information Technology,* Bradford, UK, p. 305-312 (2010).

17. Senthilkumar, M. & S. Somasundaram. Energy Aware Multiple Constraints Intelligent Multipath QoS Routing Protocol with Dynamic Mobility Prediction for MANET. In: *Proceedings of International on Process Automation, Control and Computing,* Coimbatore, India, p.1-8 (2011).
18. Leung, R., J. Liu, E. Poon, A. Chan & L. Baochun. MP-DSR: a QoS-aware multi-path dynamic source routing protocol for wireless ad-hoc networks. In: *Proceedings of 26th Annual IEEE Conference on Local Computer Networks, Tampa, Florida,* USA, p. 132-141 (2001).
19. Hwang, Y. & P. Varshney. An adaptive QoS routing protocol with dispersity for ad-hoc networks. In: *Proceedings of 36th Annual Hawaii International Conference on System Sciences,* Big Island, HI, USA, p. 6-9 (2003).
20. Wang, Y.H., C.H. Tsai, H.Z. Lin & C.A. Wang. Interference aware QoS Multipath Routing for Ad Hoc Wireless Networks. *International Journal of Computers and Applications* 29(4): 372-378 (2007).
21. Reddy, R.L. & V.S. Raghavan. SMORT: Scalable Multipath on Demand Routing in Mobile Ad Hoc Networks. *Ad Hoc Networks* 5(2): 162-188 (2007).
22. Sarma, N. & S. Nanda. A multipath QoS routing with route stability for mobile ad hoc networks. *IETE Technical Review* 27(5): 380-397 (2010).
23. Chakeres, I.D, E.M. Belding-Royer & J.P. Macker. Perceptive admission control for wireless network quality of service. *Ad Hoc Networks* 5 (7): 1129-1148 (2007).
24. IEEE Computer Society. Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications. *IEEE Std. 802.11* (2007).
25. Kurkowski, S., T. Camp & W. Navidi. Two Standards for Rigorous MANET Routing Protocol Evaluation. In: *Proceedings of 3rd IEEE Conference on Mobile Ad hoc and Sensor Systems,*Vancouver, British Columbia, Canada, p. 256-266 (2006).
26. Navidi, W. & T. Camp. Stationary distributions for the random waypoint mobility model. *IEEE Transaction on Mobile Computing* 3(1): 99-108 (2004).

Proceedings of the Pakistan Academy of Sciences 52 (3): 215–225 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

A Neuro-Fuzzy based Non-linear Control Technique for Steam Boiler using Levenberg-Marquardt Algorithm

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Abstract: The objective of this paper is to design a nonlinear control system, to stabilize the drum level and steam pressure of the industrial boiler at desired values. It is difficult to maintain the accurate control performances and to achieve the desired estimated values by using conventional proportional integral derivative (PID) control system. Based on the dynamic behavior of the boiler an Adaptive Fuzzy Logic (AFL) control strategy is designed to stabilize the drum level and steam pressure at desired values. The proposed non-linear AFL strategy is robust to meet the control objectives and to handle the uncertainties faster than traditional controllers. The simulation results show that the proposed AFL has tracking ability with better steady state error and transient response than conventional PID controller.

Keywords: Adaptive Fuzzy Logic controller (AFLC), PID controller, Levenberg-Marquardt (LM)

1. INTRODUCTION

Steam boiler is one of the vital machinery in industries used for the purpose of generating steam. Steam is being used in many industrial processes including power generation, central heating, textiles, and cement industry. To achieve the desired performance, steam boiler needs to be efficient in order to provide best quality of steam. This necessitate for the selection of a valid boiler model and suitable control strategy to obtain the desired outputs. Therefore Pellegrinetti and Bentsman [1] model is suitable for representing the non-linear behavior of steam boiler which was developed from the Astrom and Bell [2-4] model. Steam generator is highly non-linear, complex and time varying system whose parameters change with operating conditions. The model has three inputs (fuel, feedback water, and air flow) and four outputs (drum pressure, excess oxygen, steam flow rate, and drum water level). Our interest lies in the control of three outputs: drum pressure, steam flow rate, and drum water level.

Boiler efficiency can be optimized by adopting a control strategy that provides desirable outputs. Poor control of drum water level may cause shutdown of steam generator plant. The water level in steam boiler must be maintained in allowable limits in order to operate the boiler efficiently and safely [5]. Violating the specified limits may cause either moist steam at the outlet that results rusting of turbine in case of steam-turbine unit, or overheating of drum material which will cause deterioration of boiler material. Similarly steam pressure is to be controlled to regulate steam at the outlet. Steam boiler control is difficult due to certain factors including non-linear characteristics, dynamic uncertainties and load disturbances. Efficient controller is needed to provide desired output to increase the efficiency of the plant. In recent years different controllers have been used for controlling the steam generator parameters including proportional integral (PI),

Received, March 2015; Accepted, May 2015
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proportional derivative (PD), proportional integral derivative (PID), state feedback controller (SFC), linear quadratic regulator (LQR), neural network (NN) and sliding mode predictive controller (SMPC).

As for as PID controller is concerned, the entire operating range of non-linear boiler model is divided in to three linear segments and multiple PIDs or any other linear controllers are used for controlling different segments [6]. The controllers are tuned heuristically to achieve best possible results. Estimation of internal states of the process increases the possibility of better and efficient control. PID lacks the property of estimating the internal states of multivariable MIMO process [7]. PID causes wastage of energy and decreases the plant efficiency. It requires human intervention and understanding for suitable resolutions and corrective actions. Conventional PID controller has demerits of inability to understand process, lack of identifying small drifts over interval of time from ideal response and is unable to follow the desired dynamic behavior over the entire non-linear operating region, resulting in decrease in the overall efficiency and economy of the plant [8].

Alternately, state feedback controller could be a suitable technique to achieve these goals. State feedback controller has the ability of understanding process and therefore provides a good control of manipulated variables. Its implementation for industrial boiler is difficult because of non-availability of methodology of right pole placement. LQR has a good aspect of reducing controller energy and avoid saturation of actuator. But LQR and state feedback controller techniques are used for linear models [9].

Sliding mode predictive controller (SMPC) has better approach to control drum pressure than PID and Smith predictor in terms of oscillations and speed of the response, but it fails if the settling time is considered [8]. However none of these controllers can match the desired performance of a real time industrial boiler. Industrial boiler requires adaptive controller that has properties of monitoring and updating its parameters accordingly. Adaptive control strategy is used for regulating different plant parameters according to the desired time domain specification. The control process becomes more complex when considering both drum level and drum pressure control within the same system [10]. Changing reference points in such system causes change in dynamics of the entire plant.

Among the adaptive controllers, adaptive neuro-fuzzy inference system has some limitations for different applications including boilers. It needs linear model for different load conditions, i.e., 50%, 70%, and 100%. For this reason, the design of controller should be modified for each operating load in order to achieve optimal performance [6].

Overcoming the above problems require adaptation of parameters. The overall adaptability is compensated using adaptive fuzzy logic controller (AFLC). Adaptive fuzzy controller updates its parameters and organizes their values itself. It does not require to understand the physics or modeling of plant [11]. Levenberg-Marquardt (LM) technique is used to update the AFLC parameters. This technique is used to minimize quadratic, linear and nonlinear error functions and is comparatively faster and convergent.

2. MODELING OF PLANT

This paper is based on the simulation model of Pellegrinetti and Bentsman [1]. The model is obtained from steam plant at Abbott power plant in Champaign, Illinois. This is a multivariable MIMO plant having three inputs (fuel flow, air flow, and feed water flow) and four outputs (drum pressure, oxygen level, drum level, and steam flow rate). The fuel flow has influence on the steam flow rate and drum pressure. The second input air flow affects the oxygen level whereas the drum level is effected by feed water flow and steam flow rate. Respective inputs should be controlled to obtain the desired output. The model has internal perturbation and measurement noises. Perturbation effect changes with time. The schematic and block diagrams of model are shown in Fig. 1 and 2, respectively. The model can be represented mathematically by the following system of equations:

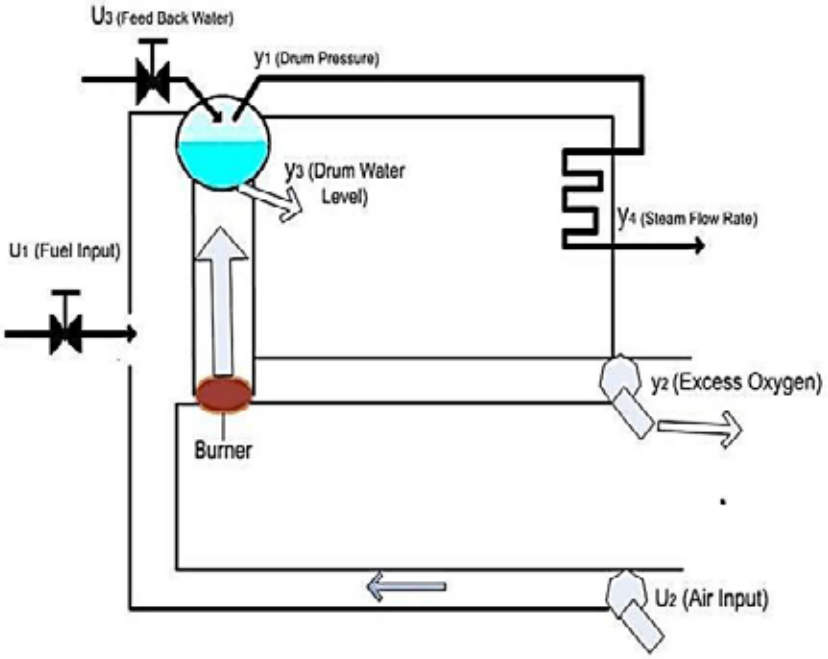

Fig. 1. Steam boiler schematic Diagram.

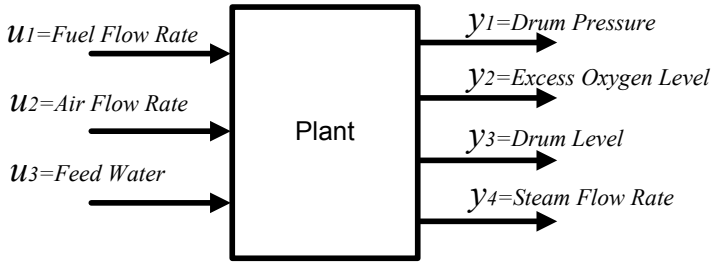

Fig. 2. Steam boiler block diagram.

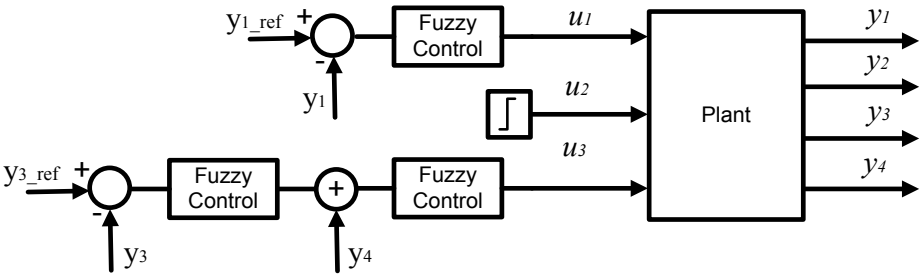

Fig. 3. Block diagram of proposed control.

$$x_1'(t) = -0.00478x_4(t)x_1^{\frac{9}{8}}(t) + 0.28u_1(t-2) - 0.01348u_3(t-3)$$

$$x_2'(t) = 0.1540357x_2(t) + \frac{\begin{bmatrix} 103.5462u_2(t-2) - u_1(t-2) \\ \{107.4835 + 1.9515x_2(t)\} \end{bmatrix}}{29.04u_2(t-2) - 1.824u_1(t-2)}$$

$$x_3'(t) = -0.00533176x_1(t) - 0.02195x_4(t)x_1(t) + 0.7317058u_3(t-3)$$

$$x_4'(t) = -0.04x_4(t) + 0.029988u_1(t-2) + 0.018088$$

$$y_1(t) = -140214x_1(t-3)$$

$$y_2(t) = x_2(t-4)$$

$$y_3(t) = -0.1048569x_1(t-10) + 0.15479x_3(t-10) + 0.4954961x_4(t-10)x_1(t-10)$$
$$-0.20797u_3(t-13) + 1.272u_1(t-12)$$
$$+\left[\frac{\{-324212.7805x_1(t-10) - 99556.24778\}\{1 - 0.001185x_3(t-10)\}}{\{x_3(t-10)\}\{x_1(t-10) - 1704.50476\}}\right] - 103.7351$$

$$y_4(t) = [0.85663x_4(t-2) - 0.18128]x_1(t-2) \quad (1)$$

Where

$x_1$is drum pressure state(kg/cm^2)

$y_1$is measured drum pressure (PSI)

$y_2$andx_2 are measured excess oxygen level and its state, respectively $(percent)$

$x_3$is system fluid density (kg/m^3)

$y_3$is drum water level $(inch)$

$x_4$is exogenous variable related to load disturbances intensity $(0-1)$

$y_4$is steam flow rate (kg/sec)

$u_1, u_2, u_3$are the fuel, air, and feed water flow rate inputs, respectively, having range [0 1].

The plant is linearized around the nominal operating points $(points\ at\ which\ x' = [x_1'x_2'x_3'x_4']^T = 0)$

$$x^0 = [22.5\ \ 1.5734733038535\ \ 621.17\ \ 0.8374]^T$$
$$y^0 = [320\ \ 2.5\ \ 1.6\ \ 12.05]^T \quad (2)$$
$$u^0 = [0.5138\ \ 0.5064\ \ 0.8127]^T$$

3. PROPOSED ADAPTIVE FUZZY LOGIC CONTROLLER

In this section the proposed fuzzy model based on Levenberg Marquardt (LM) technique is discussed. In recent years different controllers have been designed for benchmark non-linear model of steam boiler using conventional PID and state space approaches. Each approach has its own merits and demerits. None of these approaches can match the desired performance as required for the real plant. AFLC based on Levenberg-Marquardt

technique is the best approach for the non-linear, complex, and poorly understandable plants. This is referred as direct adaptive control technique. The block diagram for proposed control is shown in Fig. 3. Fuzzy adaptive controller does not require a perfect model to achieve the optimal performance. The non-linear model uncertainty is handled by the knowledge based modifier that makes fuzzy controller adaptive by modifying the center of singleton membership function [12-14].

The training of steepest descent is very slow and smaller step size makes it convergent. Gauss-Newton method is faster to minimize the cost function but the probability of divergence increases. The technique fails if the Jacobian matrix's inverse does not exist. A second order method, Levenberg-Marquardt technique is faster and stable using Jacobian matrix [15-17]. Levenberg-Marquardt algorithm provides solution to the problem called *Non-linear least square minimization*. The technique minimizes the function of the following form [18]-[19]

$$f(x) = \frac{1}{2}\sum_{j=1}^{m} r_j^2(x) \tag{3}$$

where

$$x = [x_1 x_2 x_3 \ \cdots x_n]^T \in R^{n\times 1} \tag{4}$$

$x$represents a vector belong to $R^{n\times 1}$ and eachr_j is the function from $R^n \, to \, R$. The r_j is called the residuals and it is assumed that $m \geq n$. The function f is represented as a $residual\ vector$, i.e., $r: R^n \rightarrow R^m$ defined by

$$r(x) = [r_1(x), r_2(x), \cdots, r_m(x)] \tag{5}$$

The function f in Eq. (3) can be written as

$$f(x) = \frac{1}{2}\|r(x)\|^2 \tag{6}$$

The derivative of f can be written as the Jacobian matrix J of r with respect to x

$$J(x) = \frac{\partial r_j}{\partial x_i} \quad 1 \leq j \leq m\,, \quad 1 \leq i \leq n \tag{7}$$

The learning algorithm of Levenberg-Marquardt is given by

$$\Delta w = -\lambda(J^T J + \mu I)^{-1} J e \tag{8}$$

Where Δw is the update weight, $J$is the Jacobean matrix, e is the error defined as the difference between actual value and desired value. The other two parameters λ and μ are used to control the step size and the regularization term to make it invertible and to stabilize the algorithm [20].

The update rules for Levenberg-Marquardt is given by

$$w_{k+1} = w_k - \lambda\left(J_k^T J_k + \mu I\right)^{-1} J_k e_k \tag{9}$$

Eq. (9) is used for updating different parameters.w_{k+1}shows the updated value, $w_k$is the previous value, $J_k$is the Jacobean matrix, λ is combination coefficient, and $I$is identity matrix.

3.1 Controller Design

The controller based on Levenberg-Marquardt technique minimizes both linear and non-linear functions. The design emerges from the following cost equation

$$e_m = \frac{1}{2}\left(f_m - y_{ref}\right)^2 \tag{10}$$

The output equation of controller used for defuzzification is given by

$$f_m = \left(\sum_{i=1}^{R} b_i \mu_i\left(x_j^m, k\right)\right) \Big/ \left(\sum_{i=1}^{R} \mu_i\left(x_j^m, k\right)\right) \tag{11}$$

where

$$\mu_i\left(x_j^m, k\right) = \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right) \tag{12}$$

The Eq. (12) is used for updating the input and output membership function of the fuzzy logic controller. $c_j^i$is the center of membership function, and σ_j^i represents variance.

3.1.1 Derivation of Equation for Updating Parameters

The equation for updating the parameters is derived from the error defined by Eq. (10). The derivative of Eq. (10) results in Jacobian of each term, i.e., variance, center and output membership function.

Taking the derivative of Eq. (10) with respect to b_j

$$\frac{\partial e_m}{\partial b_j} = \frac{\partial}{\partial b_j}\left(\frac{1}{2}\left(f_m - y_{ref}\right)^2\right) \tag{13}$$

$$\frac{\partial e_m}{\partial b_j} = \left(f_m - y_{ref}\right)\frac{\partial \varepsilon}{\partial b_j} \tag{14}$$

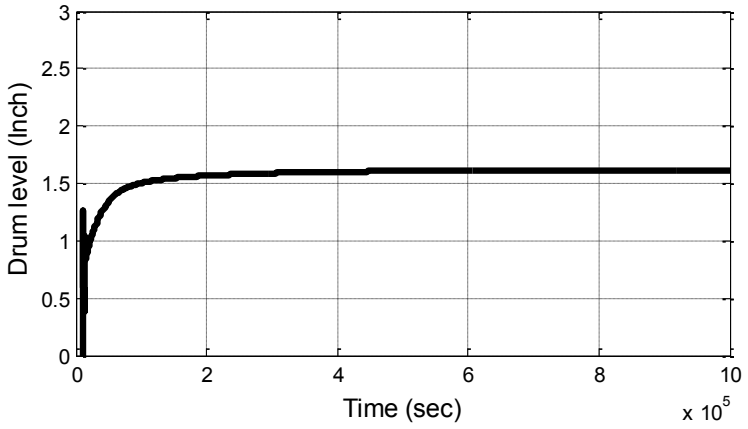

Fig. 4. Drum water level.

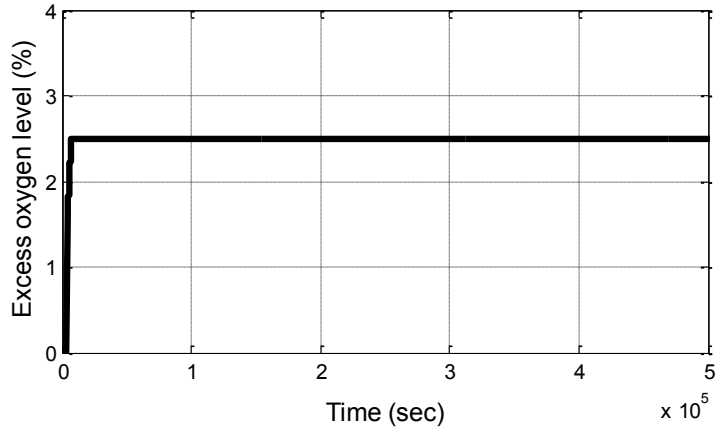

Fig. 5. Excess of oxygen.

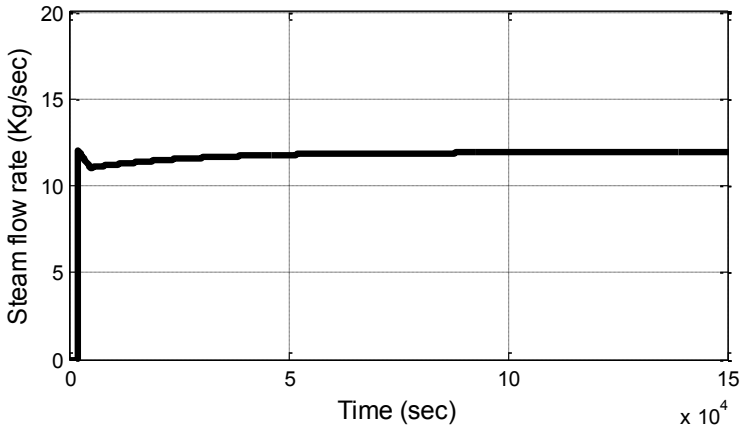

Fig. 6. Steam flow rate.

By putting the value of f_m from Eq. (10) and $\varepsilon = (f_m - y_{ref})$, we obtain

$$\frac{\partial e_m}{\partial b_j} = \varepsilon \frac{\partial}{\partial b_j}\left[\left(\sum_{i=1}^{R} b_i \mu_i(x_j^m, k)\right) \Big/ \left(\sum_{i=1}^{R} \mu_i(x_j^m, k)\right)\right] \tag{15}$$

$$\frac{\partial e_m}{\partial b_j} = \varepsilon \frac{\partial}{\partial b_j}\left[\left(\sum_{i=1}^{R} b_i \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right)\right) \Big/ \left(\sum_{i=1}^{R} \prod exp\left(-\frac{1}{2}\left(\frac{x_j^m - c_j^i}{\sigma_j^i}\right)\right)\right)\right] \tag{16}$$

The above Eq. (15) shows the Jacobian of output membership function b_j. Similarly by taking derivatives with respect to σ_j and c_j will result in Jacobian of variance and center of membership function, respectively.

3.1.2 Update Equation for Output Membership Function

This equation is used for updating the output membership function, i.e., control-output to the plant. The variable b_j represents the center of output membership function. The center of output membership function updates according to the output of the plant.

$$b_j(k+1) = b_j(k) - \lambda \left[\left[\varepsilon \left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right]\left[\left[\varepsilon \left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right]^T + \mu I\right]^{-1} \left[\varepsilon \left[\frac{\mu_i(x_j^m, k)}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\right] \varepsilon \tag{17}$$

3.1.3 Update Equation for Variance

From Eq. (12) the magnitude of membership function is inversely proportional to the variance. The higher value of variance results in lower magnitude and vice versa. Variance defines the spread of the membership function which is updated by the equation given below.

$$\sigma_i(k+1) = \sigma_i(k) - \lambda \left[\left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right]\left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right]^T + \mu I\right]^{-1} \left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)^2}{(\sigma_j^i)^3}\right)\mu_i(x_j^m, k)\right] \varepsilon \tag{18}$$

3.1.4 Update Equation for Center

The Eq. (19) updates the center of membership function. The center acquires different values according to the crisp input to controller.

$$c_i(k+1) = c_i(k)$$
$$-\lambda \left[\left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right]\left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right]^T + \mu I\right]^{-1} \left[\varepsilon \left[\frac{(\sum_{i=1}^{R} b_i) - f_m}{\sum_{i=1}^{R} \mu_i(x_j^m, k)}\right]\left(\frac{(x_j^m - c_j^i)}{(\sigma_j^i)^2}\right)\mu_i(x_j^m, k)\right] \varepsilon \tag{19}$$

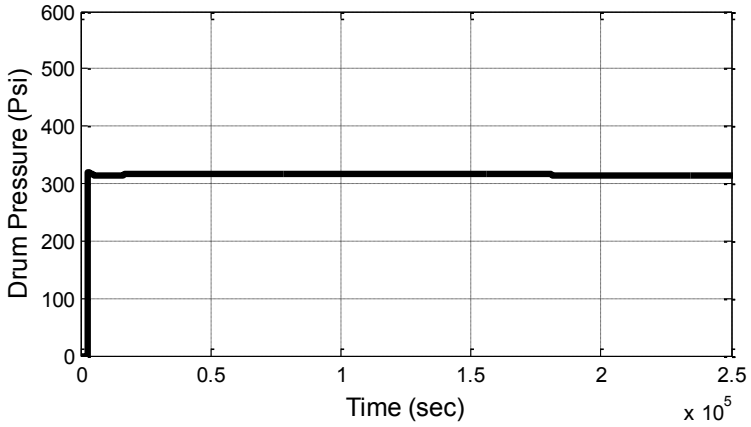

Fig. 7. Drum pressure.

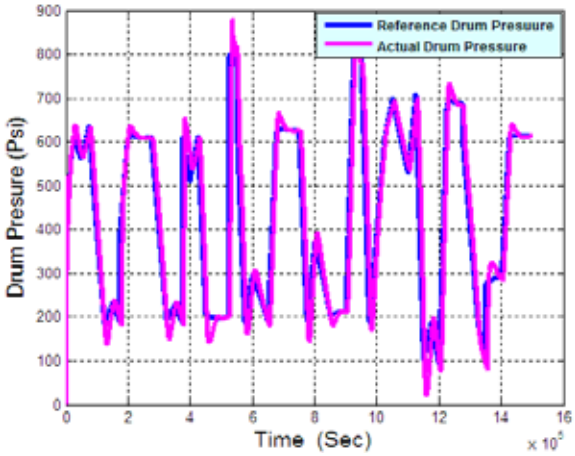

Fig. 8. Controlled drum pressure.

