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Research Article

Remarks on (1,2)*-αĝ-Homeomorphisms

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Abstract: The aim of this paper is to introduce two new class of functions called (1,2)*-αĝ-homeomorphisms and strongly (1,2)*-αĝ-homeomorphisms using (1,2)*-αĝ-closed sets and study their basic properties in bi-topological spaces.

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1. INTRODUCTION

Njastad [16] introduced α-open sets. Maki et al. [14] generalized the concepts of closed sets to α-generalized closed (briefly αg-closed) sets which are strictly weaker than α-closed sets. Veera Kumar [30] defined ĝ-closed sets in topological spaces. El Monsef et al. [1] introduced αĝ-closed sets which lie between α-closed sets and αg-closed sets in topological spaces.

Maki et al [15] introduced the notion of generalized homeomorphisms (briefly g-homeomorphism) which are generalizations of homeomorphisms in topological spaces. Subsequently, Devi et al [6] introduced two class of functions called generalized semi-homeomorphisms (briefly gs-homeomorphism) and semi-generalized homeomorphisms (briefly sg-homeomorphism). Quite recently, Zbigniew Duszynski [32] has introduced αĝ-homeomorphisms in topological spaces.

It is well-known that the above mentioned topological sets and functions have been generalized to bitopological settings due to the efforts of many modern topologists [see 7, 9, 10, 17-26]. In this present paper, we introduce two new class of bitopological functions called (1,2)*-αĝ-homeomorphisms and strongly (1,2)*-αĝ-homeomorphisms by using (1,2)*-αĝ-closed sets. Basic properties of these two functions are studied and the relation between these types and other existing ones are established.

2. PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X, Y and Z) will denote bitopological spaces.

2.1. Definition

Let S be a subset of a bitopological space X. Then S is said to be $\tau_{1,2}$-open [9] if $S = A \cup B$, where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-

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closed.

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

2.2. Definition [9]

Let S be a subset of a bitopological space X. Then

(1) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap$ {F : S $\subseteq$ F and F is $\tau_{1,2}$-closed}.

(2 the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup$ {F : F $\subseteq$ S and F is $\tau_{1,2}$-open}.

2.3 . Definition

A subset A of a bitopological space X is called

(1) (1,2)*-semi-open set [10] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)).

(2) (1,2)*-α-open set [10] if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).

(3) regular (1,2)*-open set [17] if A = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.

The (1,2)*-semi-closure (resp. (1,2)*-α-closure) of a subset A of a bitopological space X, denoted by (1,2)*-scl(A) (resp. (1,2)*-αcl(A)), is the intersection of all (1,2)*-semi-closed (resp. (1,2)*-α-closed) sets of X containing A.

2.4. Definition

A subset A of a bitopological space X is called

(1) (1,2)*-generalized closed (briefly, (1,2)*-g-closed) [19] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(2) (1,2)*-semi-generalized closed (briefly, (1,2)*-sg-closed) [21] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.

(3) (1,2)*-generalized semi-closed (briefly, (1,2)*-gs-closed) [22] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(4) (1,2)*-ĝ-closed [7] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X.

(5) (1,2)*-αg-closed [18] if (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

(6) (1,2)*-αĝ-closed [7] if (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-ĝ-open in X.

The complements of the above mentioned closed sets are called their respective open sets.

2.5. Definition

A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (1,2)*-g-open [22] (resp. (1,2)*-ĝ-open [26], (1,2)*-open [20], (1,2)*-sg-open [22], (1,2)*-gs-open [22], (1,2)*-α-open [26], (1,2)*-αg-open [23], (1,2)*-αĝ-open [26]) if the image of every $\tau_{1,2}$-open set in X is (1,2)*-g-open (resp. (1,2)*-ĝ-open, $\sigma_{1,2}$-open, (1,2)*-sg-open, (1,2)*-gs-open, (1,2)*-α-open, (1,2)*-αg-open, (1,2)*-αĝ-open) in Y.

