Seasonal Variability in Virtual Height of Ionospheric F₂ Layer at the Pakistan Atmospheric Region

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Abstract: The aim of this study was to assess the seasonal variability in virtual height of ionospheric F₂ layer for Pakistan’s atmospheric region (PAR). In this communication virtual height variations have been analyzed by the descriptive statistical techniques. These methodologies comprise an autoregressive strategy, linear regression and polynomial regression. The relevance of these models has been illuminated using predicted values of different parameters under the seasonal variation of ionospheric F₂ layer in virtual height that affect the radio wave propagation through the ionosphere. These techniques are implemented to theorize the physical process of varying the virtual heights that leads this study towards formulating the variations due to interaction of radio wave propagation with this ionospheric layer.

Keywords: autoregressive process, variability in virtual height, linear regression, polynomial regression

1. INTRODUCTION

The sun is a main source of ionization in the extraterrestrial atmosphere, so that the variability depends upon the solar activity and geomagnetic conditions. The ionosphere is very important in radio wave communications, which absorbs large quantities of radiant energy from the sun. It is earth’s upper atmosphere and the F₂ layer is the outer layer of the ionosphere. The virtual height of an ionospheric F₂ layer could be explained as the wave is refracted down gradually rather sharply. Incident and refracted rays below the ionized layer follow the same path as if the reflection has taken place from the greater height. The spot is called virtual height of this layer in other words when the incidence and returned rays are extrapolated to a vertex they meet at a height h’ is called virtual height. It has been noted that if the virtual height of the layer is known then it is quite simple to calculate the angle of incidence required for the wave to return to ground at a selected point. The F₂ layer has a permanent existence, even through the height varies on a daily basis. The frequency range of this layer is 3 to 40 MHz. It is the most important reflecting medium for high frequency radio waves and the height of F₂ layer varies from 250- 400 km. Since sun’s presence is essential, ionization can take place in any location only during the day time. Even during day time, sun’s radiation will not be from morning till evening because sun’s zenith angle changes. We may expect maximum ionization at noon, and then falling of both sides of the day’s daylight hours. Every event in the sun in respect of thermal content, magnetic field, radiation variation due to sun’s rotation around itself should be reflected in the ionospheric properties. The other
influential factors are latitude, sunspot cycle and magnetic storms [1-3].

The ionosphere seasonal variation is related to a solar zenith angle change, while its solar cycle variation is linked to a change in the solar extreme Ultraviolet (EUV) and x-ray radiation. The important feature is that the maximum electron density (N\textsubscript{mF\textsubscript{2}}) of F\textsubscript{2} layer is greater in winter than in summer, despite the fact that the solar zenith angle is smaller in summer. Specifically the summer-to-winter neutral circulation results in an increase in the O/N\textsubscript{2} ratio in the winter hemisphere and a decrease in the summer hemisphere. The day-time wavelengths are much shorter then at the night-time. The point is that the day-time electron density of the F\textsubscript{2} layer is very high and it can reflect the higher frequencies. Conversely, the night-time electron density of F\textsubscript{2} layer goes down. Thus, the time delay is used to determine the altitude of reflection and the frequency is an indicator of the electron density at that location. The height which is calculated by assuming that the waves travel with the velocity of light is called the virtual height [4-7].

The different geomagnetic storms can be significantly different and even for a given storm the system’s response can be very different in the latitudinal and longitudinal regions. However it is educational to show the ionospheric response to the large magnetic storm that was triggered by a solar flare which appeared at 1229UT on October 19, 1989 and it was 22 maximum cycles. In response to this storm, there were long-lasting electron density depletions at high latitudes as measured DG-Sonde Suparco station Karachi and worldwide Ionosondes. During the incidence of ionosphere disturbances the electron density of the F\textsubscript{2} layer takes a dip and the virtual heights go up due to the heating so that the maximum usable frequencies are decreased. The regular structure of F\textsubscript{2} layer is disrupted and strata appear in it. Through very strong ionosphere disturbances, the ionization of the F\textsubscript{2} layer may drop to a point where the layer will not reflect short waves any longer. The F\textsubscript{2} layer is destroyed during ionosphere disturbances at high geomagnetic Latitudes. We have a recorded data of the virtual height of ionosphere F\textsubscript{2} layer over a period of one year (January to December 1989) which is being used in this study [8-13].