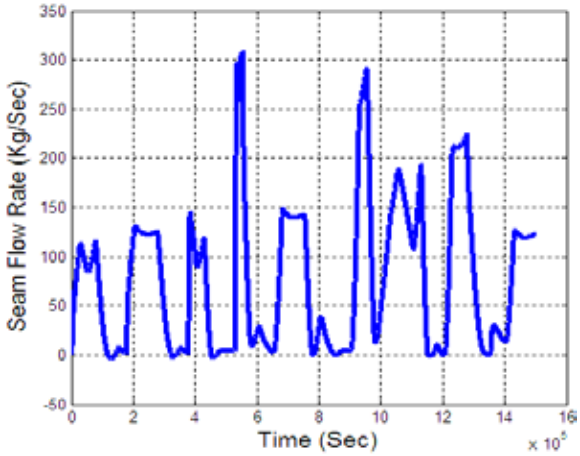

Fig. 9. Controlled steam flow rate.

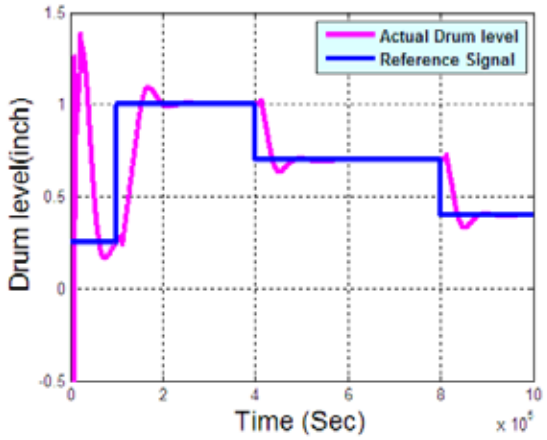

Fig. 10. Controlled drum level.

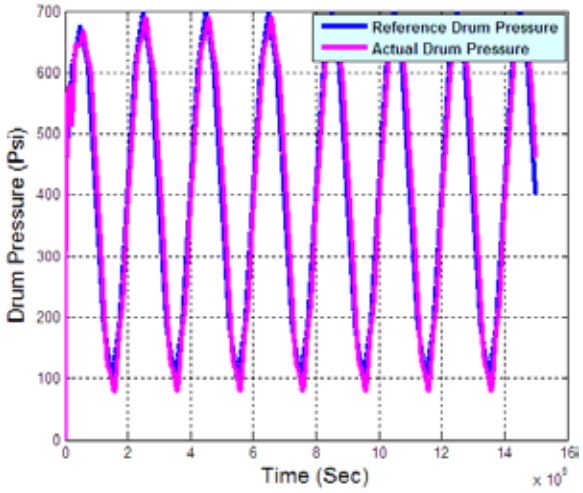

Fig. 11. Controlled drum pressure.

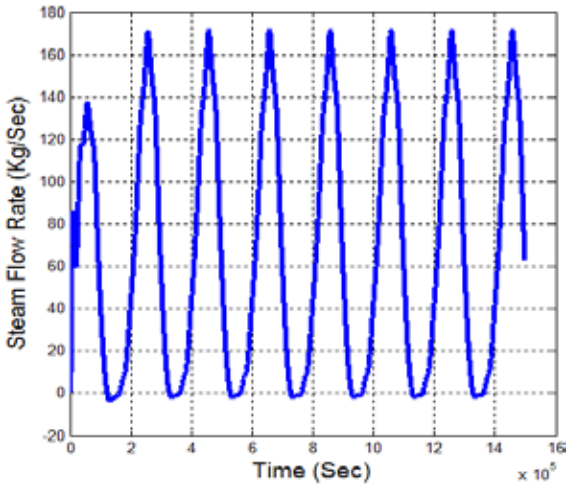

Fig. 12. Controlled steam flow rate.

4. SIMULATION RESULTS

4.1 Open Loop Responses

Fig. 4–7 show open loop responses of different parameters of steam boiler to unit step. It is observed that AFLC has the ability to understand the process and update its parameters accordingly to give desired controlled input to the plant. The drum level will rise to 1.5 inches by applying unit step at feed water input (u_3). Applying unit step at air flow input (u_2) will result the oxygen level output at 2.5%. Similarly, steam flow rate will be 12 kg/sec and drum pressure will acquire the value of 320 psi by applying step input at fuel flow (u_1).

4.2 Adaptive Fuzzy Logic Controller (AFLC) Closed Loop Responses

The simulation is performed using MATLAB, and keeping time $1500\ sec$ throughout the simulation. The Fig. 8 to 12 show controlling of different steam boiler parameters with AFLC. As the plant has dynamics of high order, as well as nonlinearities, instabilities, and time delays, for this reason multiple signals are given as reference to check different parameters responses. Fig. 8 and 9 show drum pressure and corresponding steam flow rate which approximately resemble with real time responses of Abbott power plant in Champaign, Illinois. Initially steam generator takes time to reach the required drum pressure due to burning of fuel and rise in temperature from cold start, but in fact the steam flow rate is associated with the drum pressure. Steam generator turbine needs constant steam flow to avoid fluctuation in connected load, i.e., electric generator in the case of steam generator-turbine unit. As AFLC results in oscillation free output of both drum pressure and steam flow rate, therefore AFLC is preferred over pervious controlled schemes. In addition, the overshoot in responses settles abruptly due to updating of various parameters of ALFC, resulting in smooth and controlled output.

Fig. 11 and Fig. 12 show drum pressure and corresponding steam flow rate with a sine wave as a reference signal. The output parameter, drum water level (Fig. 10) is a slow process, which is affected by fuel flow rate and steam flow rate directly. In real plant, drum level is kept constant at center of the drum throughout the operation [16]. Keeping in view the resemblance with real plant, a changing step within its limit is applied which is tracked by the output response of drum level. The actual drum level shows less settling time with AFLC. At the start of the simulation, the AFLC quickly updated its parameters and started to follow the desired drum level which was at 0.2 inch. At 400 sec the desired level of drum changes to 1 inch, the response of AFLC has 0.85% overshoot and settling time is 158 sec.

5. CONCLUSIONS

We proposed adaptive fuzzy logic controller (AFLC) for the control of multivariable steam boiler. The results of AFLC for different controlling parameters show that percentage overshoot and settling time is within allowable limits, and its response has improved both in transient and steady state region, because ALFC does not require a perfect model for its optimal performance. The self-learning and updating mechanism of adaptive fuzzy controller reduces the problem of estimating the internal states of MIMO system. Fuzzy controller removes the fluctuations from the actual response as it occur in drum pressure and steam flow rate. Throughout the simulation AFLC keeps the drum level within its allowable limit. The Gauss Newton based Levenberg-Marquardt technique enhances the process of parameter updating, thereby minimizing the computational time.

6. REFERENCES

1. Pellegrinetti, G. & J. Bentsman. Non-linear controlled oriented boiler Modeling-A benchmark Problem for controller design. *IEEE Transactions on Control Systems Technology* 4: 57-64 (1996).
2. Astrom, K.J. & R.D. Bell. Dynamic model of boiler-turbine-alternator unit: Data logs and parameters estimation for a 160 MW unit. *Department of Automation Control Lund Institute Technology Lund, Sweden Rep. LUTFD2/ (TFRT-3192),* p. 1-137 (1987).
3. Astrom, K. J. & R.D. Bell. *A Low-order Nonlinear Dynamic Model for Drum Boiler-turbine Alternator Units.* Department of Automation Control Lund Institute Technology Lund, Sweden, Rep. (TFRT-7162), (1987).
4. Astrom, K.J. & R.D. Bell. *Simple Drum-boiler Models.* Proceedings of International Federation of Automation Control (IFAC) Symposium Power system Modeling Control Application, p. 123-127 (1988).

5. Haihe, Z., W. Li & Y. Xinjun. *Fuzzy Controller of Drum Water Level for Industrial Boiler*. Proceedings International Conference on Computer Mechatronics Control and Electronic Engineering, p. 400-402 (2010).
6. Lin, B., P. Han, D. Wang & Q. Guo. *Control of Boiler-Turbine Unit Based on Adaptive Neuro-Fuzzy Inference System.* Power Engineering Department North China Electric Power University Baoding Heibei, China (2003).
7. Morilla, F. *Benchmark for PID Control based on the Boiler Control Problem*. Proceedings International Federation of Automation Control (IFAC) Conference on Advances in PID Control, p. 346-351 (2012).
8. Yuanhao, S., W. Jingcheng & Z. Yunfeng. *Sliding Mode Predictive Control of Main Steam Pressure in Coal-fired Power Plant Boiler*. Department of Automation, Jiao Tong University Shangai, Chinese Journal of Chemical Engineering 20:1107-1112 (2012).
9. Linear Quadratic Regulator, 520.454 *Control Systems Design [2000]*. The Johns Hopkins University, Department of Electrical & Computer Engineering.
10. Begum, K.G., D. Mercy, H.K. Vedi & M. Ramathilagam. An intelligent model based level control of boiler drum. *International Journal of Emerging Technology and Advanced Engineering* 3: 561-521 (2013).
11. Rucheng, Z., W. Yahui, L. Xiaofeng & T. Qichuan. An intelligent control method of thermal boiler system. *Proceedings IEEE international conference on intelligent Processing System*, p. 684-687 (1997).
12. Huang, Y.L., H.H. Lou, J.P. Gong & T.F. Edgar. Fuzzy model predictive control. *IEEE Transactions on Fuzzy Systems* 8: 665-678 (2000).
13. Wang, H.O., K. Tanaka & M.F. Griffin. An approach to fuzzy control of nonlinear systems: stability and design issues. *IEEE Transactions on Fuzzy Systems* 4: 14-23 (2002).
14. Spooner, J.T. & K.M. Passino. Stable adaptive control using fuzzy systems and neural networks. *IEEE Transactions on Fuzzy Systems* 4: 339-359 (1996).
15. Yu, H. & B.M. Wilamowski. Levenberg-Marquardt Training. *The Industrial Electronics Handbook- eng.auburn.edu,* p. (12-1)-(12-16) (2011).
16. Moon, U.C. & K.Y. Lee. Step-response model development for dynamic matrix control of a drum-type boiler–turbine system. *IEEE Transaction on Energy Conversion*. 24:423-430 (2009).
17. Wilamowski, B.M. & H. Yu. Improved Computation for Levenberg–Marquardt Training. *IEEE Transactions on Neural Network* 21: 930-937 (2010).
18. Marquardt, D. An algorithm for least-squares estimation of nonlinear parameters. *Society for Industrial & Applied Mathematics Journal on Applied Mathematics* 11: 431-441 (1963).
19. Levenberg, K. A method for the solution of certain problems in least square. *Quarterly of Applied Mathematics* 2: 164-168 (1994).
20. Chan, L.-W. & C.-C. Szeto. *Training Recurrent Network with Block-diagonal Approximated Levenberg-Marquardt Algorithm.* Computer Science and Engineering Department. The Chinese University of Hong Kong, Shatin, Hong Kong (1999).

Proceedings of the Pakistan Academy of Sciences 52 (3): 227–232 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Certain Properties of an Operator Involving the Generalized Hypergeometric Functions

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Abstract: In this work, based on the generalized derivative operator $\mathrm{K}^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)f(z)$ and by making use of the notion of subordination, two new subclasses of functions are derived. With regards to these two subclasses, some properties are discussed briefly.

Keywords: Analytic function, Hadamard product, differential operator, subordination, coefficient estimate.

1. INTRODUCTION

Historically, we were informed that John Wallis was the first ever mathematician who used hypergeometric functions and this can be found in his book entitled "Arithmetica Infinitorum" [1]. Euler also found to be in the lists of those who used hypergeometric functions as mentioned in the book entitled "Theory of hypergeometric functions" [2]. However, the first full systematic treatment was given by Carl Friedrich Gauss, and thereafter by Ernst Kummer [3]. The fundamental characterization was addressed by Bernhard Riemann for solving hypergeometric function by means of differential equation where it satisfied [4]. The importance of the hypergeometric theory is stemmed from its applications in many subjects such as, numerical analysis, dynamical system and mathematical physics.

Definition 1.1 [11]: Denote by A the class of analytic functions of the form

$$f(z)=z+\sum_{n=2}^{n=\infty}a_n z^n;\quad z\in(U=\{z\in C:|z|<1\}) \qquad (1)$$

and $\mathcal{S}$ the subclass of A consisting of univalent functions, and $S(\alpha)$, $(0<\alpha\le 1)$ denotes the subclasse of A consisting of functions that are starlike of order α in U.

Definition 1.2 [10]: For two analytic functions $f(z)=z+\sum_{n=2}^{n=\infty}a_n z^n$ and $g(z)=z+\sum_{n=2}^{n=\infty}b_n z^n$ in the open unit disk $U=\{z\in C:|z|<1\}$. The Hadamard product (or convolution) $f*g$ of f and g is defined by

$$f(z)*g(z)=(f*g)(z)=z+\sum_{n=2}^{n=\infty}a_n b_n z^n. \qquad (2)$$

Definition 1.3 [11]: Let $p(z)$ and $q(z)$ be analytic in U. Then the function $p(z)$ is said to be subordinate to $q(z)$ in U, written by

$$p(z)\prec q(z);\qquad (z\in U), \qquad (3)$$

if there exists a function $w(z)$ which is analytic in U with $w(0)=0$ and $|w(z)|<1$ with $z\in U$, and such that $p(z)=q(w(z))$ for $z\in U$. From the definition of the subordinations, it is easy to show that the subordination (3) implies that

$$p(0)=q(0) \qquad and \qquad p(U)\subset q(U) \qquad (4)$$

For complex parameters $\alpha_1,...\alpha_r$ and $\beta_1,...\beta_s$ $(\beta_j\neq 0,-1,-2,...;j=1...s)$, Dziok and

Received, March 2014; Accepted, May 2015
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Srivastava [5] defined the generalized hypergeometric function ${}_rF_s(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s;z)$ by

$$ {}_rF_S(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S;z)=\sum_{n=0}^{\infty}\frac{(\alpha_1)_n...(\alpha_r)_n}{(\beta_1)_n...(\beta_S)_n}\frac{z^n}{n!};$$
$$(r\le s+1;r,s\in N_0;z\in U), \quad (5)$$

where $(x)_n$ is the Pochhammer symbol defined, in terms of Gamma function Γ, by

$$(x)_n=\frac{\Gamma(x+n)}{\Gamma(x)}=\begin{cases}1 & \text{if} \quad n=0,\\ x(x+1)...(x+n-1) & \text{if} \quad n\in N.\end{cases} \quad (6)$$

Dziok and Srivastava [5] defined also the linear operator

$$H(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)=z+\sum_{n=2}^{\infty}\Gamma_n a_n z^n, \quad (7)$$

where

$$\Gamma_n=\frac{(\alpha_1)_{n-1}...(\alpha_r)_{n-1}}{(\beta_1)_{n-1}...(\beta_S)_{n-1}(n-1)!}. \quad (8)$$

Abbadi and Darus [6] defined the analytic function

$$\Phi^m_{\lambda_1,\lambda_2}=z+\sum_{n=2}^{\infty}\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}z^n, \quad (9)$$

where $m\in\mathbb{N}_0=\{0,1,2,....\}$ and $\lambda_2\ge\lambda_1\ge 0$.

Using the Hadamard product (2), Alhindi and Darus [8, 9] has derived the generalized derivative operator $K^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)$ as follows

$$\varphi^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)=$$
$$z+\sum_{n=2}^{\infty}\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n z^n, \quad (10)$$

where Γ_n is as given in (8).

Now, after some calculations we obtain the following equation:

$$z(K^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z))'=$$
$$\alpha_1 K^m_{\lambda_1,\lambda_2}(\alpha_1+1,...,\alpha_r;\beta_1,...,\beta_S)f(z)$$
$$-\alpha_1 K^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z). \quad (11)$$

The linear operator $\mathcal{K}^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)$ includes many other operators which were mentioned earlier in [8, 9].

If we recall the generalized Bernardi-Libera-Livingston integral operator $j_c: A\to A$ (see [13, 14, 15]), defined by

$$j_c f(z)=\frac{c+1}{z^c}\int_0^z t^{c-1}f(t)dt; \qquad (v>-1; f\in A).$$

One can easily observe that

$$j_c f(z)=\mathrm{K}^0_{0,\lambda_2}(1+c,1;c+2)$$
$$=\mathrm{K}^1_{\lambda_1,0}(1+c,1;c+2)$$
$$=\mathrm{K}^2_{0,0}(1+c,1;c+2).$$

Owa [16] introduced the fractional derivative operator by these definitions (see also [17]).

Definition 1.4 [12]: The fractional integral operator of order μ is defined, for a function f, by

$$D_z^{-\mu}f(z)=\frac{1}{\Gamma(\mu)}\int_0^z\frac{f(\eta)}{(z-\eta)^{1-\mu}}d\eta; \qquad (\mu<0), \quad (12)$$

where *f(z)* is an analytic function in a simply connected region of the z-plane containing the origin, and the multiplicity of $(z-\eta)^{\mu-1}$ is removed by requiring $\log(z-\eta)$ to be real when $z-\eta>0$.

Definition 1.5 [16]: The fractional derivative operator of order μ is defined, for a function f, by

$$D_z^{\mu}f(z)=\frac{1}{\Gamma(1-\mu)}\frac{d}{dz}\int_0^z\frac{f(\eta)}{(z-\eta)^{\mu}}d\eta; \quad (0\le\mu<1), \quad (13)$$

where *f(z)* is an analytic function in a simply connected region of the z-plane containing the origin, and the multiplicity of $(z-\eta)^{-\mu}$ is removed same as the previous definition.

Definition 1.6 [16]: Using the assumption of Definition 1.5, the fractional derivative of order $n+\mu$ is defined, for a function f, by

$$D_z^{n+\mu}f(z)=\frac{d^n}{dz^n}D_z^{\mu}f(z); \quad (0\le\mu<1; n\in\mathbb{N}_0), \quad (14)$$

Srivastava and Owa [18] (see also [19-22]) used

these definitions of fractional calculus to define the linear operator $\Omega^{\mu}: A \to A$ as follows

$$\Omega^{\mu} f(z) = \Gamma(2-\mu) z^{\mu} D_z^{\mu} f(z); \quad (\mu \neq 2,3,4,...; f \in A). \tag{15}$$

By some calculations, we can find that

$$\Omega^{\mu} f(z) = \mathrm{K}^{0}_{0,\lambda_2}(2,1;2-\mu) = \mathrm{K}^{1}_{\lambda_1,0}(2,1;2-\mu) = \mathrm{K}^{2}_{0,0}(2,1;2-\mu).$$

Kim and Srivastava [23] investigated the class of functions $f \in A$ such that $\mathcal{L}(a,c) f(z) \in S^{*}(\alpha)$,

$$a\frac{\ell(a+1,c)f(z)}{\ell(a,c)f(z)} + 1 - a \prec \frac{1+(1-2\alpha)z}{1-z}. \tag{16}$$

After that, Dziok and Srivastava [5] introduced the class $V(r,s;A,B)$ of function f with some conditions, and studied its properties.

2. THE NEW CLASS $W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$

Let us denote by $W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$ the class of functions f of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n; \qquad (a_n \geq 0; n \in \mathbb{N} \setminus 1). \tag{17}$$

with the normalization

$$f(0) = f'(0) - 1 = 0, \tag{18}$$

which also satisfy the following condition:

$$\alpha_1 \frac{K^{m}_{\lambda_1,\lambda_2}(\alpha_1+1,\alpha_2,...,\alpha_r;\beta_1,...,\beta_s)f(z)}{K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)f(z)} + 1 - \alpha_1 \prec \frac{1+Az}{1+Bz}. \tag{19}$$

in terms of subordination, where $0 \leq B \leq -1$ and $-B \leq A < B$.

In this section, the coefficient estimate for the new class $W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$ is investigated. For this purpose, two lemmas are listed. Going back to(11), for a function of the form (17) and by considering $A=1$, $B=-1$, one can notice that the condition (19) is equivalent to

$$K^{m}_{\lambda_1,\lambda_2}(\alpha_1+,\alpha_2,...,\alpha_r;\beta_1,...,\beta_s)f(z) \in S(0). \tag{20}$$

Thus we can get the following Lemma.

Lemma 2.1 If $\alpha_i = \beta_j (j = 1,...,s)$ then $W^{m}_{\lambda_1,\lambda_2}(s;1,-1) \subset S(0)$.

By the definition of the class $W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$, we can get the following lemma.

Lemma 2.2 If $A_1 \leq A_2$ and $B_1 \geq B_2$, then

$$W^{m}_{\lambda_1,\lambda_2}(r,s;A_1,B_1) \subset W^{m}_{\lambda_1,\lambda_2}(r,s;A_2,B_2) \subset W^{m}_{\lambda_1,\lambda_2}(r,s;1,-1). \tag{21}$$

Theorem 2.3 Let f of the form), then $f \in W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$ if and only if

$$\sum_{n=2}^{\infty} ((B+1)n-(A+1)) \frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^{m}} \Gamma_n a_n \leq (B-A), \tag{22}$$

where Γ_n is is defined by (8).

Proof. Firstly, Let a function f be of the form (17) belongs to the class $W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$. Using the definition of subordination and by equation (19), we can write

$$\alpha_1 \frac{K^{m}_{\lambda_1,\lambda_2}(\alpha_1+1,\alpha_2,...,\alpha_r;\beta_1,...,\beta_s)f(z)}{K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)f(z)} + 1 - \alpha_1 = \frac{1+Aw(z)}{1+Bw(z)}.$$

After some calculation, and by consider that $w(0)=0$ and $|w(z)|<1$ we can write

$$\left| \frac{\alpha_1 \{K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z) - K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)\}}{\alpha_1 B K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z) - (A+(\alpha_1-1)B)K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)} \right| < 1, \tag{23}$$

where, for convenience, we write

$$K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z) = K^{m}_{\lambda_1,\lambda_2}(\alpha_1,\alpha_2,...,\alpha_r;\beta_1,...,\beta_s)f(z),$$

and

$$K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z) = K^{m}_{\lambda_1,\lambda_2}(\alpha_1+1,\alpha_2,...,\alpha_r;\beta_1,...,\beta_s)f(z).$$

Thus, by equation (13), one can write

$$\left|\frac{\sum_{n=2}^{\infty}(n-1)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n z^{n-1}}{(B-A)-\sum_{n=2}^{\infty}(Bn-A)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n z^{n-1}}\right|<1;\quad (z\in U),$$

where Γ_n is is defined by (8). If we put $z=r$ for $0\le r<1$, we conclude that

$$\sum_{n=2}^{\infty}(n-1)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n r^{n-1}$$

$$<(B-A)-\sum_{n=2}^{\infty}(Bn-A)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n r^{n-1}$$

which yields the assertion (22) by letting $r\to 1$.

Secondly, if the function f is of the form (17) and satisfying the condition (22). Then, we are supposed to prove that $f\in W^m_{\lambda_1,\lambda_2}(r,s;A,B)$.

Using the relation (23), then it is sufficient to prove that

$$\left|\alpha_1\left\{K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z)-K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)\right\}\right|$$

$$-\left|\alpha_1 BK^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z)-(A+(\alpha_1-1)B)K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)\right|. \quad (24)$$

If we put $|z|=r$ for $0\le r<1$, then we can write

$$\left|\alpha_1\{K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z)-K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)\}\right|-$$

$$\left|\alpha_1 BK^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1+1)f(z)-(A+(\alpha_1-1)B)K^{m,r,s}_{\lambda_1,\lambda_2}(\alpha_1)f(z)\right|$$

$$=\left|\sum_{n=2}^{\infty}(n-1)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n z^n\right|-$$

$$\left|(A-B)-\sum_{n=2}^{\infty}(Bn-A)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n z^n\right|$$

$$\le\sum_{n=2}^{\infty}(n-1)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n r^n$$

$$-\left((A-B)-\sum_{n=2}^{\infty}(Bn-A)\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n a_n r^n\right)$$

$$=r\left(\sum_{n=2}^{\infty}((B+1)n-(A+1))\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n r^{n-1}-(B-A)\right)$$

$$<\sum_{n=2}^{\infty}((B+1)n-(A+1))\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n-(B-A)\le 0. \quad (25)$$

Thus, $f\in W^m_{\lambda_1,\lambda_2}(r,s;A,B)$ and the proof is complete.

Based on Theorem 2.3, the following corollary can be derived.

Corollary 2.4 If a function f is of the form (17)and $f\in W^m_{\lambda_1,\lambda_2}(r,s;A,B)$, then we can write

$$a_n\le\frac{(B-A)}{C_n};\qquad (n=2,3,4,...),$$

where

$$C_n=((B+1)n-(A+1))\frac{(1+\lambda_1(n-1))^{m-1}}{(1+\lambda_2(n-1))^m}\Gamma_n;\quad (n=2,3,4,...).$$

The result is sharp, the functions f_n of the form:

$$f_n(z)=z-\frac{A-B}{C_n}z^n;\qquad (n=2,3,4,...),$$

are the extremal functions.