2.6. Definition

A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

(1) (1,2)*-g-continuous [21] if $f^{-1}(V)$ is (1,2)*-g-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(2) (1,2)*-sg-continuous [21] if $f^{-1}(V)$ is (1,2)*-sg-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(3) (1,2)*-gs-continuous [21] if $f^{-1}(V)$ is (1,2)*-gs-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(4) (1,2)*-ĝ-continuous [23] if $f^{-1}(V)$ is (1,2)*-ĝ-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(5) (1,2)*-continuous [17] if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

2.7. Definition [22]

A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

(1) (1,2)*-g-homeomorphism if f is bijection, (1,2)*-g-open and (1,2)*-g-continuous.

(2) (1,2)*-sg-homeomorphism if f is bijection, (1,2)*-sg-open and (1,2)*-sg-continuous.

(3) (1,2)*-gs-homeomorphism if f is bijection, (1,2)*-gs-open and (1,2)*-gs-continuous.

(4) (1,2)*-homeomorphism if f is bijection, (1,2)*-open and (1,2)*-continuous.

2.8. Definition [26]

A function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called

(1) (1,2)*-α-continuous if $f^{-1}(V)$ is (1,2)*-α-open in X, for every $\sigma_{1,2}$-open set V of Y.

(2) (1,2)*-αĝ-continuous if $f^{-1}(V)$ is (1,2)*-αĝ-closed in X, for every $\sigma_{1,2}$-closed set V of Y.

(3) (1,2)*-αĝ-irresolute if $f^{-1}(V)$ is (1,2)*-αĝ-closed in X, for every (1,2)*-αĝ-closed set V of Y.

2.9. Definition [25]

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(1) pre-(1,2)*-α-closed (resp. pre (1,2)*-α-open) if the image of every (1,2)*-α-closed (resp. (1,2)*-α-open) in X is (1,2)*-α-closed (resp. (1,2)*-α-open) in Y.

(2) (1,2)*-α-irresolute if $f^{-1}(V)$ is (1,2)*-α-open in X, for every (1,2)*-α-open set V of Y.

(3) (1,2)*-gc-irresolute if $f^{-1}(V)$ is (1,2)*-g-closed in X, for every (1,2)*-g-closed set V of Y.

(4) (1,2)*-α-homeomorphism if f is bijection, (1,2)*-α-irresolute and pre-(1,2)*-α-closed.

2.10. Remark [7]

(1) Every (1,2)*-α-closed set is (1,2)*-αĝ-closed but not conversely.

(2) Every (1,2)*-αĝ-open set is (1,2)*-gs-open but not conversely.

3. (1,2)*-AĜ-HOMEOMORPHISMS

3.1. Definition

(1) A bijective function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called a strongly (1,2)*-αĝ-closed (resp. strongly (1,2)*-αĝ-open) if the image of every (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) set in X is (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) of Y.

(2) A bijective function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called an (1,2)*-αĝ-homeomorphism if f is both (1,2)*-αĝ-open and (1,2)*-αĝ-continuous.

3.2. Theorem

Every (1,2)*-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be (1,2)*-homeomorphism. Then f is bijective, (1,2)*-open and (1,2)*-continuous function. Let U be an $\tau_{1,2}$-open set in X. Since f is (1,2)*-open function, f(U) is an $\sigma_{1,2}$-open set in Y. Every $\tau_{1,2}$-open set is (1,2)*-αĝ-open and hence f(U) is (1,2)*-αĝ-open in Y. This implies f is (1,2)*-αĝ-open. Let V be a $\sigma_{1,2}$-closed set in Y. Since f is (1,2)*-continuous, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Thus $f^{-1}(V)$ is (1,2)*-αĝ-closed in X and therefore, f is (1,2)*-αĝ-continuous. Hence, f is an (1,2)*-αĝ-homeomorphism.

3.3. Remark

The converse of Theorem 3.2 need not be true as shown in the following example.

3.4. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Y. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then f is a (1,2)*-αĝ-homeomorphism but f is not a (1,2)*-homeomorphism.

3.5. Proposition

For any bijective function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

(1) $f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2)$ is (1,2)*-αĝ-continuous function.

(2) f is a (1,2)*-αĝ-open function.

(3) f is a (1,2)*-αĝ-closed function.

Proof

(1) ⇒ (2): Let U be an $\tau_{1,2}$-open set in X. Then X – U is $\tau_{1,2}$-closed in X. Since f^{-1} is (1,2)*-αĝ-continuous, $(f^{-1})^{-1}(X - U)$ is (1,2)*-αĝ-closed in Y. That is f(X – U) = Y – f(U) is (1,2)*-

$\alpha\hat{g}$-closed in Y. This implies f(U) is (1,2)*-$\alpha\hat{g}$-open in Y. Hence f is (1,2)*-$\alpha\hat{g}$-open function.

(2) ⇒ (3): Let F be a $\tau_{1,2}$-closed set in X. Then X – F is $\tau_{1,2}$-open in X. Since f is (1,2)*-$\alpha\hat{g}$-open, f(X–F) is (1,2)*-$\alpha\hat{g}$-open set in Y. That is Y – f(F) is (1,2)*- $\alpha\hat{g}$-open in Y. This implies that f(F) is (1,2)*-$\alpha\hat{g}$-closed in Y. Hence f is (1,2)*- $\alpha\hat{g}$-closed.

(3) ⇒ (1): Let V be a $\tau_{1,2}$-closed set in X. Since f is (1,2)*-$\alpha\hat{g}$-closed function, f(V) is (1,2)*-$\alpha\hat{g}$-closed in Y. That is $(f^{-1})^{-1}(V)$ is (1,2)*-$\alpha\hat{g}$-closed in Y. Hence f^{-1} is (1,2)*-$\alpha\hat{g}$-continuous.

3.6. Proposition

Let f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a bijective and (1,2)*-$\alpha\hat{g}$-continuous function. Then the following statements are equivalent:

(1) f is a (1,2)*-$\alpha\hat{g}$-open function.

(2) f is a (1,2)*-$\alpha\hat{g}$-homeomorphism.

(3) f is a (1,2)*-$\alpha\hat{g}$-closed function.

Proof

(1) ⇒ (2): Let f be a (1,2)*-$\alpha\hat{g}$-open function. By hypothesis, f is bijective and (1,2)*-$\alpha\hat{g}$-continuous. Hence f is a (1,2)*-$\alpha\hat{g}$-homeomorphism.

(2) ⇒ (3): Let f be a (1,2)*-$\alpha\hat{g}$-homeomorphism. Then f is (1,2)*-$\alpha\hat{g}$-open. By Proposition 3.5, f is (1,2)*-$\alpha\hat{g}$-closed function.

(3) ⇒ (1): It is obtained from Proposition 3.5.

3.7. Theorem

Every (1,2)*-α-homeomorphism is (1,2)*-$\alpha\hat{g}$-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a (1,2)*-α-homeomorphism. Then f is bijective, (1,2)*-α-irresolute and pre-(1,2)*-α-closed. Let F be $\tau_{1,2}$-closed in X. Then F is (1,2)*-α-closed in X. Since f is pre-(1,2)*-α-closed, f(F) is (1,2)*-α-closed in Y. Every (1,2)*-α-closed set is (1,2)*-$\alpha\hat{g}$-closed and hence f(F) is (1,2)*-$\alpha\hat{g}$-closed in Y. This implies f is (1,2)*-$\alpha\hat{g}$-closed function. Let V be a $\sigma_{1,2}$-closed set of Y. Thus V is (1,2)*-α-closed in Y. Since f is (1,2)*-α-irresolute $f^{-1}(V)$ is (1,2)*-α-closed in X. Thus $f^{-1}(V)$ is (1,2)*-$\alpha\hat{g}$-closed in X. Therefore f is (1,2)*-$\alpha\hat{g}$-continuous. Hence f is a (1,2)*-$\alpha\hat{g}$-homeomorphism.

3.8. Remark

The following Example shows that the converse of Theorem 3.7 need not be true.

3.9. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a}}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed in X and the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-$\alpha\hat{g}$-open in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-closed in X and the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-$\alpha\hat{g}$-open in Y. Moreover, the sets in {ϕ, Y, {a, b}} are called (1,2)*-α-closed in Y and the sets in {ϕ, Y, {c}} are called (1,2)*-α-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-$\alpha\hat{g}$-homeomorphism but f is not a (1,2)*-α-homeomorphism.

3.10. Remark

Next Example shows that the composition of two (1,2)*-$\alpha\hat{g}$-homeomorphisms is not always a (1,2)*-$\alpha\hat{g}$-homeomorphism.