3. THE NEW CLASS $S^*(A,B)$

In this section,a new subclass $S^*(A,B)$ of analytic functionssatisfying the following condition is defined.

Let $f\in A$, then $f\in S^*(A,B)$ if and only if

$$\frac{z\left[K^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)f(z)\right]'}{K^m_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)f(z)}\prec\frac{1+Az}{1-Bz};\quad (26)$$

where $0\le A\le 1$ and $0\le B\le 1$.

In the proceeding theorem we will study the sufficient condition for functions *f*to be in the class $S^*(A,B)$, by applying the following lemma.

Lemma 3.1 [24] Let $w(z)$ be analytic in U with $w(0)=0$. If $|w(z)|$ attains its maximum value on the circle $|z|=r<1$ at a point z_0, then

$$z_0w'(z_0)=kw(z_0),$$

where k is a real number and $k\ge 1$.

Theorem 3.2 *Suppose* $f \in A$ *which satisfying*

$$\Re\left(1+\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]''}{\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}\right) < \frac{(1+A)^2+(A+B)}{(1+A)(1-B)}; \quad (z\in U), \tag{27}$$

for some $0 \le A \le 1$ and $0 \le B \le 1$, then $f \in S^*(A,B)$.

Proof. Let $w(z)$ is defined by

$$\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}{K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)} = \frac{1+Aw(z)}{1-Bw(z)}; \quad (Bw(z)\neq 1).$$

It follows that $w(0)=0$. Moreover, $w(z)$ is analytic and after some calculations we can write

$$1+\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]''}{\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'} = \frac{(1+Aw(z))^2+zw'(z)(A+B)}{(1-Bw(z))(1+Aw(z))}.$$

Thus

$$\Re\left(1+\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]''}{\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}\right) = \Re\left(\frac{(1+Aw(z))^2+zw'(z)(A+B)}{(1-Bw(z))(1+Aw(z))}\right) < \frac{(1+A)^2+(A+B)}{(1+A)(1-B)}.$$

Next, we prove that $|w(z)|<1$. Suppose that there exists a point $z_0 \in U$ such that

$$\max_{|z|\le|z_0|} |w(z)| = |w(z_0)| = 1.$$

Suppose $w(z_0)=e^{i\theta}$ and $z_0w'(z_0)=ke^{i\theta}; k\ge 1$, then by applying Lemma 3.1 we can get

$$\Re\left(1+\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]''}{\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}\right) = \Re\left(\frac{(1+Aw(z_0))^2+z_0w'(z_0)(A+B)}{(1+Aw(z_0))(1-Bw(z_0))}\right) = \Re\left(\frac{(1+Ae^{i\theta})^2+ke^{i\theta}(A+B)}{(1+Ae^{i\theta})(1-Be^{i\theta})}\right) = \Re\left(\frac{(1+A)^2+k(A+B)}{(1+A)(1-B)}\right) \ge \frac{(1+A)^2+(A+B)}{(1+A)(1-B)}.$$

We conclude that

$$\Re\left(1+\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]''}{\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}\right) \ge \frac{(1+A)^2+(A+B)}{(1+A)(1-B)}; \quad (z\in U),$$

which contradicts our assumption. Therefore, we can obtain that $|w(z)|<1$ for all $(z\in U)$ implies

$$\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}{K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)} \prec \frac{1+Az}{1-Bz};$$

where $0 \le A \le 1$ and $0 \le B \le 1$. Thus, the proof is complete.

Corollary 3.3 Suppose that $f \in S^*(A,0)$ then we can write

$$\left|\frac{z\left[K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)\right]'}{K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)f(z)}\right| - 1 < A.$$

Putting $A=1$ implies that $K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_S)$ is starlike.

4. CONCLUSIONS

In this paper, two new subclasses

$W^{m}_{\lambda_1,\lambda_2}(r,s;A,B)$ and $S^*(A,B)$ were introduced involving the operator $K^{m}_{\lambda_1,\lambda_2}(\alpha_1,...,\alpha_r;\beta_1,...,\beta_s)$. Moreover, by considering the subordination notion, certain properties of the two subclasses were investigated.

5. ACKNOWLEDGEMENTS

The above study was supported by UKM's grant: AP-2013-009 and DIP-2013-001.

6. REFERENCES

1. Stedall, J. A. *The Arithmetic of Infinitesimals*: John Wallis 1656. New York. (2004).
2. Aomoto, K. et al. *Theory of hypergeometric functions*. Springer, Berlin (2011).
3. Kummer, E. E. "Über die hypergeometrische Reihe....(Fortsetzung)." *Journal für die reine und angewandte Mathematik* 15: 127-172 (1836).
4. Riemann, B. *Theorie der Abel'schen functionen*. Georg Reimer, (1857).
5. Dziok, J. & H.M. Srivastava. Classes of analytic functions associated with the generalized hypergeometric function. *Applied Mathematics and Computation* 103: 1-13 (1999).
6. Al-Abbadi, M.H. & M. Darus. Differential subordination for new generalized derivative operator. *Acta Universitatis Apulensis* 20: 265-280 (2009).
7. Hohlov, Yu.E. Operators and operations in the class of univalent functions. *Izvestiya Vysshikh Uchebnykh ZavedeniiMatematik*10:83-89 (1978).
8. Alhindi, K.R. & M. Darus. Certain properties for a class of analytic functions associated with hypergeometric functions."*Revue D'analyse Numerique Et De Th´ Eorie De L'approximation´* 43(2): 93–102 (2014).
9. Alhindi, K.R. & M. Darus. New class of analytic functions associated with the generalized hypergeometric functions.*Acta Universitatis Apulensis* 37: 83-91 (2014).
10. Srivastava, H.M. & S. Owa. *Univalent Functions, Fractional Calculus, and Their Applications*. Ellis Horwood, New York, USA (1989).
11. Goodman, A.W. *Univalent functions*. Vol. 2. Mariner Publishing Company (1983).
12. Duren, P.L. *Univalent Functions*. Springer-Verlag, New York, USA (1983).
13. Bernardi, S.D. Convex and starlike univalent functions. *Transactions of the American Mathematical Society* 135: 429-446 (1969).
14. Libera, R.J. Some classes of regular univalent functions.*Proceedings of the American Mathematical Society* 16: 755-758 (1965).
15. Livingston, A.E. On the radius of univalence of certain analytic functions. *Proceedings of the American Mathematical Society* 17:352-357 (1966).
16. Owa, S. On the distortion theorems. *Kyungpook Mathematical Journal* 18: 53-59 (1978).
17. Srivastava, H.M. & S. Owa (Ed.).*Univalent Functions, Fractional Calculus, and Their Applications*. Wiley, New York, USA (1989).
18. Srivastava, H.M. & S. Owa. Some characterization and distortion theorems involving fractional calculus, generalized hypergeometric functions, Hadamard products, linear operators, and certain subclasses of analytic functions. *Nagoya Mathematical Journal* 106: 1-28 (1987).
19. Srivastava, H.M. & M.K. Aouf. A certain fractional derivative operator and its applications to a new class of analytic and multivalent functions with negative coeficients. *Journal of Mathematical Analysis and Applications* 171(1): 1-13 (1992).
20. Srivastava, H.M. & M.K. Aouf. A certain fractional derivative operator and its applications to a new class of analytic and multivalent functions with negative coeficients, II. *Journal of Mathematical Analysis and Applications* 192: 673-688 (1995).
21. Srivastava, H.M. & S. Owa (Ed.).*Current Topics in Analytic Function Theory*. World Scientific, Singapore (1992).
22. Dziok, J. Classes of analytic functions involving some integral operator.*Folia Fac Sci Nat Univ Purkyn Brun. Tech. Resoviensis* 20: 21-39 (1995).
23. Kim, Y.C. & H.M. Srivastava. Fractional integral and other linear operators associated with the Gaussian hypergeometric function.*Complex Variables, Theory and Application* 34: 293-312 (1997).
24. Jack, I.S. Functions starlike and convex of order K.*Journal of the London Mathematical Society* 3: 469–474 (1971).

Proceedings of the Pakistan Academy of Sciences 52 (3): 233–237 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Antibacterial and Hemolytic Activities of Brominated 2-Phenitidine Derivatives

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Abstract: The present study describes the antibacterial and hemolytic activities of some brominated 2-phenitidine derivatives. These synthetic molecules were screened against selected bacterial strains, i.e., gram positive and negative using streptomycin as control. The cytotoxicity of these molecules was also tested using triton as reference. The study revealed that the analyzed molecules displayed moderate potential as antibacterial agents; however, these might serve as valued hemolytic agents.

Keywords: 2-phenitidine derivatives, antibacterial agents, cytotoxicity, hemolytic agents

1. INTRODUCTION

Currently, pathogenic microbes are posing life threatening infections to human health in all over the world. The microorganisms are contributing morbidity and mortality in immune-compromised patients [1]. Especially, the increasing resistance of bacterial strains is a challenge as public health crises and has to overcome [2]. One of most noteworthy Gram positive bacterial starins, *Staphylococcus aureus* is commensal and part of human microbiota. This strain has potential to establish broad spectrum interactions with human host. It is capable of colonizing at multiple body sites. It is known to ground various hospital allied diseases including wide range of skin related and tissue infections to acute issues such as endocarditic and bacteraemia. It is also recognized for food poisoning leads to gastroenteritis as well as invader to defense mechanism. It has characteristic to acquire resistance to antimicrobial agents [3-8]. An enterohemorrhagic strain of *Escherichia coli* (O157:H7) is found ubiquitously on farm lands. Such pathogens breed in gastrointestinal tract of healthy animals like cattle. In this way, *E. coli* approaches the human food chains *via* their manure. This strain produces shiga-like toxin and become causative agent of several diseases including bloody stools and hemolytic uremic syndrome [9]. This strain is showing resistance to antibiotics which is a pressing global problem nowadays [10].

Sulfonamides constitute a variety of artificial antibiotics being applied in veterinary medicine as growth promoters and for the treatment of bacterial infections such as digestive and of respiratory tract [11-12]. A variety of sulfonamides are available for therapy and prevention of specific bacterial diseases related to poultry. Sulfonamide derivatives are known for competitive antagonists of *p*-aminobenzoic acid, which is precursor of folic acid both in protozoan and bacterial cells. Folic acid a coenzyme is destined to produce nucleic acids in these cells. Therefore, sulfonamides are characterized as hindering agents of bacterial activity [13, 14]. Sulfonamides possess a bacteriostatic effect. These are useful in therapy of bacterial infections for instance eye infection and

Received, May 2014; Accepted, April 2015
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urinary tract infection [15]. This class of compounds possesses diverse pharmacological properties like antibacterial. In this way it found a key position in medicinal chemistry [16-17]. Hence, the main focus of our study was to explore some potential inhibitors of pathogenic bacteria and possible hemolytic agents to save the humans from peril of microbes.

2. MATERIALS AND METHODS

2.1. Chemistry

We have previously reported the synthesis and structural characterization of studied brominated 2-phenetidine derivatives, **1-13** (Fig. 1) [18]. Those earlier synthesized samples were subjected to current study.

2.2. Microbial Strains

Samples were tested against microbial strains in accordance with the reported method [19].

2.3. Disc Diffusion Method

The antibacterial activity was employed on compounds by the reported disc diffusion method [20].

2.4. Hemolytic Activity

The reported method [19, 21] was used to study the

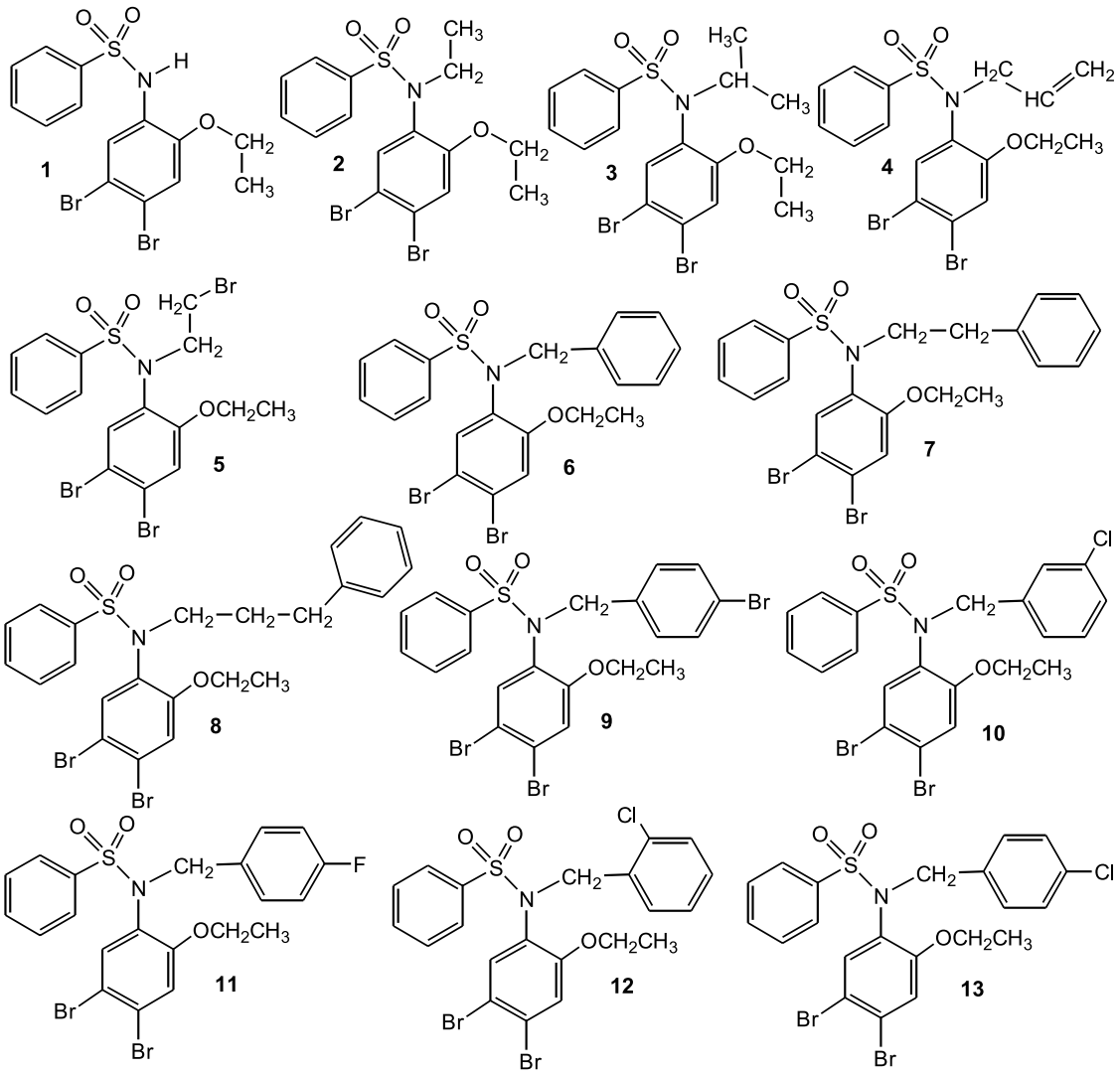

Fig. 1. Structures of brominated 2-phenetidine derivatives, **6a-m.**

hemolytic activity of the compounds. Samples were incubated at 37°C. Triton X-100 (0.1% v/v) was taken as positive control and phosphate buffer saline (PBS) was taken as negative control. Absorbance of compounds was observed at 576 nm using μ Quant (Bioteck, USA). The % RBCs lysis for each sample was calculated [21].

3. RESULTS AND DISCUSSION

3.1. Anti-bacterial Activity

Amongst the studied molecules, *N*-(4,5-dibromo-2-ethoxyphenyl)benzenesulfonamide (**1**), *N*-ethyl-*N*-(4,5-dibromo-2-ethoxyphenyl) benzenesulfonamide (**2**) and *N*-benzyl-*N*-(4,5-dibromo-2-ethoxyphenyl) benzenesulfonamide (**6**) exhibited antibacterial activity against the selected bacterial strains. The descending order of compounds activity was observed as **2** > **1** > **6** against the chosen strains of bacteria relative to standard streptomycin sulfate used throughout the assay. The greater activity of **2** as compared to **1** could be attributed to the incorporated ethyl moiety at *N*-atom in former which was not present in latter un-substituted molecule. Similarly, promising activity was shown by **6** which might be ascribed to the substitution of benzyl group on nitrogen atom in this molecule. The substantial data is evident of better activity demonstrated by compounds **1** and **2** against gram positive strain but compound **6** was comparatively more active against gram negative species *Pasturella multocida* and *Escherichia coli*. Remaining ten compounds possess very low/ no activity against bacterial species under study (Table 1).

3.2. Hemolytic Activity

N-(4-chlorobenzyl-*N*-(4,5-dibromo-2-ethoxyphenyl)benzenesulfonamide (**13**) gave high hemolytic activity value (7.54±0.340 %) but much below the positive control. It is interpreted from data that the enhanced activity of compound **13** is imparted by 4-Chlorobenzyl group. Lowest hemolytic activity was recorded by *N*-isopropyl-*N*-(4,5-dibromo-2-ethoxyphenyl) benzenesulfonamide (**3**) (0.10±0.031 %) followed

Table 1 Antibacterial against the selected bacterial species and hemolytic activity by using the human erythrocytes of the compounds.

Sr. No	Sample	*S. aureus*	*B. Subtilis*	*P. multocida*	*E. coli*	Hemolytic activity (Mean % ± S.D)
1	**1**	16	18	18	18	0.37±0.031
2	**2**	20	22	20	20	0.39±0.000
3	**3**	-	-	-	-	0.10±0.031
4	**4**	-	-	-	-	0.26±0.062
5	**5**	-	-	-	-	0.50±0.278
6	**6**	14	14	16	16	0.91±0.062
7	**7**	-	-	-	-	0.13±0.062
8	**8**	-	-	-	-	0.85±0.093
9	**9**	-	-	-	-	0.96±0.000
10	**10**	-	-	-	-	0.74±0.062
11	**11**	-	-	-	-	3.43±0.093
12	**12**	-	-	-	-	3.41±0.371
13	**13**	-	-	-	-	7.54±0.340
14	Streptomycin	28	34	34	30	
15	PBS					0.00±0.000
16	Triton (toxicity)					100±0.000

by *N*-phenylethyl-*N*-(4,5-dibromo-2-ethoxyphenyl) benzenesulfonamide (**7**) (0.13±0.062 %) but higher than the negative control phosphate-buffered saline (PBS) (Table 1). The compounds which show highest hemolytic activity might be considered for the antitumor determination and the molecules which exhibit less hemolytic activity could be targeted as valued antibacterial compounds for further studies. On the whole, the studied brominated 2-phenitidine derivatives showed much little membrane destability within the range of the given minimum and maximum values and hence have pretty low cytotoxic impact. These molecules are thus valued for further studies.

4. CONCLUSIONS

Ever growing prevalence of resisting microbial strains towards sulfonamides inculcated the need for preparation of structurally modified products owing infection combating tendency. Among the studied brominated 2-phenitidine derivatives, a few demonstrated promising activity against both bacterial strains. The compounds **1**, **2** and **6** were found more active relative to other molecules in the series. The data also showed that all the molecules were active in hemolytic analysis. The molecule **13** exhibited relatively greater cytotoxic impact and molecules **3** and **7** have very low cytotoxicity. Overall, the studied molecules are less cytotoxic and hence are very appropriate entities for further studies.

6. REFERENCES

1. Rahman, M.M., M.M.I. Sheikh, S.A. Sharmin, M.S. Islam, M.A. Rahman, M.M. Rahman, & M.F. Alam. Antibacterial activity of leaf juice and extracts of *Moringa oleifera* Lam. against some human pathogenic bacteria. *Journal of Natural Sci*ence 8(2): 219-227 (2009).
2. Cushnie, T.P.T. & A.J. Lamb. Antimicrobial Activity of flavonoids. *International Journal of Antimicrobial Agents* 26: 343–356 (2005).
3. Rahman, M.M., A. Richardson & M. Sofian-Azirun**.** Antibacterial activity of propolis and honey against *Staphylococcus aureus* and *Escherichia coli*. *African Journal of Microbiology Research* 4(16): 1872-1878 (2010).
4. Barbagelata, M.S., L.P. Alvarez, C.M. Dotto, S.M. Lattar, D.O. Sordelli, & F.R. Buzzola. Aromatic Compound-dependent *Staphylococcus aureus* is safe in a nasal colonization leukopenic murine model. *International Journal of Microbiology* 2012: 1-7 (2012).
5. Pardo, L., V. Machado, M. Mollerach, M.I. Mota, L.P.N. Tuchscherr, P. Gadea, N. Gardella, D.O. Sordelli, M. Vola, F. Schelotto & G. Varela. Characteristics of community-associated methicillin resistance *Staphylococcus aureus* (CA-MRSA) strains isolated from skin and soft-tissue infections in uruguay. *International Journal of Microbiology* 2009: 1-5 (2009).
6. Morandi, S., M. Brasca, C. Andrighetto, A. Lombardi, and R. Lodi. Phenotypic and genotypic characterization of *Staphylococcus aureus* strains from Italian dairy products. *International Journal of Microbiology* 2009: 1-7 (2009).
7. Singh, K. & V.K. Singh. Expression of four methionine sulfoxide reductases in *Staphylococcus aureus*. *International Journal of Microbiology* 2012: 1-8 (2012).
8. Rajam, G., G.M. Hammons, G.M. Carlone, J.S. Sampson & E.W. Ades. A novel innate immune-enhancement strategy combined with IVIG rescues mice from fatal *Staphylococcus aureus* septicemia. *International Journal of Microbiology* 2011: 1-5 (2011).
9. Piper, C. P.G. Casey, C. Hill, P.D. Cotter & R.P. Ross. The Lantibiotic Lacticin 3147 prevents systemic spread of *Staphylococcus aureus* in Murine infection model. *International Journal of Microbiology* 2012: 1-6 (2012).
10. Duffitt, A.D. R.T. Reber, A. Whipple, & C. Chauret. Gene expression during survival of *Escherichia coli* O157:H7 in soil and water. *International Journal of Microbiology* 2011: 1-12 (2011).
11. Liu, R.P. He, Z. Li, & R. Li. Simultaneous determination of 16 sulfonamides in animal feeds by UHPLC-MS-MS. *Journal of Chromatographic Science* 49: 640-646 (2011).
12. Shao, B., D. Dong, Y. Wu, J. Hu, J. Meng, X. Tu, & S. Xu. Simultaneous determination of 17 sulfonamide residues in porcine meat, kidney and liver by solid-phase extraction and liquid chromatography-tandem mass spectrometry. *Analytica Chimica Acta* 546: 174-181 (2005).
13. Siddiki, A.Z., M.J. Karim, & E.H. Chawdhury. Sulfonamide resistance in chicken coccidiosis: A clinico-pathological study. *Bangladesh Journal of Microbiol*ogy 25(1): 60-64 (2008).
14. Ali, E.S.H., F.I. Nassar, A.M. Badawi & S.A. Afify. Physical properties and biological applications

of novel substituted biphenyl-sulfonamides. *International Journal of Genetics and Molecular Biology* 2(5): 78-91 (2010).

15. Nagaraja, P., S.D. Naik, A.K. Shrestha & A. Shivakumar. A sensitive spectrophotometric method for the determination of sulfonamides in pharmaceutical preparations. *Acta Pharm* 57: 333-342 (2007).
16. Owa, T., T. Okauchi, K. Yoshimatsu., N.H. Sugi, Y. Ozawa, T. Nagasu, N. Koyanagi, T. Okabe, K. Kitoh & H. Yoshino. A focused compound library of novel *N-(*7-Indolyl) benzenesulfonamides for the discovery of potent cell cycle inhibitors. *Bioorganic and Medicinal Chemistry Letters* 10: 1223-1226 (2000).
17. Dauban, P. & R.H. Dodd. Intramolecular Bromine-catalyzed Aziridination: a new direct access to cyclic sulfonamides. *Tetrahedron Letters* 42: 1037-1040 (2001).
18. Abbasi, M.A., A. Saeed, A. Rehman, K.M. Khan, M. Ashraf, & S.A. Ejaz. Synthesis of brominated 2-phenitidine derivatives as valuable inhibitors of cholinesterases for the treatment of alzheimer's disease. *Iranian Journal of Pharmaceutical Research* 13 (1): 87-94 (2014).
19. Powell, W.A., C.M. Catranis & C.A. Maynard. Design of self-processing antimicrobial peptide for plant protection. *Letters in Applied Microbiology* 31(2): 163-165 (2000).
20. CLSI (The clinical Laboratory Standard Institute). Agar dilution and disk diffusion susceptibility testing of campylobacter spp. *Journal of Clinical Microbiology* 45(8): 2758-2759 (2010).
21. Sharma, P. & J.D. Sharma. *In vitro* hemolysis of human erythrocytes by plant extracts with antiplasmodial activity. *Journal of Ethnophamacology* 74: 239-243 (2001).

Proceedings of the Pakistan Academy of Sciences 52 (3): 239–246 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Some New Nonlinear Dynamical Integral Inequalities with Applications on Time Scales

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Abstract: We investigated some non-linear dynamical integral inequalities on time scales, providing explicit bounds on unknown functions. These types of inequalities, consolidate and develop some existing well-known inequalities, and can be utilized in qualitative theory of dynamical equations.

2010 Mathematics Subject Classification. 26D15.

Keywords: Integral inequalities, Gronwall integral inequalities, time scales

1. INTRODUCTION

Substantial attention has been given in recent years to the theory of dynamic equations on time scales, which was introduced by Hilger's landmark [1].For instance see [2] and the references cited therein. Therefore, several researchers have discussed numerous aspects of dynamic equations on time scales. Dynamic inequalities play a significant role in the qualitative study of dynamic equations [3-7]. Various researchers have been studied integral inequalities of different types on the time scales [8].

The primary objective of our work is to analyze some non-linear dynamic integral inequalities on time scales which not only generalized few existing well known results. But this work also came handy to determine the explicit bounds of the solutions of particular dynamical equations on time scales. Along with we provide some continuous and discrete inequalities for different time scales. As a whole in this work, we have deeply studied time scales and time scales essentials.$\mathbb{T}$ is considered to be a time scales and $\mathrm{C_{rd}}(\mathbb{T})$ denotes the set of all rd-continuous functions defined on $\mathbb{T}$. For convenience throughout the whole discussion we assume that $\mathfrak{t}_0 \in \mathbb{T}$. The work is structured as follows: Non-linear dynamic inequalities on time scales are given in section 2. In section 3 some applications to illustrate our main results are given.

2. MAIN RESULTS

In this paper, the following non-linear dynamic integral inequalities would be under consideration.

$$[\theta(\mathfrak{t})]^{\mathrm{p}} \leq \tilde{\mathrm{a}}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} \{g(\mathfrak{r})[\theta(\mathfrak{r})]^{\mathrm{q}} + h(\mathfrak{r})[\theta(\mathfrak{r})]^{r} + j(\mathfrak{r})[\theta(\mathfrak{r})]^{s}\}\Delta\mathfrak{r} \tag{1}$$

$$[\theta(\mathfrak{t})]^{\mathrm{p}} \leq \tilde{\mathrm{a}}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} \{g(\mathfrak{r})[\theta^{\sigma}(\mathfrak{r})]^{\mathrm{q}} + h(\mathfrak{r})[\theta^{\sigma}(\mathfrak{r})]^{r} + j(\mathfrak{r})[\theta^{\sigma}(\mathfrak{r})]^{s}\}\Delta\mathfrak{r} \tag{2}$$

$$[\theta(\mathfrak{t})]^{\mathrm{p}} \leq \tilde{\mathrm{a}}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} K(\mathfrak{t},\mathfrak{r})\{g(\mathfrak{r})[\theta(\mathfrak{r})]^{\mathrm{q}} + h(\mathfrak{r})[\theta(\mathfrak{r})]^{r} + j(\mathfrak{r})[\theta(\mathfrak{r})]^{s}\}\Delta\mathfrak{r} \tag{3}$$

Received, December 2014; Accepted, April 2015
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$$[\Theta(\mathfrak{t})]^{\mathrm{p}} \leq \tilde{\mathrm{a}}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} K(\mathfrak{t},\mathfrak{r})\{g(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{\mathrm{q}} + h(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{r} + j(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{s}\}\Delta\mathfrak{r} \tag{4}$$

$$[\Theta(\mathfrak{t})]^{\mathrm{p}} \leq \rho + \int_{\mathfrak{t}_0}^{\mathfrak{t}} \{g(\mathfrak{r})[\Theta(\mathfrak{r})]^{\mathrm{q}} + h(\mathfrak{r})[\Theta(\mathfrak{r})]^{r} + j(\mathfrak{r})[\Theta(\mathfrak{r})]^{s}\}\Delta\mathfrak{r} \tag{5}$$

provided that $\tilde{\mathrm{a}}, \mathfrak{b}, g, h, j, \Theta$: $\mathbb{T}^{K} \to \boldsymbol{R}^{+}(= [0,\infty)$ are rd-continuous functions, and K: $\mathbb{T}$ x $\mathbb{T}^{\mathrm{k}} \to$ $\mathbf{R}^{+}(= [0,\infty))$ is a continuous function.

Lemma 1. [2,p.255] Let n, y $\in C_{rd}(\mathbb{T})$ and $m \in \boldsymbol{R}_{+}(\mathbb{T})$, then

$$y^{\Delta}(y) \leq m(\mathfrak{t})y(\mathfrak{t}) + n(\mathfrak{t}), \quad \mathfrak{t} \in \mathbb{T},$$

implies

$$y(\mathfrak{t}) \leq y(\mathfrak{t}_0)e_m(\mathfrak{t},\mathfrak{t}_0) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} e_m\big(\mathfrak{t},\sigma(\mathfrak{r})\big)\Delta\mathfrak{r}, \quad \mathfrak{t} \in \mathbb{T}.$$

Lemma 2. [4] Let us consider $0 \leq \tilde{\mathrm{a}}$, $\mathrm{p} \geq \mathrm{q} \geq 0$, and $0 \neq \mathrm{p}$ and if we take $0 \leq \mathrm{k}$ then

$$\tilde{\mathrm{a}}^{\frac{\mathrm{p}}{\mathrm{q}}} \leq \frac{\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}} + \frac{\mathrm{p}-\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}}{\mathrm{p}}}.$$

Lemma 3. [2,p.46] Suppose K: $\mathbb{T}$ x $\mathbb{T}^{\mathrm{k}} \to$ **R**is continuous at $(\mathfrak{t},\mathfrak{t})$,then for any$\epsilon > 0, \exists\ \mathfrak{U}$ neighbourhood of $\mathfrak{t} \in \mathbb{T}^{\mathrm{k}}$, which does not depend on $\mathfrak{r} \in [\mathfrak{t}_0, \sigma(\mathfrak{t})]$, s.t

$$\left|K(\sigma(\mathfrak{t}),\mathfrak{r}) - K(s,\mathfrak{r}) - K_1^{\Delta}(\mathfrak{t},\mathfrak{r})(\sigma(\mathfrak{t}) - s)\right| \leq \epsilon|\sigma(\mathfrak{t}) - s|, \qquad s \in \mathfrak{U}$$

provided that $\mathrm{K}_1^{\Delta}(\mathfrak{t},.)$ (the derivative of K w.r.t the first variable) is rd-continuous on $[\mathfrak{t}_0, \sigma(\mathfrak{t})], \mathfrak{t} > \mathfrak{t}_0$, then

$$\mathfrak{d}(\mathfrak{t}) := \int_{\mathfrak{t}_0}^{\mathfrak{t}} K(\mathfrak{t},\mathfrak{r})\Delta\mathfrak{r} \Longrightarrow \mathfrak{d}^{\Delta}(\mathfrak{t}) = \int_{\mathfrak{t}_0}^{\mathfrak{t}} K_1^{\Delta}(\mathfrak{t},\mathfrak{r})\Delta\mathfrak{r} + K(\sigma(\mathfrak{t}),\mathfrak{t})$$

Before stating main results, some symbolic representation for the sake of brevity and compact understanding are given as:

$$\begin{aligned}\mathfrak{H}(\mathfrak{t}) := {} & g(\mathfrak{t})\left\{\frac{\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}(\mathfrak{t}) + \frac{\mathrm{p}-\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}}{\mathrm{p}}}\right\} + h(\mathfrak{t})\left\{\frac{r}{\mathrm{p}} k^{\frac{r-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}(\mathfrak{t}) + \frac{\mathrm{p}-r}{\mathrm{p}} k^{\frac{r}{\mathrm{p}}}\right\} \\ & + j(\mathfrak{t})\left\{\frac{s}{\mathrm{p}} k^{\frac{s-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}(\mathfrak{t}) + \frac{\mathrm{p}-s}{\mathrm{p}} k^{\frac{s}{\mathrm{p}}}\right\}\end{aligned} \tag{6}$$

$$\begin{aligned}\tilde{\mathfrak{H}}(\mathfrak{t}) := {} & g(\mathfrak{t})\left\{\frac{\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}^{\sigma}(\mathfrak{t}) + \frac{\mathrm{p}-\mathrm{q}}{\mathrm{p}} k^{\frac{\mathrm{q}}{\mathrm{p}}}\right\} + h(\mathfrak{t})\left\{\frac{r}{\mathrm{p}} k^{\frac{r-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}^{\sigma}(\mathfrak{t}) + \frac{\mathrm{p}-r}{\mathrm{p}} k^{\frac{r}{\mathrm{p}}}\right\} \\ & + j(\mathfrak{t})\left\{\frac{s}{\mathrm{p}} k^{\frac{s-\mathrm{p}}{\mathrm{p}}} \tilde{\mathrm{a}}^{\sigma}(\mathfrak{t}) + \frac{\mathrm{p}-s}{\mathrm{p}} k^{\frac{s}{\mathrm{p}}}\right\}\end{aligned} \tag{7}$$

$$\mathfrak{X}(\mathfrak{t}) := \mathfrak{b}(\mathfrak{t})\left\{\frac{\mathrm{q}}{\mathrm{p}} g(\mathfrak{t}) k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}} + \frac{r}{\mathrm{p}} h(\mathfrak{t}) k^{\frac{r-\mathrm{p}}{\mathrm{p}}} + \frac{s}{\mathrm{p}} j(\mathfrak{t}) k^{\frac{s-\mathrm{p}}{\mathrm{p}}}\right\} \tag{8}$$

$$\tilde{\mathfrak{X}}(\mathfrak{t}) := \mathfrak{b}^{\sigma}(\mathfrak{t})\left\{\frac{\mathrm{q}}{\mathrm{p}} g(\mathfrak{t}) k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}} + \frac{r}{\mathrm{p}} h(\mathfrak{t}) k^{\frac{r-\mathrm{p}}{\mathrm{p}}} + \frac{s}{\mathrm{p}} j(\mathfrak{t}) k^{\frac{s-\mathrm{p}}{\mathrm{p}}}\right\} \tag{9}$$

$$\widetilde{\mathfrak{H}}(\mathfrak{t}) \coloneqq \mathfrak{H}(\mathfrak{t})K(\sigma(\mathfrak{t}),\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} K^{\Delta}(\mathfrak{t},\mathfrak{r})\{\mathfrak{H}(\mathfrak{r}) + \beta(\mathfrak{r})\mathfrak{X}(\mathfrak{r})\}\Delta\mathfrak{r}. \tag{10}$$

$$\widetilde{\mathfrak{E}}(\mathfrak{t}) \coloneqq \frac{\mathfrak{q}.\, g(\mathfrak{t})k^{\frac{\mathfrak{q}}{\mathfrak{p}}} + r.\, h(\mathfrak{t})k^{\frac{r}{\mathfrak{p}}} + s.\, j(\mathfrak{t})k^{\frac{s}{\mathfrak{p}}}}{k\mathfrak{p}} \tag{11}$$

$$\widetilde{\mathfrak{D}}(\mathfrak{t}) \coloneqq \frac{\rho - k}{k\mathfrak{p}}\left[\mathfrak{q}.\, g(\mathfrak{t})k^{\frac{\mathfrak{q}}{\mathfrak{p}}} + r.\, h(\mathfrak{t})k^{\frac{r}{\mathfrak{p}}} + s.\, j(\mathfrak{t})k^{\frac{s}{\mathfrak{p}}}\right] + g(\mathfrak{t})k^{\frac{\mathfrak{q}}{\mathfrak{p}}} + h(\mathfrak{t})k^{\frac{r}{\mathfrak{p}}} + j(\mathfrak{t})k^{\frac{s}{\mathfrak{p}}} \tag{12}$$

$$\widehat{\mathfrak{Y}}(\mathfrak{t}) \coloneqq \widetilde{\mathfrak{H}}(\mathfrak{t})K(\sigma(\mathfrak{t}),\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} K^{\Delta}(\mathfrak{t},\mathfrak{r})\widetilde{\mathfrak{H}}(\mathfrak{r})\Delta\mathfrak{r}. \tag{13}$$

$$\tilde{\mathfrak{z}}(\mathfrak{t}) \coloneqq y^{\sigma}(\mathfrak{t})K(\sigma(\mathfrak{t}),\mathfrak{t})\widetilde{\mathfrak{X}}(\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} K^{\Delta}(\mathfrak{t},\mathfrak{r})y^{\sigma}(\mathfrak{r})\widetilde{\mathfrak{X}}(\mathfrak{r})\Delta\mathfrak{r}. \tag{14}$$

$$\mathfrak{X}_1(\mathfrak{t}) \coloneqq \frac{\widetilde{\mathfrak{X}}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})}{1 - \mu(\mathfrak{t})\widetilde{\mathfrak{X}}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})} \tag{15}$$

$$\widetilde{\mathfrak{Y}}(\mathfrak{t}) \coloneqq \frac{\widehat{\mathfrak{Y}}(\mathfrak{t})}{1 - \mu(\mathfrak{t})\widehat{\mathfrak{Y}}(\mathfrak{t})} \tag{16}$$

$$\widetilde{\mathfrak{X}}(\mathfrak{t}) \coloneqq \mathfrak{X}(\mathfrak{t})K(\sigma(\mathfrak{t}),\mathfrak{t}) \tag{17}$$

The main result of this paper is as follow.

Theorem 1. Let $\tilde{a}, \mathfrak{b}, g, h, j, \Theta: \mathbb{T}^{K} \to \mathbf{R}$ be non-negative rd-continuous functions and $(0 \neq)\mathfrak{p}, \mathfrak{q}, r, s$ are constant with $\mathfrak{p} \geq \mathfrak{q}$; $\mathfrak{p} \geq r$ and $\mathfrak{p} \geq s$ such that (1) holds, then

$$\Theta(\mathfrak{t}) \leq \sqrt[\mathfrak{p}]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} e_{\mathfrak{X}}(\mathfrak{t},\sigma(\mathfrak{r}))\mathfrak{H}(\mathfrak{r})\Delta\mathfrak{r}}, \tag{18}$$

where $\mathfrak{H}$ and $\mathfrak{X}$ are defined above in equation (6) and (8), respectively.

Proof. Define a function

$$z(\mathfrak{t}) = \int_{\mathfrak{t}_0}^{\mathfrak{t}} \{g(\mathfrak{r})[\Theta(\mathfrak{r})]^{\mathfrak{q}} + h(\mathfrak{r})[\Theta(\mathfrak{r})]^{r} + j(\mathfrak{r})[\Theta(\mathfrak{r})]^{s}\}\Delta\mathfrak{r},$$

so that $z(\mathfrak{t}_0) = 0$ and z is non decreasing

$$z^{\Delta}(\mathfrak{t}) = g(\mathfrak{t})[\Theta(\mathfrak{t})]^{\mathfrak{q}} + h(\mathfrak{t})[\Theta(\mathfrak{t})]^{r} + j(\mathfrak{t})[\Theta(\mathfrak{t})]^{s} \tag{19}$$

$$\Rightarrow \Theta(\mathfrak{t}) \leq \sqrt[\mathfrak{p}]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})} \tag{20}$$

Direct application of Lemma 2 and inequality (20) in (19) yield*:*

$$z^{\Delta}(\mathfrak{t}) \leq g(\mathfrak{t})[\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})]^{\frac{\mathfrak{q}}{\mathfrak{p}}} + h(\mathfrak{t})[\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})]^{\frac{r}{\mathfrak{p}}} + j(\mathfrak{t})[\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})]^{\frac{s}{\mathfrak{p}}}$$

$$\leq g(\mathfrak{t})\left[\frac{\mathfrak{q}}{\mathfrak{p}}k^{\frac{\mathfrak{q}-\mathfrak{p}}{\mathfrak{p}}}\left(\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})\right) + \frac{\mathfrak{p}-\mathfrak{q}}{\mathfrak{p}}k^{\frac{\mathfrak{q}}{\mathfrak{p}}}\right] + h(\mathfrak{t})\left[\frac{r}{\mathfrak{p}}k^{\frac{r-\mathfrak{p}}{\mathfrak{p}}}\left(\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})z(\mathfrak{t})\right) + \frac{\mathfrak{p}-r}{\mathfrak{p}}k^{\frac{r}{\mathfrak{p}}}\right]$$

$$+j(\mathfrak{t})\left[\frac{s}{\mathrm{p}}k^{\frac{s-\mathrm{p}}{\mathrm{p}}}\left(\tilde{\mathrm{a}}(\mathfrak{t})+\mathfrak{b}(\mathfrak{t})z(\mathfrak{t})\right)+\frac{\mathrm{p}-s}{\mathrm{p}}k^{\frac{s}{\mathrm{p}}}\right]\ \leq\ \mathfrak{H}(\mathfrak{t})+z(\mathfrak{t})\mathfrak{X}(\mathfrak{t}). \tag{21}$$

Application of Lemma 1 yields:

$$z(\mathfrak{t})\leq\int_{\mathfrak{t}_0}^{\mathfrak{t}}e_{\mathfrak{X}}\left(\mathfrak{t},\sigma(\mathfrak{r})\right)\mathfrak{H}(\mathfrak{r})\Delta\mathfrak{r}.$$

Hence, the result.

Theorem 2. Let $\tilde{\mathrm{a}},\mathfrak{b},\mathrm{g},\mathrm{h},\mathrm{j},\Theta:\mathbb{T}^{\mathrm{K}}\rightarrow\mathbf{R}$ be non-negative rd-continuous functions and $(0\neq)\mathrm{p},\mathrm{q},\mathrm{r},\mathrm{s}$ are constants with $\mathrm{p}\geq\mathrm{q};\ \mathrm{p}\geq\mathrm{r}$ and $\mathrm{p}\geq\mathrm{s}$. If $1>\mu(\mathfrak{t})\tilde{\mathfrak{X}}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})$ s.t (2) satisfied,then

$$\Theta(\mathfrak{t})\leq\sqrt[\mathrm{p}]{\tilde{\mathrm{a}}(\mathfrak{t})+\mathfrak{b}(\mathfrak{t})\int_{\mathfrak{t}_0}^{\mathfrak{t}}\left(\tilde{\mathfrak{H}}\oplus\mathfrak{X}_1\right)(\mathfrak{r})\,\Delta\mathfrak{r}}, \tag{22}$$

provided that ω is defined in the proof.Functions $\tilde{\mathfrak{H}}$ and $\mathfrak{X}_1$ are defined by equation (6) and (15), respectively.

Proof. Define a function

$$\omega(\mathfrak{t})\ =\int_{\mathfrak{t}_0}^{\mathfrak{t}}\{g(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{\mathrm{q}}+h(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{r}+j(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{s}\}\Delta\mathfrak{r},$$

so that $\omega(\mathfrak{t}_0)=0$ and ω is non decreasing

$$\omega^{\Delta}(\mathfrak{t})=g(\mathfrak{t})[\Theta^{\sigma}(\mathfrak{r})]^{\mathrm{q}}+h(\mathfrak{t})[\Theta^{\sigma}(\mathfrak{r})]^{r}+j(\mathfrak{t})[\Theta^{\sigma}(\mathfrak{r})]^{s} \tag{23}$$

$$\Longrightarrow\Theta(\mathfrak{t})\leq\sqrt[\mathrm{p}]{\tilde{\mathrm{a}}(\mathfrak{t})+\mathfrak{b}(\mathfrak{t})\omega(\mathfrak{t})} \tag{24}$$

Direct application of Lemma 2 and inequality (24) in (23) yield:

$$\omega^{\Delta}(\mathfrak{t})\leq g(\mathfrak{t})[\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})]^{\frac{\mathrm{q}}{\mathrm{p}}}+h(\mathfrak{t})[\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})]^{\frac{r}{\mathrm{p}}}+j(\mathfrak{t})[\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})]^{\frac{s}{\mathrm{p}}}$$

$$\leq g(\mathfrak{t})\left[\frac{\mathrm{q}}{\mathrm{p}}k^{\frac{\mathrm{q}-\mathrm{p}}{\mathrm{p}}}\left(\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})\right)+\frac{\mathrm{p}-\mathrm{q}}{\mathrm{p}}k^{\frac{\mathrm{q}}{\mathrm{p}}}\right]+h(\mathfrak{t})\left[\frac{r}{\mathrm{p}}k^{\frac{r-\mathrm{p}}{\mathrm{p}}}\left(\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})\right)+\ \ \frac{\mathrm{p}-r}{\mathrm{p}}k^{\frac{r}{\mathrm{p}}}\right]$$

$$+j(\mathfrak{t})\left[\frac{s}{\mathrm{p}}k^{\frac{s-\mathrm{p}}{\mathrm{p}}}\left(\tilde{\mathrm{a}}^{\sigma}(\mathfrak{t})+\mathfrak{b}^{\sigma}(\mathfrak{t})\omega^{\sigma}(\mathfrak{t})\right)+\frac{\mathrm{p}-s}{\mathrm{p}}k^{\frac{s}{\mathrm{p}}}\right]\leq\ \tilde{\mathfrak{H}}(\mathfrak{t})+\omega^{\sigma}(\mathfrak{t})\tilde{\mathfrak{X}}(\mathfrak{t}). \tag{25}$$

It is observed that:

$$\tilde{\mathfrak{X}}(\mathrm{t})\omega^{\sigma}(\mathrm{t})=\frac{\mathfrak{X}_1(\mathrm{t})}{1+\mu(\mathrm{t})\mathfrak{X}_1(\mathrm{t})}$$

$\Longrightarrow$ (25) becomes

$$\omega^{\Delta}(\mathfrak{t})\leq\tilde{\mathfrak{H}}(\mathfrak{t})+\frac{\mathfrak{X}_1(\mathfrak{t})}{1+\mu(\mathfrak{t})\mathfrak{X}_1(\mathfrak{t})}\left(=(\tilde{\mathfrak{H}}\oplus\mathfrak{X}_1)(\mathfrak{t})\right)$$

Hence, the result.

The following two theorems are the weighted variants of the last theorems respectively.

Theorem 3. Let $\tilde{a}, b, g, h, j, \Theta: \mathbb{T}^K \to \mathbf{R}$ be non-negative rd-continuous functions and $(0 \neq) p, q, r, s$ are constants with $p \geq q$; $p \geq r$ and $p \geq s$.Let $K(t, s)$ be a weight function as defined in lemma 3 s.t $K_1^{\Delta}(t, s) \geq 0$ for $t \geq s$ such that (3) holds, then

$$\Theta(t) \leq \sqrt[p]{\tilde{a}(t) + b(t) \int_{t_0}^{t} e_{\tilde{\mathfrak{X}}}(t, \sigma(\tau)) \tilde{\tilde{\mathfrak{H}}}(\tau) \Delta\tau},$$

where $\tilde{\tilde{\mathfrak{H}}}$ and $\tilde{\tilde{\mathfrak{X}}}$ are defined above in equation (10) and (17), respectively.

Proof. Define a function

$$\beta(t) = \int_{t_0}^{t} \alpha(t, \tau) \Delta\tau,$$

provided that

$$\alpha(t,\tau) = K(t, \tau)\{g(\tau)[\Theta(\tau)]^q + h(\tau)[\Theta(\tau)]^r + j(\tau)[\Theta(\tau)]^s\}$$

so that $\beta(t_0) = 0$ and $\beta(t)$ is non decreasing.

$$(3) \Rightarrow \Theta(t) \leq \sqrt[p]{\tilde{a}(t) + b(t)\beta(t)}. \tag{27}$$

Direct application of lemma 2 and inequality (19), (21) and (27) yield:

$$\beta^{\Delta}(t) = \alpha(\sigma(t), t) + \int_{t_0}^{t} \alpha_1^{\Delta}(t, \tau) \Delta\tau$$

$$= K(\sigma(t), t)\{g(t)[\Theta(t)]^q + h(t)[\Theta(t)]^r + j(t)[\Theta(t)]^s\}$$

$$+ \int_{t_0}^{t} K^{\Delta}(t, \tau)\{g(\tau)[\Theta(\tau)]^q + h(\tau)[\Theta(\tau)]^r + j(\tau)[\Theta(\tau)]^s\} \Delta\tau$$

$$= K(\sigma(t), t) z^{\Delta}(t) + \int_{t_0}^{t} K^{\Delta}(t, \tau) z^{\Delta}(\tau) \Delta\tau \leq \tilde{\tilde{\mathfrak{H}}}(t) + \beta(t) \tilde{\tilde{\mathfrak{X}}}(t). \tag{28}$$

application of lemma 1 yields the required result.

Theorem 4. Under the assumptions of theorem 3 for $1 > \mu(t)\, \hat{\mathfrak{Y}}(t)$ such that (4) holds, then

$$\Theta(t) \leq \sqrt[p]{\tilde{a}(t) + b(t) \int_{t_0}^{t} \left(\tilde{\mathfrak{z}} \oplus \tilde{\mathfrak{Y}}\right)(\tau)\, \Delta\tau}, \tag{29}$$

where $\hat{\mathfrak{Y}}, \tilde{\mathfrak{z}}$ and $\tilde{\mathfrak{Y}}$ are defined by equation (13), (14) and (16), respectively.