3.11. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called (1,2)*-$\alpha\hat{g}$-open in X. Let Y= {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Let Z = {a, b, c}, η_1 = {ϕ, Z} and η_2 = {ϕ, Z, {a, b}}. Then the sets in {ϕ, Z, {a, b}} are called $\eta_{1,2}$-open and the sets in {ϕ, Z, {c}} are called $\eta_{1,2}$-closed. Also the sets in {ϕ, Z, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Z and the sets in {ϕ, Z, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Z. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be two identity functions. Then both f and g are (1,2)*-αĝ-homeomorphisms. The set {a, c} is $\tau_{1,2}$-open in X, but (g o f)({a, c}) = {a, c} is not (1,2)*-αĝ-open in Z. This implies that g o f is not (1,2)*-αĝ-open and hence g o f is not (1,2)*-αĝ-homeomorphism.

3.12. Theorem

Every (1,2)*-αĝ-homeomorphism is (1,2)*-gs-homeomorphism but not conversely.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a (1,2)*-αĝ-homeomorphism. Then f is a bijective, (1,2)*-αĝ-open and (1,2)*-αĝ-continuous function. Let U be an $\tau_{1,2}$-open set in X. Then f(U) is (1,2)*-αĝ-open in Y. Every (1,2)*-αĝ-open set is (1,2)*-gs-open and hence, f(U) is (1,2)*-gs-open in Y. This implies f is (1,2)*-gs-open function. Let V be $\sigma_{1,2}$-closed set in Y. Then $f^{-1}(V)$ is (1,2)*-αĝ-closed in X. Hence $f^{-1}(V)$ is (1,2)*-gs-closed in X. This implies f is (1,2)*-gs-continuous. Hence f is (1,2)*-gs-homeomorphism.

3.13. Remark

The following Example shows that the converse of Theorem 3.12 need not be true.

3.14. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-gs-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Moreover, the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-g-closed and (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-gs-homeomorphism but f is not a (1,2)*-αĝ-homeomorphism.

3.15. Remark

The following Examples show that the concepts of (1,2)*-αĝ-homeomorphisms and (1,2)*-g-homeomorphisms are independent of each other.

3.16. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in X. Moreover, the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-αĝ-open and (1,2)*-ĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {b}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {b}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {c}, {a, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {b, c}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = a and f(c) = c. Then f is a (1,2)*-αĝ-homeomorphism but f is not a (1,2)*-g-homeomorphism.

3.17. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-g-closed in X. Moreover, the sets in {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-αĝ-closed and (1,2)*-α$\hat{g}$-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-gs-closed and (1,2)*-gs-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = c, f(c) = a. Then f is a (1,2)*-g-homeomorphism but f is not a (1,2)*- α$\hat{g}$-homeomorphism.

3.18. Remark

(1,2)*-α$\hat{g}$-homeomorphisms and (1,2)*-sg-homeomorphisms are independent of each other as shown below.

3.19. Example

The function f defined in Example 3.16 is (1,2)*- α$\hat{g}$-homeomorphism but not (1,2)*-sg-homeomorphism.

3.20. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and (1,2)*-α$\hat{g}$-open in X; the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed and (1,2)*-α$\hat{g}$-closed in X. Also, the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-sg-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b, c}}. Then the sets in {ϕ, Y, {a}, {b, c}} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {a}, {b, c}} are called (1,2)*-α$\hat{g}$-closed and (1,2)*-α$\hat{g}$-open in Y. Moreover, the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-sg-closed and (1,2)*-sg-open in Y. Define a function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = a and f(c) = c. Then f is (1,2)*-sg-homeomorphism but not (1,2)*-α$\hat{g}$-homeomorphism.

4. STRONGLY (1,2)*-A$\hat{G}$-HOMEOMORPHISMS

4.1. Definition

A bijection f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is said to be strongly (1,2)*-α$\hat{g}$-homeomorphism if f is (1,2)*-α$\hat{g}$-irresolute and its inverse f^{-1} is also (1,2)*-α$\hat{g}$-irresolute.