Proof. Consider

$$y(t) = \int_{t_0}^{t} K(t, \tau)\{g(\tau)[\Theta^{\sigma}(\tau)]^q + h(\tau)[\Theta^{\sigma}(\tau)]^r + j(\tau)[\Theta^{\sigma}(\tau)]^s\} \Delta\tau = \int_{t_0}^{t} z(t, \tau) \Delta\tau$$

where, $z(\mathfrak{t},\mathfrak{r}) = K(\mathfrak{t},\mathfrak{r})\{g(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{\mathfrak{q}} + h(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{r} + j(\mathfrak{r})[\Theta^{\sigma}(\mathfrak{r})]^{s}\}$,

so that $y(\mathfrak{t}_0) = 0$ and $y(\mathfrak{t})$ is non-decreasing.

$$(4) \Longrightarrow \quad \Theta(\mathfrak{t}) \leq \sqrt[p]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t})y(\mathfrak{t})}. \tag{30}$$

Direct application of lemma 2 and (7),(9),(13),(19) yield:

$$y^{\Delta}(\mathfrak{t}) = z(\sigma(\mathfrak{t}),\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} z_1^{\Delta}(\mathfrak{t},\mathfrak{r})\Delta\mathfrak{r} \leq K(\sigma(\mathfrak{t}),\mathfrak{t})\left[\tilde{\mathfrak{H}}(\mathfrak{t}) + y^{\sigma}(\mathfrak{t})\tilde{\mathfrak{X}}(\mathfrak{t})\right] + \int_{\mathfrak{t}_0}^{\mathfrak{t}} K^{\Delta}(\mathfrak{t},\mathfrak{r})\left[\tilde{\mathfrak{H}}(\mathfrak{r}) + y^{\sigma}(\mathfrak{r})\tilde{\mathfrak{X}}(\mathfrak{r})\right]\Delta\mathfrak{r}$$

$$= \tilde{\mathfrak{Y}}(\mathfrak{t}) + K(\sigma(\mathfrak{t}),\mathfrak{t})y^{\sigma}(\mathfrak{t})\tilde{\mathfrak{X}}(\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} K^{\Delta}(\mathfrak{t},\mathfrak{r})y^{\sigma}(\mathfrak{r})\tilde{\mathfrak{X}}(\mathfrak{r})\Delta\mathfrak{r}. \tag{31}$$

For (14), the above inequality (31) has the form

$$y^{\Delta}(\mathfrak{t}) \leq \tilde{\mathfrak{z}}(\mathfrak{t}) + \frac{\tilde{\mathfrak{Y}}(\mathfrak{t})}{1 + \mu(\mathfrak{t})\tilde{\mathfrak{Y}}(\mathfrak{t})} = \left(\tilde{\mathfrak{z}} + \tilde{\mathfrak{Y}}\right)(\mathfrak{t}).$$

Hence, the result.

3. APPLICATIONS

Now we discuss here the few utilizations of the main results for the special cases $\mathbb{T} = \mathbf{R}$, $\mathbb{Z}$.

Corollary 1.(Continuous case)

Let $\mathbb{T} = \mathbf{R}$ and $\tilde{a}, \mathfrak{b}, g, h, j, \Theta : [\mathfrak{t}_0,\infty) \to \mathbf{R}_+$ be continuous functions; Let $(0 \neq)p, q, r, s$ are constants such that $p \geq q$; $p \geq r$ and $p \geq s$, then (18) implies

$$\Theta(\mathfrak{t}) \leq \sqrt[p]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} exp\left(\int_{\mathfrak{r}}^{\mathfrak{t}} \mathfrak{X}(\eta)d\eta\right)\mathfrak{H}(\mathfrak{r})d\mathfrak{r}}, \tag{32}$$

provided that $\mathfrak{X}$ and $\mathfrak{H}$ are defined as in theorem 1.

Corollary 2.(Discrete case)

Let $\mathbb{T} = \mathbb{Z}$ and $\tilde{a}, \mathfrak{b}, g, h, j, \Theta : \mathfrak{A} \to \mathbf{R}_+, \mathfrak{A} = \{\mathfrak{t}_0, \mathfrak{t}_0 + 1, \ldots\}$; let $(0 \neq)p, q, r, s$ are constants s.t $p \geq q$; $p \geq r$ and $p \geq s$, then (18) implies

$$\Theta(\mathfrak{t}) \leq \sqrt[p]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \sum_{\mathfrak{r}=\mathfrak{t}_0}^{\mathfrak{t}-1} \prod_{\eta=\mathfrak{r}+1}^{\mathfrak{t}-1} \left(1 + \mathfrak{X}(\eta)\right)\mathfrak{H}(\mathfrak{r})} \tag{33}$$

provided that $\mathfrak{X}$ and $\mathfrak{H}$ are defined just like in Theorem 1.

Corollary 3. (Continuous case)

Let $\mathbb{T} = \mathbf{R}$ and $\tilde{a}, \mathfrak{b}, g, h, j, \Theta : [\mathfrak{t}_0,\infty) \to \mathbf{R}_+$ be continuous functions; let $(0 \neq)p, q, r, s$ are constants s.t $p \geq q$; $p \geq r$ and $p \geq s$. Let $K(\mathfrak{t}, s)$ be a weight function (defined in Lemma 3) s.t $\frac{\partial K(\mathfrak{t},s)}{\partial \mathfrak{t}} \geq 0$ for $\mathfrak{t} \geq s$ then (29) implies

$$\Theta(\mathfrak{t}) \leq \sqrt[p]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \int_{\mathfrak{t}_0}^{\mathfrak{t}} \left(\tilde{\mathfrak{z}} \oplus \tilde{\mathfrak{Y}}\right)(s)\, ds}, \tag{34}$$

where $\tilde{\mathfrak{Y}}$ and $\tilde{\mathfrak{z}}$ are as defined in Theorem 4.

Corollary 4. (Discrete case)
Assume $\mathbb{T} = \mathbb{Z}$ and $\tilde{a}, \mathfrak{b}, g, h, j, \Theta : \mathfrak{A} \to \mathbf{R}_+$; let $(0 \neq) p, q, r, s$ are constants s.t $p \geq q; p \geq r$ and $p \geq s$. Let $K(\mathfrak{t}, s)$ be a weight function as defined in lemma 3 s.t $K(\mathfrak{t}+1, s) - K(\mathfrak{t}, s) \geq 0$ for $\mathfrak{t} \geq s$ and $1 > \hat{\mathfrak{Y}}(\mathfrak{t})$, then (29)⇒

$$\Theta(\mathfrak{t}) \leq \sqrt[p]{\tilde{a}(\mathfrak{t}) + \mathfrak{b}(\mathfrak{t}) \sum_{s=\mathfrak{t}_0}^{\mathfrak{t}-1} \left(\tilde{\mathfrak{z}} \oplus \tilde{\mathfrak{Y}}\right)(s)}, \tag{35}$$

provided

$$\hat{\mathfrak{Y}}(\mathfrak{t}) = K(\mathfrak{t}+1, \mathfrak{t})\tilde{\mathfrak{H}}(\mathfrak{t}) + \sum_{s=\mathfrak{t}_0}^{\mathfrak{t}-1} [K(\mathfrak{t}+1, s) - K(\mathfrak{t}, s)]\tilde{\mathfrak{H}}(s),$$

And $\tilde{\mathfrak{Y}}, \tilde{\mathfrak{H}}$ and $\tilde{\mathfrak{z}}$ are as defined in Theorem 4.
Note 1. Let $\tilde{a}, g, \Theta: \mathbb{T}^K \to \mathbf{R}$ be non-negative rd-continuous functions and $p = 1 = q$ s.t $\mathfrak{b}(t) = 1, h \equiv 0 \equiv k$.Then, (18) implies

$$\Theta(\mathfrak{t}) \leq \tilde{a}(\mathfrak{t}) + \int_{\mathfrak{t}_0}^{\mathfrak{t}} e_g(\mathfrak{t}, \sigma(\mathfrak{r}))g(\mathfrak{r})\tilde{a}(\mathfrak{r})\Delta\mathfrak{r}\,, \tag{36}$$

which is nothing except that [2, Theorem 6.4].

Corollary 5. Assume that g, h, j, Θ are non-negative rd-continuous functions and $(0 \neq) p, q, r, s$ are constant with $p \geq q$; $p \geq r$ and $p \geq s$. If $\rho \geq 0$ is a real constant, then (5) implies

$$\Theta(\mathfrak{t}) \leq \sqrt[p]{\rho + \int_{\mathfrak{t}_0}^{\mathfrak{t}} e_{\tilde{\mathfrak{E}}}\left(\mathfrak{t}, \sigma(\mathfrak{r})\right)\tilde{\mathfrak{D}}(\mathfrak{r})\Delta\mathfrak{r}}. \tag{37}$$

Proof. By using Theorem 1, (37) follows from (5).
Finally, to illustrate our main results we give an application to initial value dynamical equation. Let us examine the following IVP on time scales

$$[\Theta^p(\mathfrak{t})]^\Delta = \mathfrak{G}\left(\mathfrak{t}, \emptyset_q\left(\Theta(\mathfrak{t})\right), \emptyset_r\left(\Theta(\mathfrak{t})\right), \emptyset_s\left(\Theta(\mathfrak{t})\right)\right), \Theta^p(\mathfrak{t}_0) = \Theta_0, \tag{38}$$

where, $\emptyset_q(\zeta) = |\zeta|^q . \mathrm{sgn}\zeta$ and $\mathfrak{G}: \mathbb{T} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is a continuous functions.t

$$|\mathfrak{G}(\mathfrak{t}, \Theta^q(\mathfrak{t}), \Theta^r(\mathfrak{t}), \Theta^s(\mathfrak{t}))| \leq g(\mathfrak{t}).|\Theta(\mathfrak{t})|^q + h(\mathfrak{t}).|\Theta(\mathfrak{t})|^r + j(\mathfrak{t}).|\Theta(\mathfrak{t})|^s,$$

provided that g, h, j are non-negative rd-continuous functions on $\mathbb{T}^K$, then

$$\Theta^p(\mathfrak{t}) = C + \int_{\mathfrak{t}_0}^{\mathfrak{t}} \mathfrak{G}\left(\mathfrak{r}, \emptyset_q\left(\Theta(\mathfrak{r})\right), \emptyset_r\left(\Theta(\mathfrak{r})\right), \emptyset_s\left(\Theta(\mathfrak{r})\right)\right)\Delta\boldsymbol{\tau}\,,$$

$$\Rightarrow |\Theta^p(\mathfrak{t})| \leq |C| + \int_{\mathfrak{t}_0}^{\mathfrak{t}} \left|\mathfrak{G}\left(\mathfrak{r}, \emptyset_q\left(\Theta(\mathfrak{r})\right), \emptyset_r\left(\Theta(\mathfrak{r})\right), \emptyset_s\left(\Theta(\mathfrak{r})\right)\right)\right| \Delta\mathfrak{r}$$

$$\leq |C| + \int_{\mathfrak{t}_0}^{\mathfrak{t}} \{g(\mathfrak{r}).|\Theta(\mathfrak{r})|^q + h(\mathfrak{r}).|\Theta(\mathfrak{r})|^r + j(\mathfrak{r}).|\Theta(\mathfrak{r})|^s\}\Delta\mathfrak{r}.$$

In fact, $\mathtt{C} = \Theta^{\mathrm{p}}(\mathtt{t}_0)$, then Corollary 5 yields:

$$|\Theta(\mathtt{t})| \leq \sqrt[\mathrm{p}]{\Theta^{\mathrm{p}}(\mathtt{t}_0) + \int_{\mathtt{t}_0}^{\mathtt{t}} e_{\widetilde{\mathfrak{C}}}\big(\mathtt{t}, \sigma(\mathtt{\tau})\big)\widetilde{\mathfrak{D}}(\mathtt{\tau})\Delta\mathtt{\tau}},$$

provided that $\Theta(\mathtt{t})$ is a solution of (38).

4. REFERENCES

1. Sun, Y. & T. Hassan. Some nonlinear dynamic integral inequalities on time scales, *Applied Mathematics and Computation* 220: 221-225 (2013).
2. Bohner, M. & A. Peterson. *Dynamic Equations on Time Scales: An Introduction with Application*. Birkhäuser, Boston, USA (2001).
3. Li, W.N. Some new dynamic inequalities on time scales. *Journal of Mathematical Analysis and Applications* 319: 802-814 (2006).
4. Feng, Q., F. Meng & Y. Zhang. Some new finite difference inequalities arising in the theory of difference equations. *Advances in Difference Equations* 2011, 2011:21 doi: 10. 1186/1687-1847 (2011).
5. Pachpatte, B.G. Inequalities applicable in the theory of finite difference equations. *Journal of Mathematical Analysis and Applications* 222: 438-459 (1998).
6. Meng, F.W. & W.N. Li. On some new nonlinear discrete inequalities and their applications. *Journal of Computational and Applied Mathematics* 158: 407-417 (2003).
7. Pachpatte, B.G. On some new inequalities related to certain inequality arising in the theory of differential equations. *Journal of Mathematical Analysis and Applications* 251: 736-751 (2000).
8. Pachpatte, D.B. Explicit estimate on integral inequalities with time scales. *Journal of Inequalities in Pure and Applied Mathematics* 7 (4, article 143), 8 pp. (2006).

Proceedings of the Pakistan Academy of Sciences 52 (3): 247–254 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Dynamics of Bosons in Two Wells of an External Trap

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Abstract: We study the dynamics of the atoms of Bose-Einstein condensate in a double well potential by deriving the two mode model for the well known Gross-Pitaevskii equation. The symmetric and anti-symmetric basis functions have been used for the development of the two mode model. The stability of these basis functions has been investigated. It is found that both solutions are stable. The time dependent Gross-Pitaevskii equation and the two mode approximations are solved numerically and then compare the results. It is shown that the solution obtained from two mode model demonstrates good agreement with the solution of the Gross-Pitaevskii equation.

Keywords: Bose-Einstein condensate, double well potential, Gross-Pitaevskii equation, Josephson tunneling.

1. INTRODUCTION

Multi-component Hamiltonian systems have gained a lot of attention in the past few years due to the development of theoretical and experimental results in coupled Bose-Einstein condensates (BECs) [1] and coupled nonlinear optical systems [2]. In BECs, mixtures of distinct spin states of rubidium [3,4] and sodium [5] were created experimentally. The two components BECs with different atomic species were also formed in laboratories, e.g. potassium-rubidium [6] and lithium-cesium [7]. Due to these experiments, many theoretical studies have been done to investigate the ground state solutions [8, 9] and the small amplitude excitations [8, 10, 11]. Several other nonlinear structures were also formed such as domain walls [12, 13, 14], dark-dark and dark-bright solitons [15, 16], vortex rings [17] and so on.

In 1962, Josephson presented the idea of electron tunneling between two superconductors which were separated by a thin insulator [18]. The effect of tunneling was named as Josephson tunneling. Since weak coupling is the only requirement for the effect of Josephson tunneling, it was thought that the weakly linked macroscopic quantum samples may admit such tunneling. In BECs, such tunneling was predicted by Smerzi and coresearchers [19, 20, 21]. The experimental realization of Josephson tunneling for a single [22, 23] and array of short Bose-Josephson junction [24] were made. Kaurov and Kuklov [25, 26] extended the idea of Bose-Josephson junction to long Bose-Josephson junction. This junction was analogues to long superconducting Josephson junction. They proposed that atomic vortices could be seen in weakly coupled BECs and that these vortices are similar to Josephson fluxons in superconducting long Josephson junction [27]. Further it was shown that due to the presence of a critical coupling, atomic Josephson vortices can be transformed to a dark soliton and vice versa. Josephson tunneling of dark solitons in a double-well potential was studied in [28].

The dynamics of Josephson tunneling in BECs was explained using a two-mode approximation in [29, 30, 31, 32, 33]. The coupled-mode equations were modified and improved in [34]. In this paper, we study the validity of the coupled-mode equations. The stability of the basis functions

Received, January 2015; Accepted, June 2015
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(which are used for the approximations) is also studied by investigating the eigenvalues structures.

2. MATHEMATICAL MODEL AND DESCRIPTION

Let us consider the atoms of BECs at very low temperature that is nearly at zero Kelvin. If U(x, t) is the wave function of the atoms of BECs which are interacting with each other, then the equation that describes the dynamics of atoms of BECs is the famous Gross-Pitaevskii (GP) equation. The GP equation in the dimensionless form is given as

$$i\frac{\partial \mathrm{U}}{\partial t} = -\frac{1}{2}\frac{\partial^2 \mathrm{U}}{\partial x^2} + \sigma|\mathrm{U}|^2\mathrm{U} + V\mathrm{U}, \qquad (1)$$

where x and t are the space and time variables, σ is the nonlinearity coefficient and $i = \sqrt{-1}$. V is the external potential which in our case is a combination of a harmonic potential with Gaussian barrier and is given as

$$V = \frac{1}{2}\Omega^2 x^2 + Ae^{-(x/b)^2}, \qquad (2)$$

with Ω representing the frequency of oscillation and A and b are respectively the height and width of the Gaussian barrier.

To obtain the two mode approximations, we use a pair of real symmetric and real antisymmetric functions which are denoted by φ_e and φ_o respectively. It is easy to see that if we substitute $U(x,t) = \sqrt{N}e^{-it\gamma_{e,o}}\varphi_{e,o}(x)$ into eq.(1), (where γ_e and γ_o are constants which represent the chemical potential in each well) the basis functions φ_e and φ_o will satisfy the following steady state equations

$$\gamma_e\varphi_e = -\frac{1}{2}\frac{d^2\varphi_e}{dx^2} + V\varphi_e + \sigma_1{\varphi_e}^3, \qquad (3)$$

$$\gamma_o\varphi_o = -\frac{1}{2}\frac{d^2\varphi_o}{dx^2} + V\varphi_o + \sigma_1{\varphi_o}^3, \qquad (4)$$

where $\sigma_1 = \sigma N$.

To seek the solution φ_e numerically, we discretize eq. (3) and approximate the second order derivative by the central difference approximation so that a system of nonlinear algebraic equations is obtained. The system can be solved using Newton's method with the Neumann boundary conditions to obtain the solution φ_e which is shown in Fig. 1. Similarly, from eq. (4) we get the solution φ_o and is depicted in Fig. 2. The stability of these basis functions will be discussed later.

One can now express the wave function $U(x,t)$ as [30]

$$U(x,t) = \sqrt{N}[\varphi_1(x)U_1(t) + \varphi_2(x)U_2(t)]. \qquad (5)$$

Here, $\varphi_1(x) = \left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right)$ and $\varphi_2(x) = \left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)$ with $\int_{-\infty}^{\infty}\varphi_i\varphi_j dx = \delta_{ij}, i,j = e,o$, and δ denotes the Kronecker delta function. N represents the number of boson atoms such that $\int_{-\infty}^{\infty}|U(x,t)|^2 = N$. So, we have

$$U = \sqrt{N}\left[U_1\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right) + U_2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right].$$

Substituting this value in (1), we get

$$\sqrt{N}\left[\dot{U}_1\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right) + \dot{U}_2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right]$$

$$= -\frac{\sqrt{N}}{2}\left[U_1\left(\frac{\acute{\varphi}_e+\acute{\varphi}_o}{\sqrt{2}}\right) + U_2\left(\frac{\acute{\varphi}_e-\acute{\varphi}_o}{\sqrt{2}}\right)\right]$$

$$+ V\sqrt{N}\left[U_1\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right) + U_2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right]$$

$$+ \sigma N\sqrt{N}\left[U_1^2\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right)^2 + U_2^2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)^2\right.$$

$$\left. + 2U_1U_2\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right)\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right]\left[\bar{U}_1\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right)\right.$$

$$\left. + \bar{U}_2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right],$$

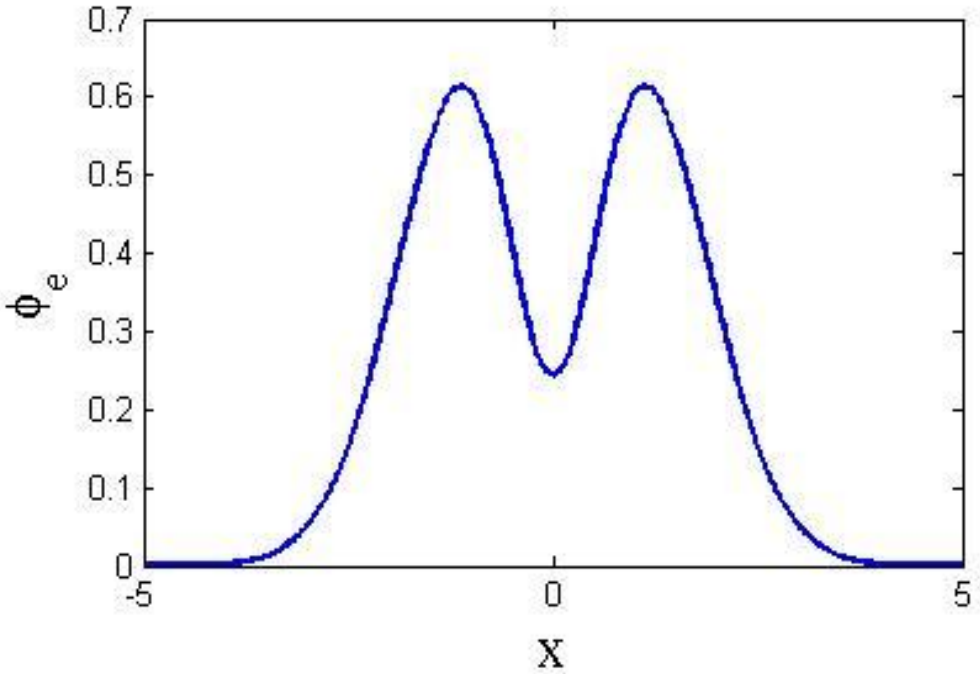

Fig. 1. Numerically obtained solution $\boldsymbol{\varphi_e}$ for the parameter values $\boldsymbol{\sigma_1}$ = 1, $\boldsymbol{\gamma_e}$ = 1.