4.2. Theorem

Every strongly (1,2)*-α$\hat{g}$-homeomorphism is (1,2)*-α$\hat{g}$-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be strongly (1,2)*-α$\hat{g}$-homeomorphism. Let U be $\tau_{1,2}$-open in X. Then U is (1,2)*-α$\hat{g}$-open in X. Since f^{-1} is (1,2)*-α$\hat{g}$-irresolute, $(f^{-1})^{-1}$(U) is (1,2)*-α$\hat{g}$-open in Y. That is f(U) is (1,2)*-α$\hat{g}$-open in Y. This implies f is (1,2)*-α$\hat{g}$-open function. Let F be a $\sigma_{1,2}$-closed in Y. Then F is (1,2)*-α$\hat{g}$-closed in Y. Since f is (1,2)*-α$\hat{g}$-irresolute, f^{-1}(F) is (1,2)*-α$\hat{g}$-closed in X. This implies f is (1,2)*-α$\hat{g}$-continuous function. Hence f is (1,2)*-α$\hat{g}$-homeomorphism.

4.3. Remark

The following Example shows that the converse of Theorem 4.2 need not be true.

4.4. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, c}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {b}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, b}, {b, c}} are called (1,2)*-α$\hat{g}$-closed in X and the sets in {ϕ, X, {a}, {c}, {a, b}, {a, c}} are called (1,2)*-α$\hat{g}$-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-α$\hat{g}$-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-α$\hat{g}$-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-α$\hat{g}$-homeomorphism but f is not a strongly (1,2)*-α$\hat{g}$-homeomorphism.

4.5. Theorem

The composition of two strongly (1,2)*-α$\hat{g}$-homeomorphisms is a strongly (1,2)*-α$\hat{g}$-homeomorphism.

Proof

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) and g : (Y, σ_1, σ_2) → (Z, η_1, η_2) be two strongly (1,2)*-α$\hat{g}$-homeomorphisms. Let F be a (1,2)*-α$\hat{g}$-closed set in Z. Since g is (1,2)*-α$\hat{g}$-irresolute, g^{-1}(F) is (1,2)*-α$\hat{g}$-closed in Y. Since f is a (1,2)*-α$\hat{g}$-irresolute, $f^{-1}(g^{-1}(F))$ is (1,2)*-α$\hat{g}$-closed in X. That is $(g \text{ o } f)^{-1}$(F) is (1,2)*-α$\hat{g}$-closed in X. This implies that g o f : (X, τ_1, τ_2) → (Z,

η_1, η_2) is (1,2)*-αĝ-irresolute. Let V be a (1,2)*-αĝ-closed in X. Since f^{-1} is a (1,2)*-αĝ-irresolute, $(f^{-1})^{-1}(V)$ is (1,2)*-αĝ-closed in Y. That is f(V) is (1,2)*-αĝ-closed in Y. Since g^{-1} is a (1,2)*-αĝ-irresolute, $(g^{-1})^{-1}(f(V))$ is (1,2)*-αĝ-closed in Z. That is g(f(V)) is (1,2)*-αĝ-closed in Z. So, (g o f)(V) is (1,2)*-αĝ-closed in Z. This implies that $((g \circ f)^{-1})^{-1}(V)$ is (1,2)*-αĝ-closed in Z. This shows that $(g \circ f)^{-1}$:(Z, η_1, η_2) → (X, τ_1, τ_2) is (1,2)*-αĝ-irresolute. Hence g o f is a strongly (1,2)*-αĝ-homeomorphism.

We denote the family of all strongly (1,2)*-αĝ-homeomorphisms from a bitopological space (X, τ_1, τ_2) onto itself by (1,2)*-sαĝ-h(X).

4.6. Theorem

The set (1,2)*-sαĝ-h(X) is a group under composition of functions.

Proof

By Theorem 4.5, g o f ∈ (1,2)*-sαĝ-h(X) for all f, g ∈ (1,2)*-sαĝ-h(X). We know that the composition of functions is associative. The identity function belonging to (1,2)*-sαĝ-h(X) serves as the identity element. If f ∈ (1,2)*-sαĝ-h(X), then f^{-1} ∈ (1,2)*-sαĝ-h(X) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of (1,2)*-sαĝ-h(X). Hence (1,2)*-sαĝ-h(X) is a group under the composition of functions.

4.7. Theorem

Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be a strongly (1,2)*-αĝ-homeomorphism. Then f induces an (1,2)*-isomorphism from the group (1,2)*-sαĝ-h(X) onto the group (1,2)*-sαĝ-h(Y).