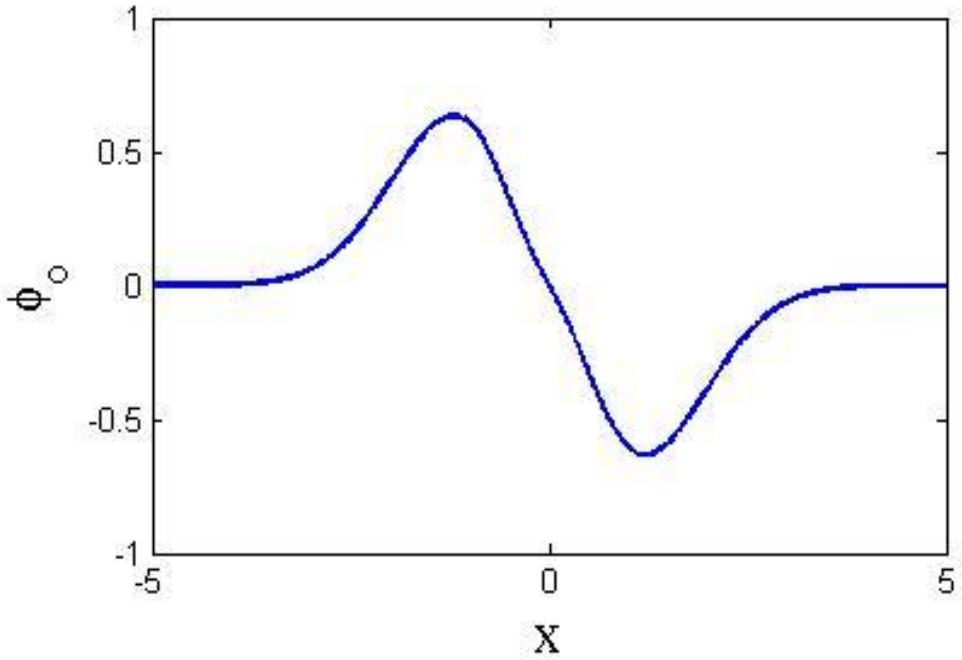

Fig. 2. Numerically obtained solution $\boldsymbol{\varphi_o}$ for the parameter values $\boldsymbol{\sigma_1}$ = 1, $\boldsymbol{\gamma_e}$ = 1.

where bar represents the complex conjugate, primes are used for the second order derivative with respect to x and dot for derivative with respect to time.

$$i\left[\dot{U}_1(\varphi_e+\varphi_o)+\dot{U}_2(\varphi_e-\varphi_o)\right]$$
$$=-\frac{1}{2}\left[U_1\left(\acute{\varphi}_e+\acute{\varphi}_o\right)+U_2\left(\acute{\varphi}_e-\acute{\varphi}_o\right)\right]+V\left[U_1\left(\acute{\varphi}_e+\acute{\varphi}_o\right)+U_2\left(\acute{\varphi}_e-\acute{\varphi}_o\right)\right]$$
$$+\sigma_1\left[U_1^2\left(\frac{\varphi_e^2+\varphi_o^2+2\varphi_e\varphi_o}{2}\right)+U_2^2\left(\frac{\varphi_e^2+\varphi_o^2-2\varphi_e\varphi_o}{2}\right)\right.$$
$$\left.+2U_1U_2\left(\frac{\varphi_e^2-\varphi_o^2}{2}\right)\right]\left[\overline{U}_1\left(\frac{\varphi_e+\varphi_o}{\sqrt{2}}\right)+\overline{U}_2\left(\frac{\varphi_e-\varphi_o}{\sqrt{2}}\right)\right].$$

$$i\left[\dot{U}_1(\varphi_e+\varphi_o)+\dot{U}_2(\varphi_e-\varphi_o)\right]$$
$$=(U_1+U_2)\left(-\frac{1}{2}\acute{\varphi}_e+V\varphi_e\right)+(U_1-U_2)\left(-\frac{1}{2}\acute{\varphi}_o+V\varphi_o\right)$$
$$+\frac{\sigma_1}{2}[U_1^2(\varphi_e^2+\varphi_o^2+2\varphi_e\varphi_o)+U_2^2(\varphi_e^2+\varphi_o^2-2\varphi_e\varphi_o)$$
$$+2U_1U_2(\varphi_e^2-\varphi_o^2)][\overline{U}_1(\varphi_e+\varphi_o)+\overline{U}_2(\varphi_e-\varphi_o)].$$

Using eq.(3) and eq.(4), we get

$$i\left[\dot{U}_1(\varphi_e+\varphi_o)+\dot{U}_2(\varphi_e-\varphi_o)\right]$$
$$=(U_1+U_2)(\gamma_e\varphi_e-\sigma_1{\varphi_e}^3)+(U_1-U_2)(\gamma_o\varphi_o-\sigma_1{\varphi_o}^3)$$
$$+\frac{\sigma_1}{2}[|U_1|^2U_1(\varphi_e^3+3\varphi_e\varphi_o^2+3\varphi_e^2\varphi_o+\varphi_o^3)+U_1^2\overline{U}_2(\varphi_e^3-\varphi_e\varphi_o^2+\varphi_e^2\varphi_o-\varphi_o^3)$$
$$+\overline{U}_1U_2^2(\varphi_e^3-\varphi_e\varphi_o^2-\varphi_e^2\varphi_o+\varphi_o^3)+|U_2|^2U_2(\varphi_e^3+3\varphi_e\varphi_o^2-3\varphi_e^2\varphi_o-\varphi_o^3)$$
$$+2|U_1|^2U_2(\varphi_e^3-\varphi_e\varphi_o^2+\varphi_e^2\varphi_o-\varphi_o^3)+2U_1|U_2|^2(\varphi_e^3-\varphi_e\varphi_o^2-\varphi_e^2\varphi_o+\varphi_o^3)] \quad (6)$$

Multiplying both sides by $(\varphi_e+\varphi_o)$ and integrating with respect to x from -∞ to ∞ and using $\int_{-\infty}^{\infty}\varphi_i\varphi_j dx=\delta_{ij}, i,j=e,o$, we obtain

$$2i\dot{U}_1=(U_1+U_2)[\gamma_e-\mu_{ee}]+(U_1-U_2)[\gamma_o-\mu_{oo}]$$
$$+\frac{1}{2}[\mu_{ee}(|U_1|^2U_1+U_1^2\overline{U}_2+\overline{U}_1U_2^2+|U_2|^2U_2+2|U_1|^2U_2+2U_1|U_2|^2)$$
$$+\mu_{00}(|U_1|^2U_1-U_1^2\overline{U}_2+\overline{U}_1U_2^2-|U_2|^2U_2-2|U_1|^2U_2+2U_1|U_2|^2)$$
$$+\mu_{eo}(6|U_1|^2U_1-2\overline{U}_1U_2^2-4U_1|U_2|^2)],$$

where $\mu_{ij}=\sigma_1\int\varphi_i^2(x)\varphi_j^2(x)dx,\ i,j=e,o$, and the integrals with odd powers of φ_e and φ_o will be zero. Since $\int_{-\infty}^{\infty}|U(x,t)|^2=N\Rightarrow|U_1|+|U_2|=1$. Using this equation, the above equation can be written as

$$2i\dot{U}_1 = (U_1+U_2)[\gamma_e-\mu_{ee}]+(U_1-U_2)[\gamma_o-\mu_{oo}]$$
$$+\frac{1}{2}[\mu_{ee}\{2(U_1+U_2)-|U_1|^2U_1-|U_2|^2U_2+U_1^2\bar{U}_2+\bar{U}_1U_2^2\}$$
$$+\mu_{oo}\{2(U_1-U_2)-|U_1|^2U_1+|U_2|^2U_2-U_1^2\bar{U}_2+\bar{U}_1U_2^2\}$$
$$+\mu_{eo}(-4U_1+10|U_1|^2U_1-2\bar{U}_1U_2^2)].$$

$$i\dot{U}_1 = \left[\frac{\gamma_e+\gamma_o}{2}-\left(\frac{\mu_{ee}+\mu_{oo}}{2}\right)+\frac{1}{4}(2\mu_{ee}+2\mu_{oo}-4\mu_{eo})\right]U_1+\frac{1}{4}(-\mu_{ee}-\mu_{oo}+10\mu_{eo})|U_1|^2U_1$$
$$+\frac{1}{4}(\mu_{ee}-\mu_{oo})U_1^2\bar{U}_2+\left[\frac{\gamma_e-\gamma_o}{2}-\left(\frac{\mu_{ee}-\mu_{oo}}{2}\right)+\frac{1}{4}(2\mu_{ee}-2\mu_{oo})\right]U_2$$
$$+\frac{1}{4}(-\mu_{ee}+\mu_{oo})|U_2|^2U_2+\frac{1}{4}(\mu_{ee}+\mu_{oo}-2\mu_{eo})\bar{U}_1U_2^2$$

$$i\dot{U}_1 = \left[\frac{\gamma_e+\gamma_o}{2}-\mu_{eo}\right]U_1+\left(\frac{10\mu_{eo}-\mu_{ee}-\mu_{oo}}{4}\right)|U_1|^2U_1+\left(\frac{\mu_{ee}-\mu_{oo}}{4}\right)U_1^2\bar{U}_2+\left(\frac{\gamma_e-\gamma_o}{2}\right)U_2$$
$$-\left(\frac{\mu_{ee}-\mu_{oo}}{4}\right)|U_2|^2U_2+\left(\frac{\mu_{ee}+\mu_{oo}-2\mu_{eo}}{4}\right)\bar{U}_1U_2^2$$

$$i\dot{U}_1 = \left(B+C|U_1|^2+\frac{\Delta\mu}{4}U_1\bar{U}_2\right)U_1+\left(\frac{\Delta\gamma}{2}-\frac{\Delta\mu}{4}|U_2|^2+D\bar{U}_1U_2\right)U_2. \quad (7)$$

Similarly, multiplying both sides of eq.(6) by $(\varphi_e-\varphi_o)$ and integrating with respect to x from -∞ to ∞ and following the same procedure as before, we obtain

$$i\dot{U}_2 = \left(B+C|U_2|^2+\frac{\Delta\mu}{4}U_2\bar{U}_1\right)U_2+\left(\frac{\Delta\gamma}{2}-\frac{\Delta\mu}{4}|U_1|^2+D\bar{U}_2U_1\right)U_1, \quad (8)$$

where

$$B=\frac{\gamma_e+\gamma_o}{2}-\mu_{eo},$$
$$C=\frac{10\mu_{eo}-\mu_{ee}-\mu_{oo}}{4},$$
$$D=\frac{\mu_{ee}+\mu_{oo}-2\mu_{eo}}{4},$$
$$\Delta\mu=\mu_{ee}-\mu_{oo},$$
$$\Delta\gamma=\gamma_e-\gamma_o.$$

Thus, eq. (7) and eq. (8) represent a system of two ordinary differential equations of first order. These equations describe the dynamics of atoms of BEC in each well of the external potential. We solve this system of differential equations using Runge-Kutta method of order 4 to get U_1 and U_2. Substituting these solutions U_1 and U_2 into eq. (5), we obtain the solution U which is shown in Fig. 3 by dotted line. We then solve eq. (1) numerically and the solution obtained is shown in Fig. 3 by solid line. Figure shows that the solution obtained through two mode model is very close to the numerical solution of the GP equation and hence justifies the validity of the two mode model.

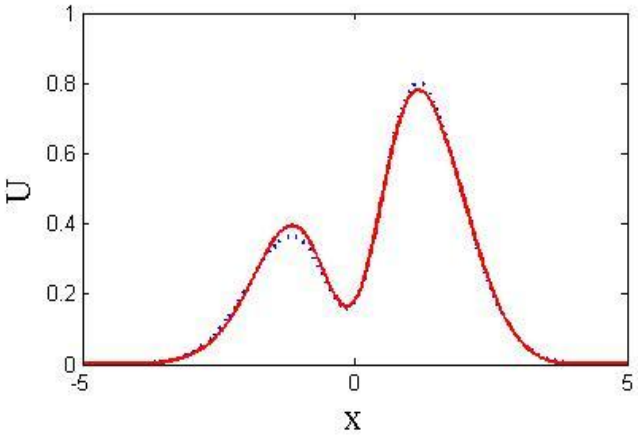

Fig. 3. Comparison of the two solutions. The solid curve represents the solution of eq. (1) while dotted curve is the solution obtained through two mode approximation. The graph shows very good agreement between the two solutions.

3. STABILITY OF BASIS FUNCTIONS

Let us discuss the stability of the solutions φ_e and φ_o. To do so, we first substitute

$$U(x,t) = e^{-it\gamma_e}\,\widetilde{U}(x,t)$$

into eq. (1) to obtain

$$i\frac{\partial \widetilde{U}}{\partial t} = -\frac{1}{2}\frac{\partial^2 \widetilde{U}}{\partial x^2} + \sigma|\widetilde{U}|^2\widetilde{U} + V\widetilde{U} - \gamma_e\widetilde{U}. \tag{9}$$

We now perturb the solution φ_e by adding a small perturbation $\eta(x,t)$ in it, i.e.

$$\widetilde{U}(x,t) = \varphi_e(x) + \eta(x,t), \tag{10}$$

where we assume that the perturbation η is so small that its squares and higher power terms can be neglected. Substituting the value of $\widetilde{U}(x,t)$ from eq. (10) into eq. (9) and using eq. (3), we get

$$i\frac{\partial \eta}{\partial t} = -\frac{1}{2}\frac{\partial^2 \eta}{\partial x^2} + (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\eta + \sigma\,{\varphi_e}^2\bar{\eta}. \tag{11}$$

Taking complex conjugate of eq. (11) to obtain

$$-i\frac{\partial \bar{\eta}}{\partial t} = -\frac{1}{2}\frac{\partial^2 \bar{\eta}}{\partial x^2} + (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\bar{\eta} + \sigma\,{\varphi_e}^2\eta. \tag{12}$$

For simplicity, we denote η by α and $\bar{\eta}$ by β so that eq. (11) and eq. (12) can be written as

$$i\frac{\partial \alpha}{\partial t} = -\frac{1}{2}\frac{\partial^2 \alpha}{\partial x^2} + (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\alpha + \sigma\,{\varphi_e}^2\beta. \tag{13}$$

$$i\frac{\partial \beta}{\partial t} = \frac{1}{2}\frac{\partial^2 \beta}{\partial x^2} - (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\beta - \sigma\,{\varphi_e}^2\alpha. \tag{14}$$

Eq. (13) and eq. (14) can also be written as

$$-\frac{1}{2}\frac{\partial^2 \alpha}{\partial x^2} + (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\alpha + \sigma\,{\varphi_e}^2\beta = \lambda\alpha, \tag{15}$$

$$\frac{1}{2}\frac{\partial^2 \beta}{\partial x^2} - (2\sigma\,{\varphi_e}^2 + V - \gamma_e)\beta - \sigma\,{\varphi_e}^2\alpha = \lambda\beta. \tag{16}$$

Discretizing eq. (15) and eq. (16) with step size h and using the Neumann boundary conditions yield an eigenvalue problem $AX = \lambda X$ with eigenvalues λ and

$$A = \begin{bmatrix} A_1 & -D_1 \\ D_1 & -A_1 \end{bmatrix},$$

where

$$A_1 = \begin{bmatrix} \frac{-1}{h^2} - \left(2\sigma\,{\varphi_{e,1}}^2 + V - \gamma_e\right) & \frac{1}{2h^2} & 0 & 0 & \dots & \frac{1}{2h^2} \\ \frac{1}{2h^2} & \frac{-1}{h^2} - \left(2\sigma\,{\varphi_{e,2}}^2 + V - \gamma_e\right) & \frac{1}{2h^2} & 0 & \dots & 0 \\ 0 & \frac{1}{2h^2} & \frac{-1}{h^2} - \left(2\sigma\,{\varphi_{e,3}}^2 + V - \gamma_e\right) & \frac{1}{2h^2} & \dots & 0 \\ & \ddots & & \ddots & \ddots & \\ & & \ddots & & \ddots & \\ \frac{1}{2h^2} & 0 & \dots & \frac{1}{2h^2} & & \frac{-1}{h^2} - \left(2\sigma\,{\varphi_{e,N}}^2 + V - \gamma_e\right) \end{bmatrix},$$

$$D_1 = \begin{bmatrix} \sigma\,{\varphi_{e,1}}^2 & 0 & 0 & \dots & 0 \\ 0 & \sigma\,{\varphi_{e,2}}^2 & 0 & \dots & 0 \\ 0 & 0 & \sigma\,{\varphi_{e,3}}^2 & \dots & 0 \\ & & \ddots & & \\ & & & \ddots & \\ 0 & 0 & 0 & \dots & \sigma\,{\varphi_{e,N}}^2 \end{bmatrix}.$$

The solution will be stable if all eigenvalues are real. We find the eigenvalues of above matrix A for the solution φ_e shown in Fig. (1). It is found that the imaginary parts of all eigenvalues are zero, i.e. all eigenvalues are real as they all are lying on the horizontal axis as shown in Fig. (4). This shows that solution φ_e is stable.

Following the same procedure as above, we found the eigen values structure for the solution φ_o depicting that the solution φ_o is also stable.

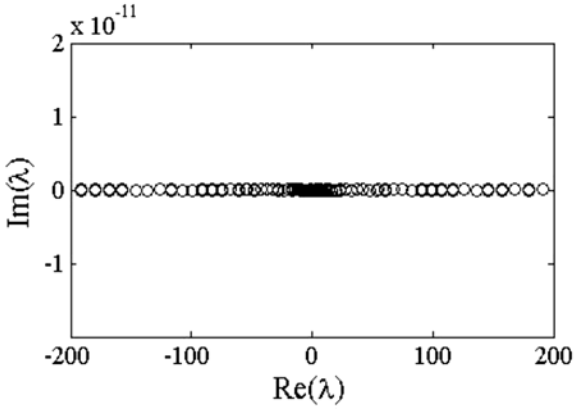

Fig. 4 . The eigenvalues structure for the solution $\boldsymbol{\varphi_e}$. All eigenvalues are lying on the horizontal axis showing the stability of the solution.

4. CONCLUSIONS

In this paper, we have presented the derivation of a two mode model using a symmetric and an anti-symmetric basis functions. It was found that the solutions obtained through two mode model and that from the time-dependent GP equation are very close to each other and validated the two mode model. The two mode model can be used to describe the dynamics of bosons in each well of the external potential. We also studied the stability of the basis functions by perturbing the solutions. Both solutions were found to be stable.

5. REFERENCES

1. Pitaevskii, L.P. & S. Stringari. *Bose-Einstein Condensation*. Oxford University Press, Oxford, UK (2003).
2. Kivshar, Yu.S. & G.P. Agrawal. *Optical Solitons: From Fibers to Photonic Crystals*. Academic Press, San Diego, USA (2003).
3. Myatt, C.J., E.A. Burt, R.W. Ghrist, E.A. Cornell & C.E. Wieman. Production of two overlapping Bose-Einstein condensates by sympathetic cooling. *Physical Review Letters* 78: 586-589 (1997).
4. Hall, D.S., M.R. Matthews, J.R. Ensher, C.E. Wieman & E.A. Cornell. Dynamics of component separation in a binary mixture of Bose-Einstein

condensates. *Physical Review Letters* 81: 1539-1542 (1998).
5. Stamper-Kurn, D.M., M.R. Andrews, A.P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, & W. Ketterle. Optical confinement of a Bose-Einstein condensate. *Physical Review Letters* 80: 2027-2030 (1998).
6. Modugno, G., G. Ferrari, G. Roati, R.J. Brecha, A. Simoni & M. Inguscio. Bose-Einstein condensation of potassium atoms by sympathetic cooling. *Science* 294: 1320-1322 (2001).
7. Mudrich, M., S. Kraft, K. Singer, R. Grimm, A. Mosk, & M. Weidemüller. Sympathetic cooling with two atomic species in an optical trap. *Physical Review Letters* 88: 253001 (2002).
8. Pu, H. & N.P. Bigelow. Properties of two-species Bose condensates. *Physical Review Letters* 80: 1130-1133 (1998).
9. Ho, T.-L. & V.B. Shenoy. Binary mixtures of Bose condensates of alkali atoms. *Physical Review Letters* 77: 3276-3279 (1996).
10. Busch, Th., J.I. Cirac, V.M. Pérez-García & P. Zoller. Stability and collective excitations of a two-component Bose-Einstein condensed gas: A moment approach. *Physical Review A* 56: 2978-2983 (1997).
11. Esry, B.D. & C.H. Greene. Low-lying excitations of double Bose-Einstein condensates. *Physical Review A* 57: 1265-1271 (1998).
12. Trippenbach, M. K. Góral, K. Rzazewski, B. Malomed & Y.B. Band. Structure of binary Bose-Einstein condensates. *Journal of Physics B: Atomic, Molecular and Optical Physics* 33: 4017-4032 (2000).
13. Coen, S. & M. Haelterman. Domain wall solitons in binary mixtures of Bose-Einstein condensates. *Physical Review Letters* 87: 140401 (2001).
14. Malomed, B.A., H.E. Nistazakis, D.J. Frantzeskakis & P.G. Kevrekidis. Static and rotating domain-wall cross patterns in Bose-Einstein condensates. *Physical Review A* 70: 043616 (2004).
15. Öhberg, P. & L. Santos, Dark solitons in a two-component Bose-Einstein condensate. *Physical Review Letters* 86: 2918-2921 (2001).
16. Busch, Th. & J.R. Anglin. Dark-bright solitons in inhomogeneous Bose-Einstein condensates. *Physical Review Letters* 87: 010401 (2001).
17. Berloff, N.G. Solitary wave complexes in two-component condensates. *Physical Review Letters* 94: 120401 (2005).
18. Josephson, B.D. Possible new effects in superconductive tunneling. *Physics Letters* 1: 251-253 (1962).
19. Smerzi, A., S. Fantoni , S. Giovanazzi & S.R. Shenoy. Quantum coherent atomic tunneling between two trapped Bose-Einstein condensates. *Physical Review Letters* 79: 4950-4953 (1997).
20. Raghavan, S., A. Smerzi, S. Fantoni & S.R. Shenoy. Coherent oscillations between two weakly coupled Bose-Einstein condensates: Josephson effects, π oscillations, and macroscopic quantum self-trapping. *Physical Review A* 59: 620-633 (1999).
21. Giovanazzi, S., A. Smerzi & S. Fantoni. Josephson effects in dilute Bose-Einstein condensates. *Physical Review Letters* 84: 4521-4524 (2000).
22. Albiez, M., R. Gati, J. Folling, S. Hunsmann, M. Cristiani & M.K. Oberthaler. Direct observation of tunneling and nonlinear self-trapping in a single Bosonic Josephson junction. *Physical Review Letters* 95: 010402 (2005).
23. Levy, S., E. Lahoud, I. Shomroni & J. Steinhauer. The a.c. and d.c. Josephson effects in a Bose–Einstein condensate. *Nature* 449: 579-583 (2007).
24. Cataliotti, F.S., S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi & M. Inguscio. Josephson Junction Arrays with Bose-Einstein Condensates. *Science* 293: 843-846 (2001).
25. Kaurov, V.M. & A.B. Kuklov. Josephson vortex between two atomic Bose-Einstein condensates. *Physical Review A* 71: 011601 (2005).
26. Kaurov, V.M. & A.B. Kuklov. Atomic Josephson vortices. *Physical Review A* 73: 013627 (2006).
27. Ustinov, A.V. Solitons in Josephson junctions. *Physica D: Nonlinear Phenomena* 123: 315-329 (1998).
28. Susanto, H., J. Cuevas & P. Kruger. Josephson tunnelling of dark solitons in a double-well potential. *Journal of Physics B: Atomic Molecular & Optical Physics* 44: 095003 (2011).
29. Sacchetti, A. Nonlinear time dependent one-dimensional Schrödinger equation with double well potential. *SIAM Journal on Mathematical Analysis* 35: 1160-1176 (2003).
30. Sacchetti, A. Nonlinear time-dependent Schrödinger equations: the Gross-Pitaevskii equation with double-well potential. *Journal of Evolution Equations* 4: 345-369 (2004).
31. Ostrovskaya, E.A., Yu.S. Kivshar, M. Lisak, B. Hall, F. Cattani & D. Anderson. Coupled-mode theory for Bose-Einstein condensates. *Physical Review A* 61: 031601 (2000).
32. Ananikian, D. & T. Bergeman. Gross-Pitaevskii equation for Bose particles in a double-well potential: Two-mode models and beyond. *Physical Review A* 73: 013604 (2006).
33. Jia, X.Y., Li W.D. & J.Q. Liang. Nonlinear correction to the Boson Josephson-junction model. *Physical Review A* 78: 023613 (2008).
34. Julia-Diaz, B., J. Martorell, M. Mele-Messeguer & A. Polls. Beyond standard two-mode dynamics in Bosonic Josephson junctions. *Physical Review A* 82: 063626 (2010).

Proceedings of the Pakistan Academy of Sciences 52 (3): 255–261 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Effect of ZrO_2 on Physiochemical Properties of SiO_2-Al_2O_3-CaO-MgO-ZnO Glass-ceramic System

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Abstract: In present study, monoclinic-ZrO_2 (m-ZrO_2) was used as a nucleating agent with SiO_2-Al_2O_3-CaO-MgO system, to evaluate microstructural change, crystallization tendency, mechanical and chemical properties of glass ceramics as a function of zirconia addition. It was found that zirconia not only helps in crystallization of glass ceramic but also improves mechanical properties of glass ceramic. With increase in concentration of ZrO_2 in the system the chemical durability of against acid and alkali and Vickers's hardness increases, 1.64% ZrO_2 showed superior chemical durability and mechanical strength. Therefore, the experimental results provided strong evidence that this material had all the potential properties to be used as building material, or as an engine component.

Keywords: Glass-ceramic, crystallization, nucleating agent, diopside, zirconia

1. INTRODUCTION

Glass-ceramics are fine-grained polycrystalline solids normally obtained by controlled crystallization (devitrification) of amorphous solids [1-5]. The bulk chemical composition of the crystalline phases and microstructure resulting from the nucleation and growth sequence are the key factors that influence the properties of the final material. Glass-ceramics find applications in various fields of aerospace, medical and nuclear industry [1, 6].

Glass-ceramics in the SiO_2-Al_2O_3-CaO-MgO system have been widely investigated for its various useful properties like low cost, superior mechanical strength, good chemical durability and excellent wear resistance. Diopside CaO-MgO-$2SiO_2$ crystalline phase generally precipitates in SiO_2-Al_2O_3-CaO-MgO system which has desirable mechanical strength that may also be machined to some extent [6-8]. Diopside ($CaMgSi_2O_6$) glass-ceramics are well known for their mechanical and electrical properties, i.e., high mechanical strength, low dielectric constant, high quality factor and low temperature sinterability [9]. From previous studies it is well known that pure form of zirconia has three different crystallographic forms; monoclinic, tetragonal and cubic. The monoclinic zirconia remains stable up to 1150 ^oC when it transforms to tetragonal symmetry. At temperatures above 2300 ^oC the cubic form exists. The transformation tetragonal-to-monoclinic phase change is of technological importance as this stress-induced martensitc phase transformation at crack tip makes the material resistant to crack propagation, which is accompanied by a volume expansion of about 4% of material [10-12]. Therefore, glass-ceramics having secondary phase of tetragonal zirconia is supposed to improve strength of material as it is well known

Received, February 2015; Accepted, May 2015
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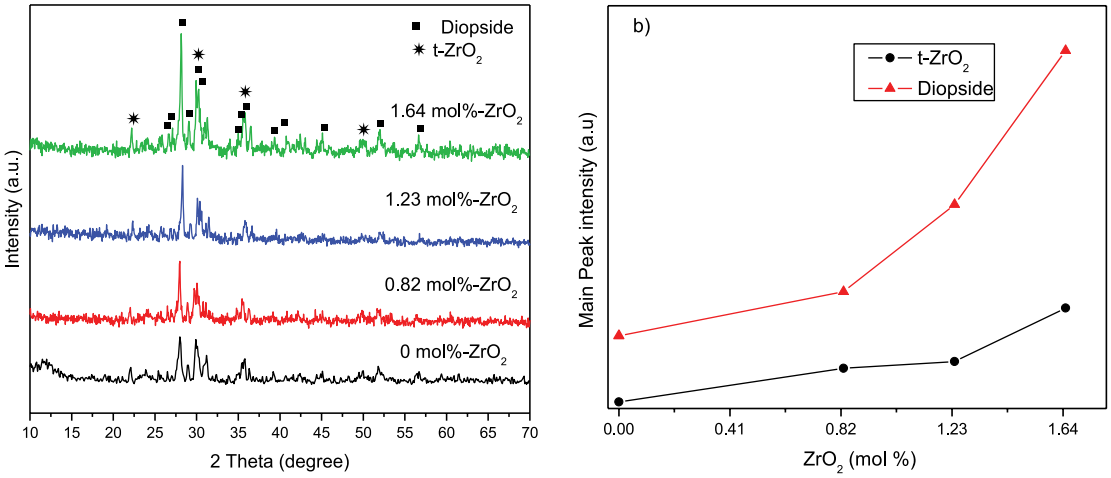

Fig. 1: (a) X-ray diffraction patterns of $CaMgSi_2O_6$ added with different amount of ZrO_2 after sintered at 1150 °C for 4 h; **(b)** Phase amount of $CaMgSi_2O_6$ and t-ZrO_2 in specimens at 1150 °C for 4 h.

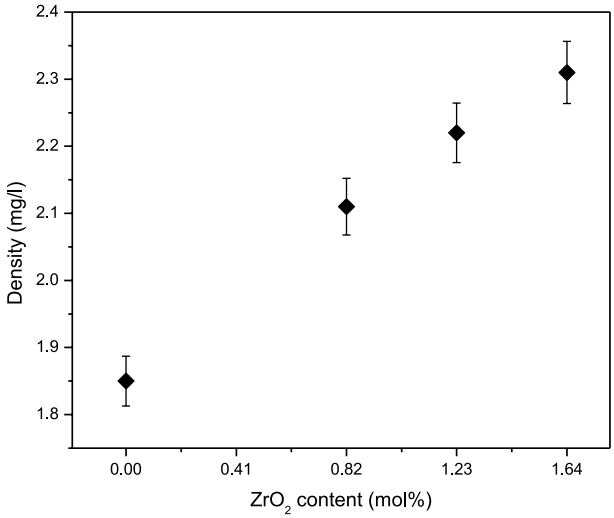

Fig. 2. The density of glass ceramic specimens sintered at 1150 °C for 2h as function of ZrO_2 concentration.

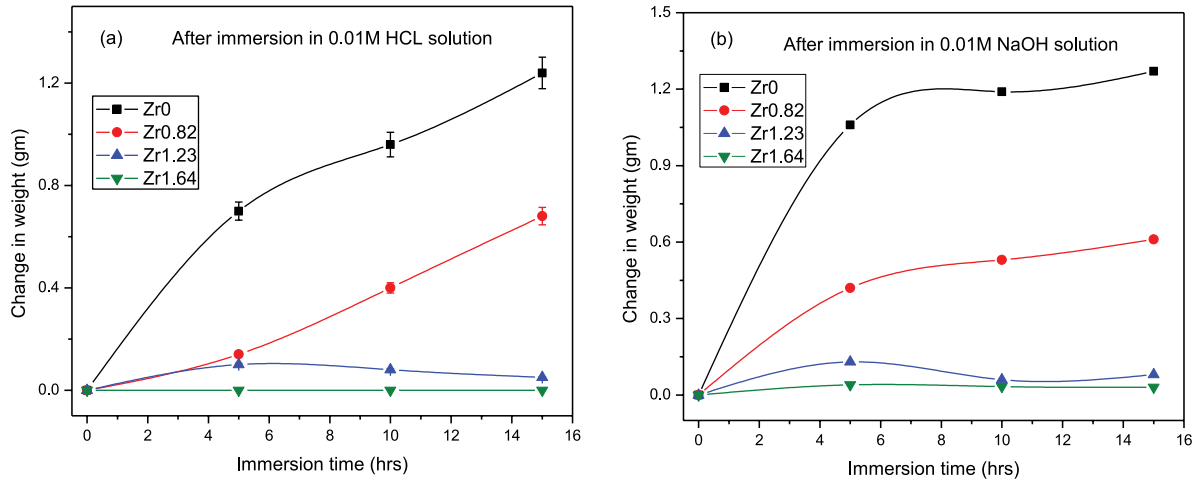

Fig. 3(a), 3(b): The chemical resistance against acid and alkali studied for various time intervals for all glass ceramic samples.

that addition of second strong phase can improve mechanical properties of material [7, 10, 13].

In present study, monoclinic-ZrO_2 was used with SiO_2-Al_2O_3-CaO-MgO system, to evaluate microstructural change, crystallization tendency, mechanical and chemical properties of glass ceramics as a function of zirconia addition.

2. EXPERIMENTAL PROCEDURE

Three batch mixtures in SiO_2-Al_2O_3-CaO-MgO-ZrO_2 system with nominal composition mentioned in Table 1 was prepared from reagent grade chemicals. A small amount of ZnO was also added to system which is known to enhance chemical durability and to provide gloss to the glass-ceramic [9]. Raw materials were weighed and thoroughly mixed for one hour. Premixed batches were melted in Pt crucible, using preheated muffle furnace (Model 5132 Lindberg, USA) at temperature range 1450-1500 °C for 2hrs and then poured in cold water to obtain transparent glass frits, which was then dried and milled with agate mortar and pestle. The glasses were pulverized to particle size ≤40μm. Pellets (15mm in diameter) used for crystallization experiment and test pieces (3 × 4 × 40 mm) used for property measurements were formed by uniaxial pressing at 56 MPa.

Glass batches were converted into ceramic derivatives by controlled crystallization in order to achieve the desired crystalline structure at significant growth rate [15]. The pellets were sintered at 1150 °C for 2h for crystallization. The crystal phases grown in samples were determined by X-ray diffraction (XRD) (Model Bruker D8 Discover, Germany) using CuKα radiation in 2θ range from 10° to 70° and average crystallite size was measured by the Scherer's formula:

$$t = (K\lambda)/(B\cos\theta)$$

The bulk densities of glass-ceramics were measured by the Archimedes method using water as buoyant. The micro-hardness of glass-ceramics was measured via Vickers indentation (SHIMADZU, Japan) with indentation of 9.8N for 15 sec using diamond indenter; three valid indentations (no evident cracks or other defects) were considered to calculate a mean value.

The chemical durability of the glass-ceramics against acidic and alkaline conditions were examined, and especially in acidic condition because these materials are considered to have potential application as building material or as an engine component where it has to face serious threats of leaching and weight loss due to exposure to acid rains (occur in many countries) and chemical reactions inside engine, so all glass ceramics were examined for 0.01M HCL and 0.01M NaOH solutions at room temperature for various time intervals as a function of weight and density gradient.

The microstructure of glass-ceramic samples were observed by SEM-EDX (microscope FEI-QUANTA-200) operated in scanning electron emission mode.

3. RESULTS AND DISCUSSION

The investigation of the glass-ceramic materials Zr0.82, Zr1.23 and Zr1.64 by X-ray diffraction analysis (Fig. 1(a)) revealed that diopside ($CaMgSi_2O_6$) as primary phase developed in all

Table 1. Nominal composition of batches in mol %.

Sample No.	Oxide Composition (mol %)					
	SiO_2	Al_2O_3	CaO	MgO	ZnO	ZrO_2
Zr0	48.64	7.5	31.36	9.0	3.5	0
Zr0.82	47.82	7.5	31.36	9.0	3.5	0.82
Zr1.23	46.59	7.5	31.36	9.0	3.5	1.23
Zr1.64	46.18	7.5	31.36	9.0	3.5	1.64

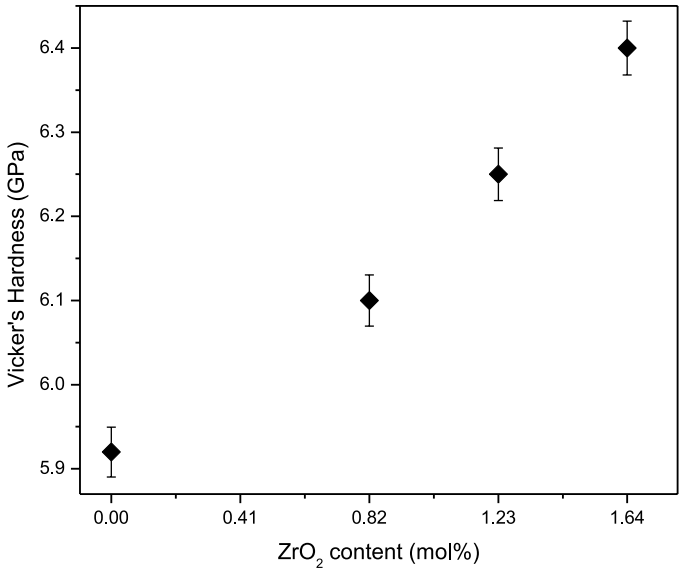

Fig. 4. Hardness of glass ceramic as function of zirconia content.

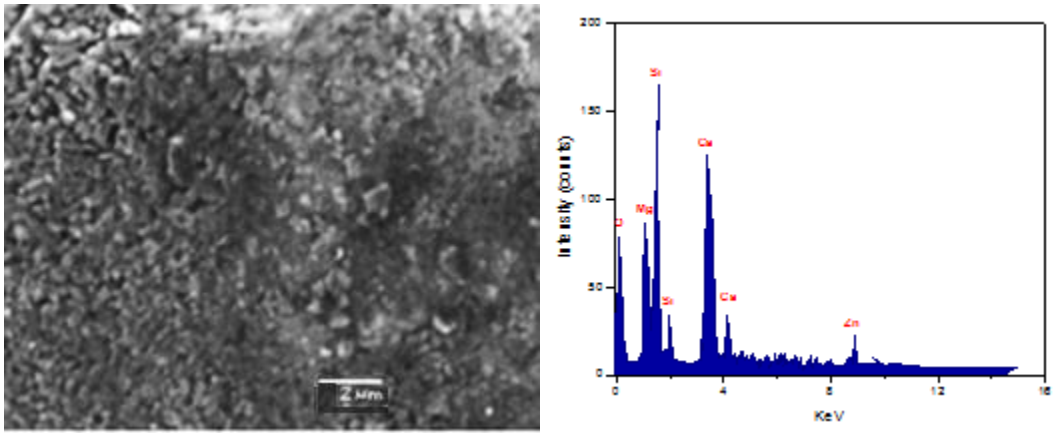

Fig. 5(a). SEM-EDX images of different magnification at Zr0.

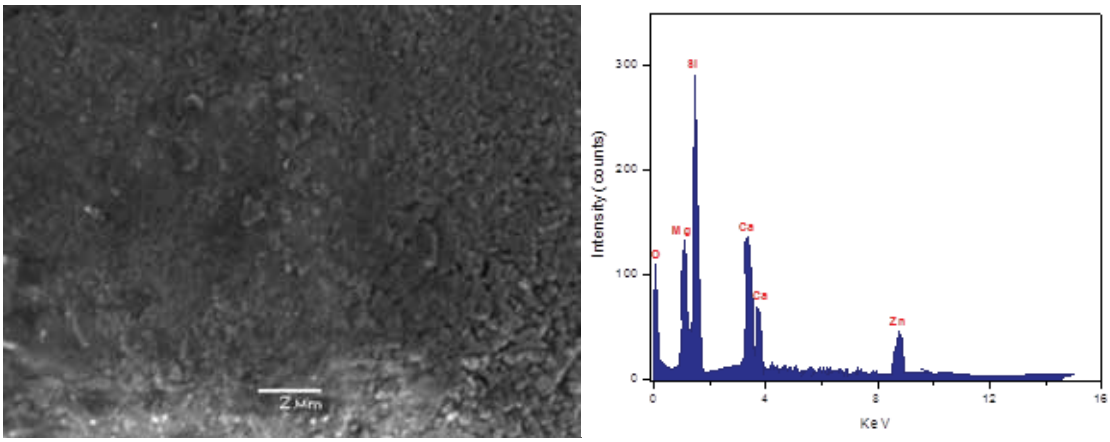

Fig. 5(b). SEM images of different magnification at Zr0.82.

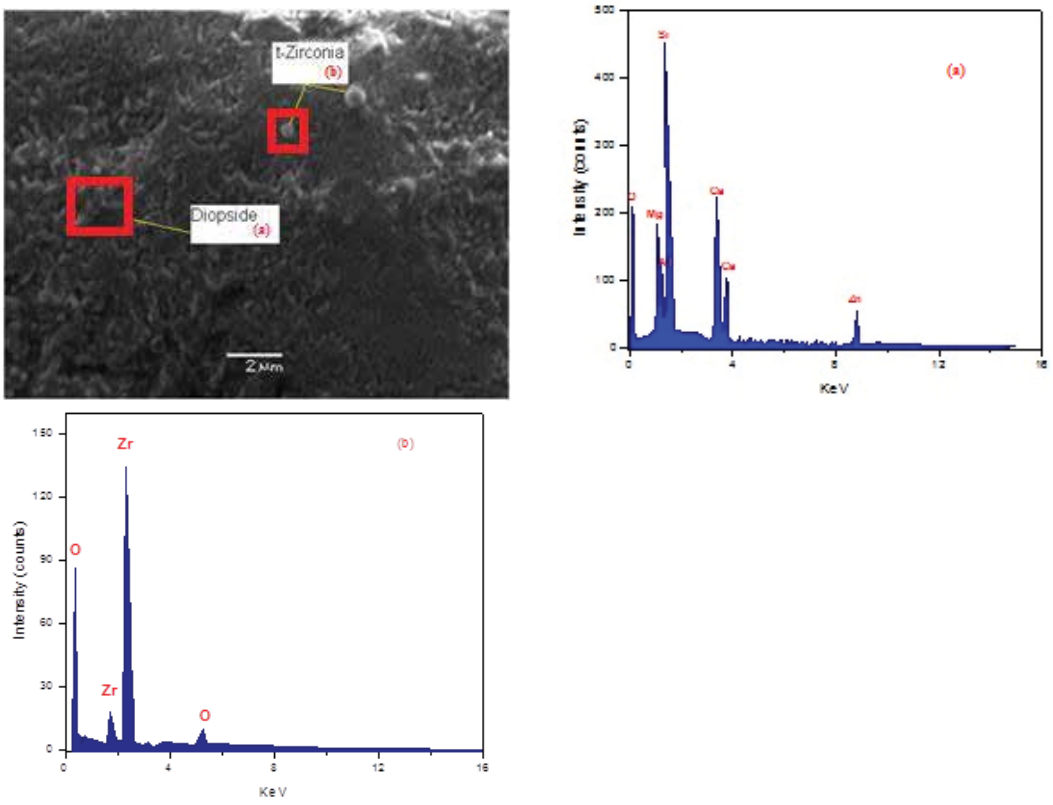

Fig. 5(c). SEM-EDX images of different magnification at Zr1.23.

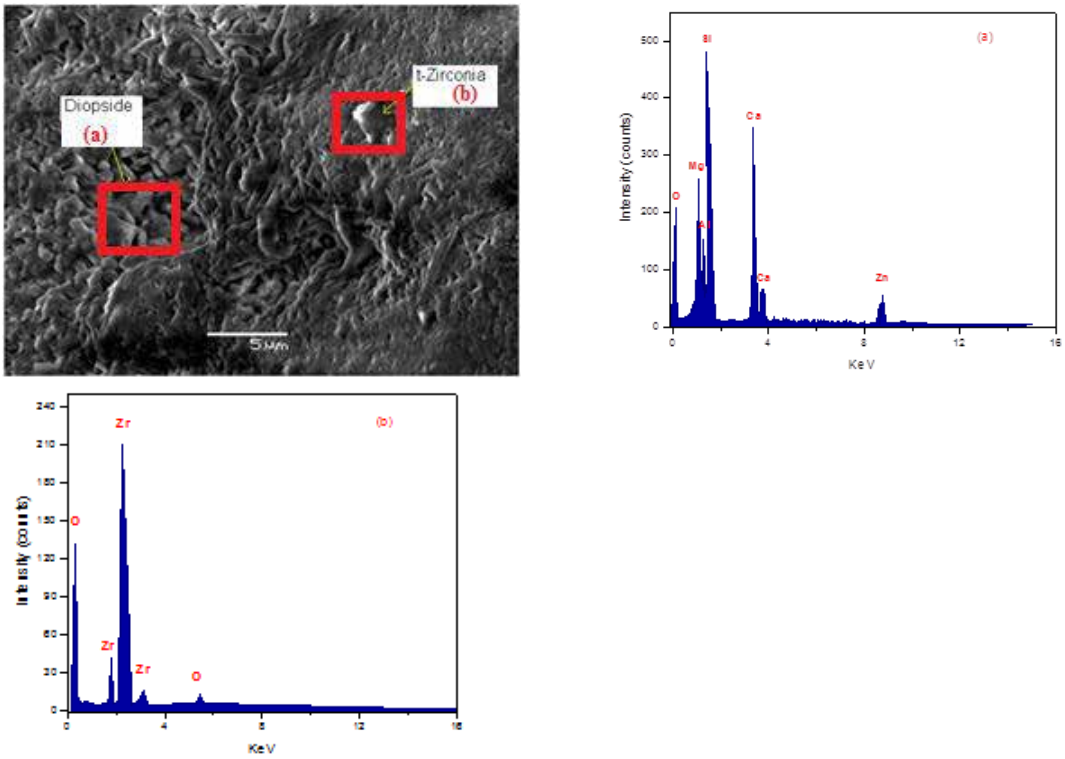

Fig. 5(d). SEM-EDX images of different magnification at Zr1.64.

glass ceramics and tetragonal-Zirconia (t-ZrO_2) was crystallized as secondary minor phase when glass was sintered at 1150 °C for 2 hours holding time. The phase intensity of diopside ($CaMgSi_2O_6$) increases by progressive increment of monoclinic-ZrO_2 (m-ZrO_2) concentration in the system therefore it was found that ZrO_2 stimulates crystallization. It may be understood as increasing ZrO_2 content in the system the peaks became sharper indicating more ordered crystalline structure of diopside phase. The amount of diopside phase increases illustrated by increase in peak intensity and amount of phase shown in (Fig. 1(b)), similar trend had been observed earlier by Feng et.al that Zirconia play a positive role in nucleating crystallization of diopside based glass ceramic [14]. The average crystallite size was ≈ 0.04117 μm (41.17 nm) measured by Scherer's formula.

The density of glass ceramics as a function of ZrO_2 concentration after sintering at 1150 °C was shown in (Fig. 2). However, a small change in densities of samples was observed which was therefore due to reduction in porosity during heat treatment process. It was also observed that reinforcement of t-ZrO_2 in diopside crystalline structure enhanced the densification of glass ceramics.

The chemical durability against 0.01M HCL and 0.01M NaOH solutions was found to improve with increase in concentration of ZrO_2 in the system illustrated by the change in weight of specimens after soaking in respective solutions shown in (Figure 3(a)), (Figure 3(b)) and Figure 3(c) respectively. This result indicated that addition of ZrO_2 to SiO_2-Al_2O_3-CaO-MgO system the provided it resistance against acidic and alkaline solution, as ZrO_2 is known to be good chemical stability [15].

The Vickers's hardness shown in (Fig. 4) was measured 5.92±1 GPa for Zr0, 6.1±2 GPa for Zr0.82, while Zr1.64 had 6.4±3 GPa, it show that secondary phase improved its mechanical properties. As it is well known that a small monoclinic-tetragonal (m-t) phase transformation would therefore play its role in enhancement of mechanical properties of glass ceramic [16].

Fig. 5(a), Fig. 5(b), Fig. 5(c) and Fig. 5(d) illustrate the SEM-EDX results of Zr0, Zr0.82, Zr1.23 and Zr1.64 ,respectively; it was found that with increase in the concentration of zirconia enhancement of crystal nucleation, grain growth in diopside phase and significantly reduction of porosity can clearly be identified in SEM images. The Zr1.64 glass ceramic can be seen more densified and grains of diopside also became enlarged. A few spherical shape grains of t-ZrO_2 were also visible in the SEM micrograph confirmed by the EDX spectra.

4. CONCLUSIONS

From our experimental results it was revealed that zirconia play stimulatory role in crystallization of glass ceramic in SiO_2-Al_2O_3-CaO-MgO-ZnO system. Zirconia not only enhanced densification process but also induced certain improved chemical and mechanical properties, which was therefore due to monoclinic to tetragonal (m-t) phase transformation of zirconia.

5. REFERENCES

1. Yekta, B.E., P. Alizadeh & L. Rezazadeh. Synthesis of glass-ceramic glazes in the ZnO-Al_2O_3-SiO_2-ZrO_2 system. *Journal of European Ceramic Society* 27: 2311–2315 (2007).
2. Alizadeh, P. & V.K. Marghussian. The effect of compositional changes on the crystallization behavior mechanical properties of diopside±wollastonite glass-ceramics in the SiO_2±CaO±MgO Na_2O system. *Journal of European Ceramic Society* 20: 765-773 (2000).
3. Tulyaganov, D.U., M.J. Ribeiro & J.A. Labrincha. Development of glass-ceramics by sintering crystallization of fine powders of calcium-magnesium-aluminosilicate glass. *Ceramic International* 28: 515-520 (2002).
4. Rezvani, M. Effects of various nucleation agents on crystallization kinetic of LAS glass ceramic. *Iranian Journal of Science and Technology* 8(4): 41-49 (2011).
5. Alizadeh, P.B., E. Yekta & T. Javadi. Sintering behavior mechanical properties of the mica–diopsidemachinable glass-ceramics. *Journal of European Ceramic Society* 28: 1569-1573 (2008).
6. Wu, C., J. Chang. J. Wang. S. Ni & W. Zhai.

Preparation characteristics of a calcium magnesium silicate bredigite bioactive ceramics. *Key Engineering Material* 26: 2925-2931 (2005).

7. Gubicza, J. Characterization of glasses ceramics by continuous indentation Tests. *Key Engineering Material* 103: 217-220 (1995).
8. Choi, B.K. & E. S. Kim. Effects of crystallization behavior on microwave dielectric properties of $CaMgSi_2O_6$ glass-ceramics. *Journal of Korean Ceramic Society* 50(1): 70-74 (2013).
9. Upadhyaya, G.S. *Sintered Metallic Ceramic Materials Preparation, Properties Applications.* Wiley Publishers, Canada, p. 501-503 (2000).
10. Salman, S.M., S.N. Salama, H. Darwish & H.A. Abo-Mosallam. In vitro bioactivity of glass-ceramics of the $CaMgSi_2O_6$-$CaSiO_3$-$Ca_5PO_{43}F$-Na_2SiO_3 system with TiO_2 or ZnO additives. *Ceramic International* 35:1083-1093 (2009).
11. Abolfazli, M., T.F. Morteza. A. Kaveh & N. Rahim. Study the effect of zirconia to increase the abrasion resistance density in alumina-zirconia system. *Journal of Basic and Applied Science Research* 3(5): 40-48 (2103).
12. Saridag, S., O. Tak & G. Alniacik. Basic properties types of zirconia: An overview. *World Journal of Stomatology* 2(3): 40-47 (2013).
13. Feng, K.C., Y.H. Su. C.C. Chou. Z.M. Liu & L.W. Chu. Defect analysis in $CaMgSi_2O_6$ glass-ceramic under reduction atmosphere. *Chinese Journal of Physics* 50(6): 932-938 (2012).
14. Masai, H., Y. Takahashi. & T. Fujiwara. Glass-ceramics containing nano-crystallites of oxide semiconductor. In: *Ceramic Materials*. Sciyo. W. Wunderlich (Ed). InTech Europe (www.intechopen.com) p. 29-48 (2010).
15. Montazerian, M., P. Alizadeh & B.E. Yekta. Pressureless sintering mechanical properties of mica glass–ceramic/Y-PSZ composite. *Journal of European Ceramic Society* 28(14): 2687–2692 (2008).

Proceedings of the Pakistan Academy of Sciences 52 (3): 263–269 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Research Article

Solution of 7 Bar Tress Model Using Derivative Free Methods

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Abstract: The focus of this research is to formulate optimization model of 7-bar trusses along with stress, stability and deflection constraints. The derivative free methods are used for the optimization of engineering design problems. These methods are basically designed for unconstrained optimization problems. In formulated optimization truss problems the constraints are handled by using exterior penalty functions. The results of the truss optimization model are obtained by using MATLAB which demonstrate the effectiveness and applicability of these derivative free methods.

Keywords: Derivative free methods, penalty function, structural optimization, truss structure, unconstrained optimization

1. INTRODUCTION

The optimization phenomenon appears in very nearly all ranges of life like assembling, scheduling, engineering and business. Utilizing optimization procedures the best results of the problem are attempted to get by using least measure of restricted assets [1].

Two principle procedures of optimization, specifically, derivative based and derivative free are, no doubt utilized frequently. Among the direct search methods we concentrated on Hooke and Jeeves (HJ) strategy [2], Nelder and Mead (NM) strategy [3-5] and Multi-Directional search (MDS) technique [6]. These methods are intended for unconstrained optimization issues. They can additionally connected to constrained optimization problems by changing them into unconstrained optimization problems by utilizing the penalty function [7, 8]. The structures of the penalty function together with views for alter penalty parameters at the end of each one unconstrained minimization step describe specific scheme or strategy.

In the early years when the derivatives of functions were weigh down to calculate, the direct search methods were prevalent, yet as of late, we have various devices for strong and automatic differentiation [9] and additionally modeling languages[10] that cost derivatives consequently. In spite of this, direct search methods having their importance. Especially the development of simulation-based optimization [11] has made it hard to utilize derivative based methods. In addition, the objective function which is not numeric in nature can't be simplified by derivative based approaches.

For calculating different sorts of optimization issues a lot of direct search methods have been produced by the analysts. A definite investigation of these systems, with recorded foundation, might be found in [12].The consideration of this system is that change the constrained optimization problem to an unconstrained one by adding/subtracting the value of or from the objective function focused around constraint present in the result [13]. Specialist's effort to improve the preliminary structure of equipment and strive to upgrade the operation of that supply once it is introduced to understand the biggest generation, the best benefit, the base cost and the minimum energy utilization [14].