Proof

Using the function f, we define a function θ_f : (1,2)*-sαĝ-h(X) → (1,2)*-sαĝ-h(Y) by $\theta_f(k) = f \circ k \circ f^{-1}$ for every k ∈ (1,2)*-sαĝ-h(X). Then θ_f is a bijection. Further, for all k_1, k_2 ∈ (1,2)*-sαĝ-h(X), $\theta_f(k_1 \circ k_2) = f \circ (k_1 \circ k_2) \circ f^{-1} = (f \circ k_1 \circ f^{-1}) \circ (f \circ k_2 \circ f^{-1}) = \theta_f(k_1) \circ \theta_f(k_2)$. Therefore θ_f is an (1,2)*-isomorphism induced by f.

4.8. Remark

The concepts of strongly (1,2)*-αĝ-homeomorphisms and (1,2)*-α-homeomorphisms are independent notions as shown in the following examples.

4.9. Example

Let X = {a, b, c}, τ_1 = {ϕ, X} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a, b}} are called $\tau_{1,2}$-open and (1,2)*-α-open; and the sets in {ϕ, X, {c}} are called $\tau_{1,2}$-closed and (1,2)*-α-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, b}} are called (1,2)*-αĝ-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {b}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}} are called $\sigma_{1,2}$-open and (1,2)*-α-open; and the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed and (1,2)*-α-closed in Y. Also the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-αĝ-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a strongly (1,2)*-αĝ-homeomorphism but f is not (1,2)*-α-homeomorphism.

4.10. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in X. Moreover, the sets in {ϕ, X, {b}, {c}, {b, c}} are called (1,2)*-α-closed in X and the sets in {ϕ, X, {a}, {a, b}, {a, c}} are called (1,2)*-α-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y} and σ_2 = {ϕ, Y, {a}}. Then the sets in {ϕ, Y, {a}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, b}, {a, c}, {b, c}} are called (1,2)*-αĝ-closed in Y and the sets in {ϕ, Y, {a}, {b}, {c}, {a, b}, {a, c}} are called (1,2)*-αĝ-open in Y. Moreover, the sets in {ϕ, Y, {b}, {c}, {b, c}} are called (1,2)*-α-closed in Y and the sets in {ϕ, Y, {a}, {a, b}, {a, c}} are called (1,2)*-α-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-α-homeomorphism but not strongly (1,2)*-αĝ-homeomorphism.

4.11. Definition

A bijective function f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) is called (1,2)*-gc-homeomorphism if f is (1,2)*-gc-irresolute and f^{-1} is (1,2)*-gc-irresolute.

4.12. Remark

The concepts of strongly (1,2)*-$\alpha\hat{g}$-homeomorphisms and (1,2)*-gc-homeomorphisms are independent of each other as the following examples show.

4.13. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {a, b}}. Then the sets in {ϕ, X, {a}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-$\alpha\hat{g}$-open in X. Moreover, the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in X and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {b}, {a, b}} and σ_2 = {ϕ, Y, {a}, {a, c}}. Then the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed and (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-$\alpha\hat{g}$-open and (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a strongly (1,2)*-$\alpha\hat{g}$-homeomorphism but not (1,2)*-gc-homeomorphism.

4.14. Example

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}} and τ_2 = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called $\tau_{1,2}$-closed. Also the sets in {ϕ, X, {c}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed and (1,2)*-g-closed in X, and the sets in {ϕ, X, {a}, {b}, {a, b}} are called (1,2)*-$\alpha\hat{g}$-open and (1,2)*-g-open in X. Let Y = {a, b, c}, σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y, {a, b}}. Then the sets in {ϕ, Y, {a}, {a, b}} are called $\sigma_{1,2}$-open and the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called $\sigma_{1,2}$-closed. Also the sets in {ϕ, Y, {b}, {c}, {a, c}, {b, c}} are called (1,2)*-$\alpha\hat{g}$-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}, {a, c}} are called (1,2)*-$\alpha\hat{g}$-open in Y. Moreover, the sets in {ϕ, Y, {c}, {a, c}, {b, c}} are called (1,2)*-g-closed in Y and the sets in {ϕ, Y, {a}, {b}, {a, b}} are called (1,2)*-g-open in Y. Let f : (X, τ_1, τ_2) → (Y, σ_1, σ_2) be the identity function. Then f is a (1,2)*-gc-homeomorphism but not strongly (1,2)*-gc-homeomorphism.

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