Structural Optimization Problem: The structural optimization problem [15] minimizing the objective function (expense, weight, volume)

Received, March 2015; Accepted, June 2015
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subject to demands on mechanical constraints. The aggregate structural volume (or weight) is typically allocated as the target capacity, in light of the fact that it is an elementary prerequisite to reduce the weight of the aeronautics and mechanical structures. For the structures in architectural engineering and civil engineering, reduction of weight toward oneself for the most part stimulates decreasing of the shape weights, and consequently encourages lower cost. Structural optimization may be sub-divided into shape optimization and topology optimization [16]. Structural optimization concerns could be attractively easy to figure, might be collected as, Find *x* to minimize subject to g*(x) ≤ 0*. Here f is the objective function and g is the constraints. Problem of this sort are called numerical programming problems

$$Min\ f(x)$$

$$Subject\ to\ g(x) \leq 0$$

Structural Design & Size: Derivative free strategies inspect instruments to make structural optimization that is prepared for size and shape streamlining of truss and edge structures. Limit is extended by including graphical overview utilities for structure visualization and enhancement process. The objective of the structural streamlining is the minimization of volume with stress and displacement soft-constraints.

Truss Structures: Truss parts are one dimensional in their close-by encourages structure and passes on simply axial loads in view of their pin relationship at nodes. This moreover infers that a truss node is simply allowed translational degrees of freedom. A truss segment needs simply a cross sectional region (A) to expose its geometry as a result of the critical load limit, and its length is controlled by the range of its end nodes. A three-dimensional truss segment has two nearby degrees of freedom and six global degrees of freedom, with three translational degrees of inflexibility at every one end of the components [17].

2. MATERIALS AND METHODS

2.1. Development of N Bar Truss Model

Consider N bar trusses, in these trusses we try to optimize the weight under stress constraint. Cross sectional area is considered as design variables.

Objective Function

In this problem, the objective function we have considered is weight of the general truss. The ρ is the parameters of material thickness and Li is the parameters of length of i^{th} part, respectively.

$$f(A) = \sum_{i}^{n} \rho A_i L_i$$

$$A_i \geq 0, \qquad i = 1, \dots, n$$

Constraints

Firstly, points out the area and the amount of fundamental nodes for supports and loads. Accordingly, a feasible truss must have all the fundamental nodes.

Secondly, the truss must not deflect more than the allowable limit due to the application of loads.

$G_2 = \sum \delta_k^{max} - \sum \delta_k (A)$, k = 1,2,…,n

Thirdly, in a feasible truss, all parts must have focuses inside the suitable quality of the material. Since, typically a truss is subjected to different loading conditions connected independently; these demands must be utilized for each one loading condition. Since the trusses of different topologies are made on the fly, some of them may be statically determinate and some of them may be statically uncertain. Hence, we have utilized derivative free strategies to compute the stress and deflection.

$G_3 = \sum S_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

Finally, in a feasible truss all members must have stresses within the allowable strength of the material. Some bar trusses have compressive force and these become compressive stress constraint and some have tensile force and these become tensile stress constraint.

$G_4 = \sum T_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

$G_5 = \sum C_j - \sum \sigma_j(A) \geq 0$, j = 1,2,…,m

In the above NLP problem where ρ is the density of the material, it may be focused that this specific objective function does not depend on any state variable, as design constraints. The parameter S_j is the allowable strength of the material, T_j is the allowable tensile of the material, δ_k^{max} is the allowable deflection in the truss and C_j is the allowable compressive strength of the material. We recommended that the cross sectional areas must, for obvious physical reason, be non-negative $A_i \geq 0, i = 1, \dots, n$.

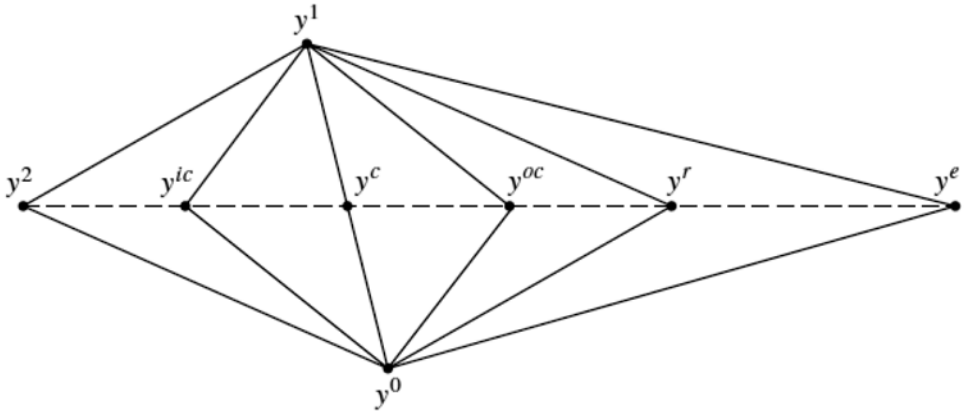

Fig. 1. Algorithm of Nelder–Mead method.

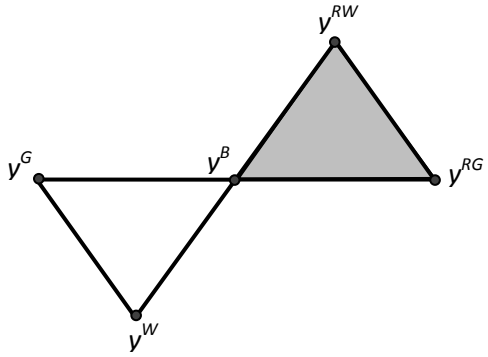

Fig. 2. Reflection.

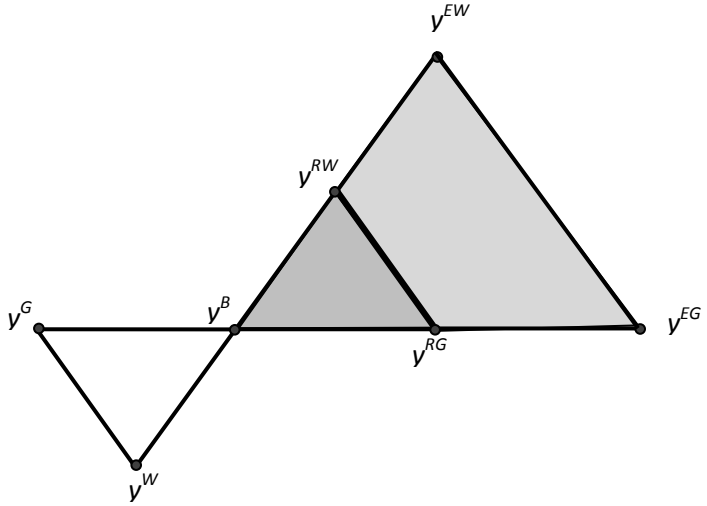

Fig. 3. Expansion.

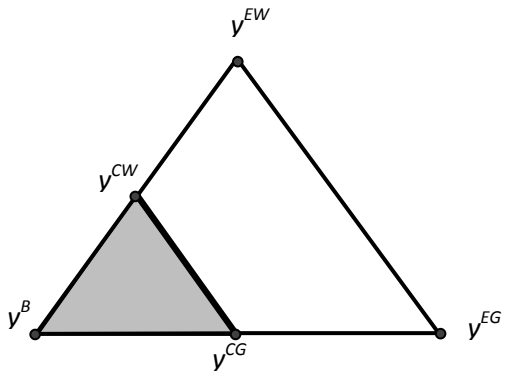

Fig. 4. Contraction.

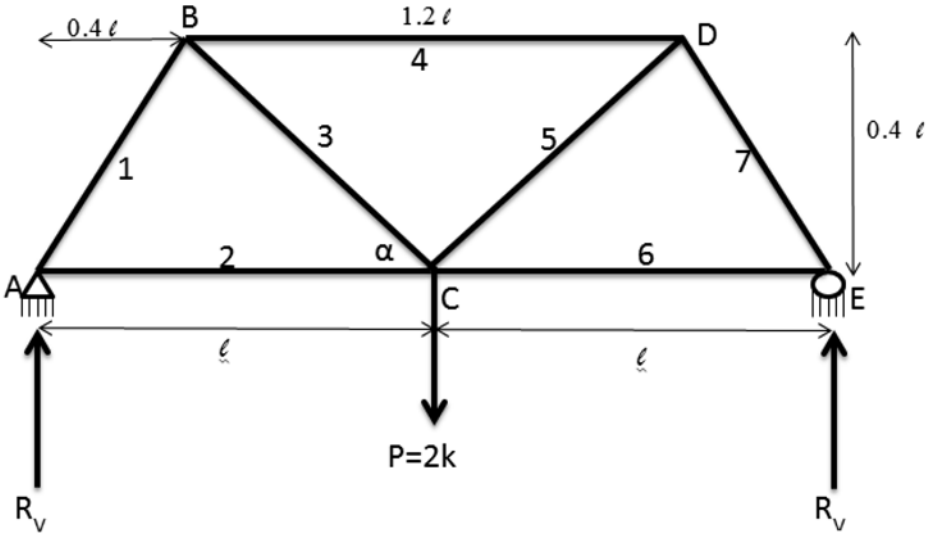

Fig. 5. Seven bar truss.

Hooke and Jeeves Method

This method starts with an initial point. In N-dimensional problem a Set of N linearly independent search directions generate 2N points.

Exploratory Move: Exploratory move is performed on the current point systematically to find the best point around the current point.

Pattern Move: When exploratory move success then pattern move is perform, a new point is found by jumping from the current base point along a direction connecting to the previous.

Nelder-Mead Simplex Method

The method uses the following operations

Reflection: Reflect the worst vertex over the centroid.

Expansion: If the function value at the reflect point is less than best point the expansion is performed.

Contraction: If the function value of the reflection point lies between the good and best vertex then

Inner Contraction: If function value greater than the best point then inner contraction is performed.

Outer Contraction: If the function values less than the best point then outer contraction is performed as shown in Fig. 1.

Shrink: If no one from the above conditions is satisfied then shrink produced.

Multi-directional Search Method

In N- dimensional problem method starts with a simplex of N+1 points. The method generates N points along N linearly independent search

directions. The method uses the following operations

Reflection: The worst and good point is reflectede at the best point as shown in Fig. 2.

Expansion: If the value of the reflection points is less than the best point then expansion is performed as shown in Fig. 3.

Inner Contraction: If the values of the reflection points is not less than the best point then contraction is performed as shown in Fig. 4.

3. FORMULATION OF SEVEN BAR TRUSS MODEL

Consider a seven bar truss. The bars AB and DE, AC and CE , BC and DC have similar length but the bar BD have different length from the other bars and young modulus E. We are to minimize the weight under stress constraint. The design variables are the cross sectional areas$A_1, A_2, A_3, A_4, A_5, A_6, A_7$. Due to symmetry

$$A_1 = A_7, A_2 = A_6, A_3 = A_5 .$$

Thus there are particularly four design variables A_1, A_2, A_3, A_4 The objective function i.e the total weight of the truss becomes

$$f(A) = (A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7)\rho l$$

In this issue the amount of bars equivalent the amount of the level of flexibility, which infers that the bar constrains or burdens, may be gotten specifically from the harmony mathematical statements. We say that the truss is statically determinate.

Objective

In this problem we are interested to minimizing the weight of the truss structure.

Objective Function

Minimize $1.132A_1l+2A_2l+1.42A_3l+1.2A_4l$

Subject to Constraints

$\frac{P\text{Csc}\theta}{2A_1} \leq S_{c1}$ (Compressive Stress Constraint)

$\frac{P(\text{Cot}\theta + \text{Cot}\alpha)}{2A_4} \leq S_{c4}$ (Compressive Stress Constraint)

$\frac{P\text{Cot}\theta}{2A_2} \leq S_{t2}$ (Tensile Stress Constraint)

$\frac{P\text{csc}\theta}{2A_3} \leq S_{t3}$ (Tensile Stress Constraint)

$\frac{P}{2\text{Sin}\theta} \leq \frac{\pi EA_1^2}{1.28l^2}$ (Stability Constraint)

$\frac{P(\text{Cot}\theta + \text{Cot}\alpha)}{2} \leq \frac{\pi EA_4^2}{5.76l^2}$ (Stability Constraint)

$\frac{Pl}{E}\left(\frac{0.566}{A_1} + \frac{0.500}{A_2} + \frac{2.236}{A_3} + \frac{2.700}{A_4}\right) \leq \delta_{max}$

$(Deflection\ constraint)$

$10mm^2 \leq A_1, A_2, A_3, A_4 \leq 500mm^2$

4. RESULTS AND DISCUSSION

The result which we get from seven bar truss by applying HJ method is that, when we take the initial guess in the range of 1 to 4. We have taken

Table 1. Parameters of seven bar truss problem.

Parameter	Description	Value
S_{ci}	Allowable Compressive strength in bar i	500 Mpa
S_{ti}	Allowable Tensile strength in bar i	500 Mpa
δ_{max}	Allowable deflection	2 mm
E	Modulus of Elasticity	200 Gpa

Table 2. The Result of seven bar problem by applying Hooke and Jeeves method.

Initial guess	Function value	Final point	Function value	No. of Function Evaluations
1,3,2,2	22.856	0.0724, 0.0110, 0.0174, 0.1528	0.3120	232

Table 3. The Result of seven bar problem by applying Nelder-Mead method.

Initial guess	Function value	Final point	Function value	No. of function Evaluations
2,2,1,1,1,3 ,3,1,2,1,2,2, 3,3,3,1,1,3, 2,2	235.70	(0.0722, 0.0717, 0.0722, 0.0722, 0.0722, 0.0063, 0.0063, 0.0057, 0.0063, 0.0063, 0.0237, 0.0237, 0.0237, 0.0231, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)	0.3100	217

Table 4. The Result of seven bar problem by applying Multi-directional search method.

Initial guess	Function value	Final point	Function value	No. of Function Evaluations
2,2,1,1,1,3 ,3,1,2,1,2,2, 3,3,3,1,1,3, 2,2	237.70	0.0712, 0.0712, 0.0622, 0.0522, 0.0222, 0.0043, 0.0043, 0.0047, 0.0033, 0.0033, 0.0235, 0.0235, 0.0235, 0.0232, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523	0.2522	325

lot of points between this range and apply this method the result does not show the consistent performance. And the final solution which we get is feasible because it satisfied all the constraints and no constraint is active at this solution. The function value is 0.3120 at the points (0.0724, 0.0110, 0.0174, 0.1528).

The best result which we get from seven bar truss by applying NM method is that, when we take the initial guess in the range of 1 to 10. We have taken lot of points between this range and apply this method the result does not show the consistent performance. And we do not take the better point in this range. But when we take the initial guess in the range 1 to 5, it also show not consistent performance but we got a point which is converges and the solution which we get from this is feasible and satisfied all the constraint and no active constraint at this solution. The function value is 0.3100 at the points (0.0722, 0.0717, 0.0722, 0.0722, 0.0722, 0.0063, 0.0063, 0.0057, 0.0063, 0.0063, 0.0237, 0.0237, 0.0237, 0.0231, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)

The best result which we get from seven bar truss by applying MDS method is that, when we take the initial guess in the range of 1 to 10. We have taken lot of points between this range and applied this method the result does not show the consistent performance. And we do not take the better point in this range. But when we take the initial guess in the range 1 to 5, it also show not consistent performance but we got a point which is convergent and the solution which we get from this is feasible and satisfied all the constraint and there is no active constraint at this solution. The function value is 0.2522 at the points (0.0712, 0.0712, 0.0622, 0.0522, 0.0222, 0.0043, 0.0043, 0.0047, 0.0033, 0.0033, 0.0235, 0.0235, 0.0235, 0.0232, 0.0237, 0.1518, 0.1518, 0.1518, 0.1518, 0.1523)

5. CONCLUSIONS

We applied Hooke and Jeeves method, Nelder-Mead method and Multi-directional search method on seven bar truss optimization problems. We implemented these three methods in MATLAB on the formulated problems for many times at various initial guesses and for a number of step sizes. Observing all tables we can conclude that result of Nelder and Mead is not acceptable due to its far away convergence even its number of function evaluations is smaller than number of function evaluations of MDS method. Function value of MDS method is comparatively much better than

function value of Nelder and Mead method. By comparing the function values obtained by these three methods we conclude that the performance of N&M method is worse than the other two methods. From these tables we conclude that the performance of Multi-directional Search method is better than the other two methods because the function value is smaller than the function value of other two methods.

6. REFERENCES

1. Rao, S.S. *Engineering Optimization Theory and Practice*. John Wiley & Sons, Canada (2009).
2. Hooke, R. & T.A. Jeeves. Direct Search Solution of Numerical and Statistical Problems. *Journal of the Association for Computing Machinery* 8: 212-229 (1961).
3. Price, C.J., I. D. Coope & D. Byatt. A Convergent Variant of the Nelder-Mead algorithm. *Journal of Optimization Theory and Applications*113: 5 –19 (2002).
4. Mckinnon K.I.M. Convergence of the Nelder–Mead Simplex Method to a Non-Stationary Point. *Society for Industrial and Applied Mathematics* 9: 148–158 (1998).
5. Lagarias J. C., J. A. Reeds, M. H. Wright & P. E. Wright. Convergence Properties of the Nelder–Mead Simplex Method in Low Dimensions. *SIAM Journal of Optimization* 9: 112–147 (1998).
6. Torczon, V.J. *Multidirectional Search: A Direct Search Algorithm for Parallel Machines*. PhD dissertation, Rice University, Houston, Texas, USA (1989).
7. Conn, A. R., K. Scheinberg & L.N. Vicente. *Introduction to Derivative-Free Optimization*. MPS-SIAM Series on Optimization. MPS-SIAM, Philadelphia (2009).
8. Tabassum, M.F. Solution of War Planning Problem Using Derivative Free Methods. *Science International* 27: 395-398 (2015).
9. Grippo, L. On Automatic differentiation, in Mathematical Programming: Recent Developments and Applications. M. Iri & K. Tanabe (Ed.). Kluwer Academy, p. 83-108 (1989).
10. Brooke, A., D. Kendrick, & A. Meeraus. *GAMS: A User Guide*. The Scientific Press, Danvers, USA (1988).
11. Andrad'ottir, S. Handbooks of Simulation: Principles, Methodology, Advances, Applications and Practice. John Wiley & Sons, New York (1998).
12. Lewis, R.M., V. Torczon, & M.W. Trosset. *Direct Search Methods: Then and NowNASA/CR--210125, ICASE Report No. 2000-26.* Prepared for Langley Research Center under Contact NAS1-97046 (2000).
13. Ashok, D. B. &T. R. Chandrugupta. *Optimization Concepts and Applications in Engineering*. Cambridge University Press, USA (2011).
14. Edger, Himmalblau & Lesdon. *Optimization Of Chemical Processes*. *The McGraw-Hill Companies*, New York (2001).
15. Isaac E. &M. Ohsaki. *Optimization and Anti-Optimization of Structures Under Uncertainty*. Imperial College Press, London (2010).
16. William D.N. *Multidisciplinary Structural Design and Optimization for Performance, Cost, and Flexibility.* University of California, Los Angeles, CA, USA (2001).
17. Brian, J.A. Size and Shape Optimization of Frame and Truss Structures through Evolutionary Methods.MS thesis, University of Idaho, Moscow (2005).

Proceedings of the Pakistan Academy of Sciences 52 (3): 271 (2015)

ISSN: 0377 - 2969 (print), 2306 - 1448 (online)

Pakistan Academy of Sciences

Corrigendum

We regret for an inadvertent error in Fig. 1 of the following Research Article:

Ashraf, Arshad, and Zulfiqar Ahmad (2015) aquifer ressponse to variable ground pumpage scenarios in Indus Basin through finite element modeling. *Proceedings of the Pakistan Academy of Sciences* Volume 52(2), June 2015 issue, Pages 125–134.

The correct Fig. 1 is as under:

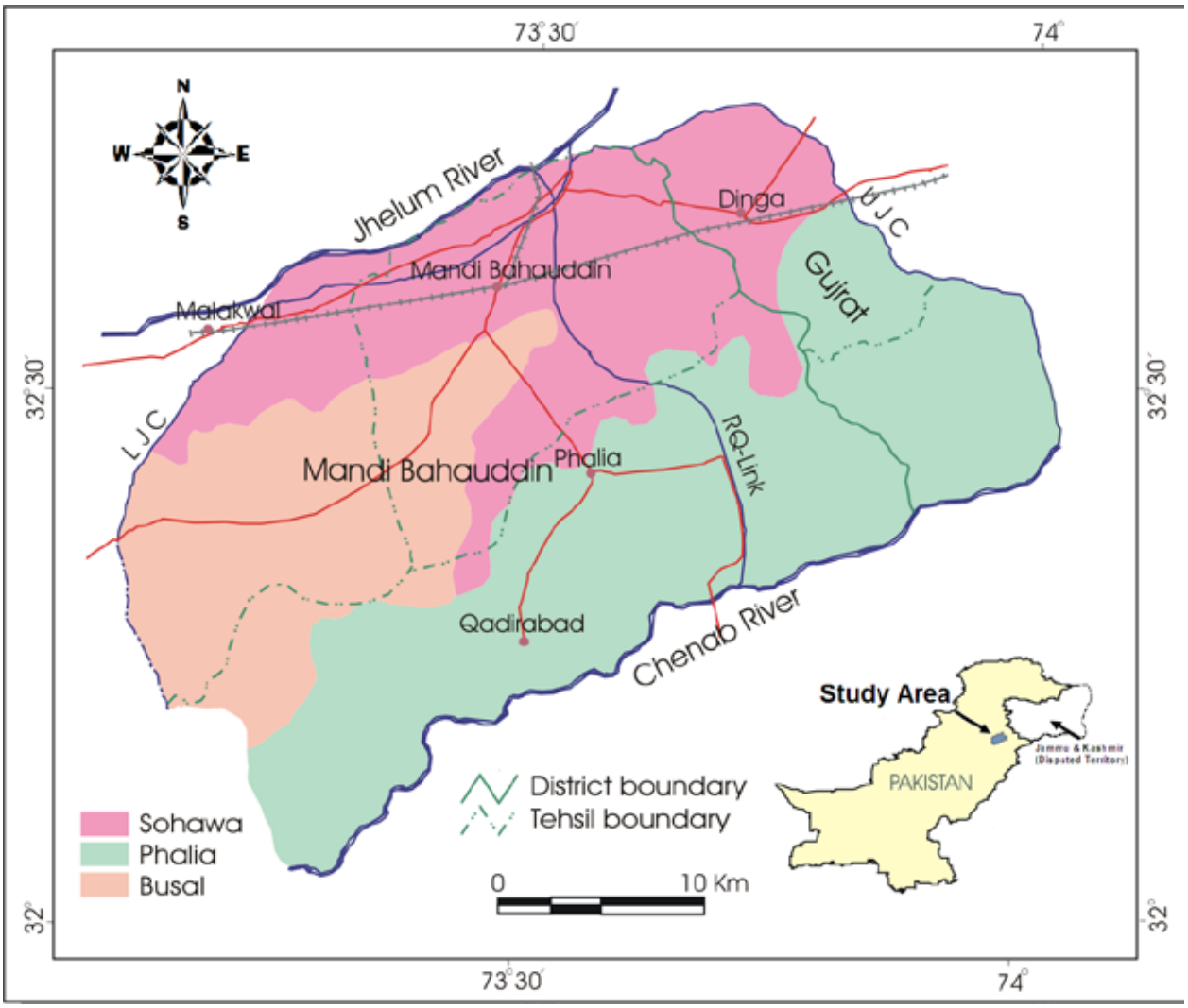

Fig. 1. Location map of the study area.

Editor-in-Chief

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1. Golding, I. Real time kinetics of gene activity in individual bacteria. *Cell* 123**:** 1025–1036 (2005).
2. Bialek, W. & S. Setayeshgar. Cooperative sensitivity and noise in biochemical signaling. *Physical Review Letters* 100**:** 258–263 (2008).
3. Kay, R.R. & C.R.L. Thompson. Forming patterns in development without morphogen gradients: differentiation and sorting. *Cold Spring Harbor Perspectives in Biology* 1: doi: 10.1101/cshperspect.a001503 (2009).

b. **Books**

4. Luellen, W.R. *Fine-Tuning Your Writing*. Wise Owl Publishing Company, Madison, WI, USA (2001).
5. Alon, U. & D.N. Wegner (Ed.). *An Introduction to Systems Biology: Design Principles of Biological Circuits*. Chapman & Hall/CRC, Boca Raton, FL, USA (2006).

c. **Book Chapters**

6. Sarnthein, M.S. & J.D. Stanford. Basal sauropodomorpha: historical and recent phylogenetic developments. In: *The Northern North Atlantic: A Changing Environment.* Schafer, P.R. & W. Schluter (Ed.), Springer, Berlin, Germany, p. 365–410 (2000).
7. Smolen, J.E. & L.A. Boxer. Functions of Europhiles. In: *Hematology, 4th ed*. Williams, W.J., E. Butler & M.A. Litchman (Ed.), McGraw Hill, New York, USA, p. 103–101 (1991).

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