2. MATERIALS AND METHODS

The study was based on the analysis of seasonal variability in virtual height and electron concentration (Ne) for Pakistan's atmospheric region. We have used some statistical tools such as Minitab and Statistica for analyzing time series. For this purpose we have utilized ionospheric data recorded on 256 Digisonde for evolution, analysis and forecasting. Spectral analysis, autocorrelation and partial autocorrelation functions have been effectively implemented on the basis of these statistical software packages.

3. RESULTS AND DISCUSSION

The results presented in this study establish that Auto regressive AR (1) order one character of modeling, the seasonal variability in virtual height of ionospheric F\textsubscript{2} layer as a physical process is an appropriate position. The reported data suggest quantitatively that the seasonal variability in virtual height of ionospheric F\textsubscript{2} layer is occurring Fig. 1 is an original time plot of virtual height of F\textsubscript{2} layer data, Fig. 2 illustrates negative correlation between the electron concentration and the seasonal variability in virtual height of h'F\textsubscript{2} layer Fig. 3 is a scatter plot of virtual height of h'F\textsubscript{2} layer data. From this graph it appears that the seasonal variability in virtual height of h'F\textsubscript{2} layer in the period (t-1) is useful in predicting the value of the total value of virtual height deliberation in period t. It seems that X\textsubscript{t} can be explained as significance of X\textsubscript{t-1}. To identify the idea we can illustrate the case of autoregressive model which is normally used in time series model.

\[ X_t = \alpha_o + \phi X_{t-1} + \epsilon_t \]  \hspace{1cm} (1)

where, X\textsubscript{t} is expressed as a linear combination of its two immediately preceding values, \epsilon\textsubscript{t} is an error value and \alpha\textsubscript{o} is constant (parameters) value.

\[ X_t = \phi X_{t-1} + \alpha_o \]  \hspace{1cm} (2)

Regression analysis describes the assessment of the unknown value of one variable from the known value of the other variable. We have the data which consists of two variables x and y that we want to find the linear function of electron concentration (Ne) of F\textsubscript{2} layer and the virtual height of h'F\textsubscript{2}
Variability in Virtual Height of Ionospheric $F_2$ Layer

Fig. 1. The temporal variation of virtual height of ionospheric $F_2$ layer on 19 October 1989.

Fig. 2. Correlation between the $h'F_2$ and Ne.
Fig. 3. Virtual height of $h'F_2$ layer data plotted past values $X_{t-1}$ and present $X_t$.

Fig. 4. Periodogram to classify randomness in the seasonal variability of $h'F_2$.
Variability in Virtual Height of Ionospheric F2 Layer

The linear regression analysis is the key for building forecasting models to produce forecast of seasonal variability of virtual height of ionosphere h’F2 layer.

\[ \overline{X} = 296.615 - 0.2837 X_{t-1} \]

\( t \) statistics: (11.421) (12.074)

\( R^2 = 3.2\% \)

\( R^2 \) is the amount of shared variance between the two variables and useful for the fit of the model.

The periodogram illustrated in Fig. 4 is used to classify randomness in the data series. Also it helps in identifying seasonality in the given time series, and in recognizing the predominance of positive or negative autocorrelation (for positive autocorrelation low-frequency amplitudes should dominate, and for negative autocorrelation, high frequencies should dominate. Thus the following inferences are incurred:

(a) Autoregressive processes may be established—pattern of autocorrelation, of partials, and within the line scale, will show a explanation of a potential model.

(b) The graph of the data set is a visual support to recognize the behavior of the pattern. The autocorrelations and the line scale are the review of the pattern presented in the data. They can expose a great agreement about the data and their characteristics.

(c) The model can be used in the present case to state the dependence between \( X_t \) and \( X_{t-1} \) in the pair (\( X_t, X_{t-1} \)), and to thus relate \( X_t \) with \( X_{t-1}, X_{t-2}, \) and so on. The plot of \( X_t \) and \( X_{t-1} \) for \( t = 2, 3, n = 365 \) is depicted in Fig. 5. It can be examined that the points are scattered around a straight line. The above model expresses the dependence of the variable on itself at different times for model under consideration \( \alpha_t \), at different \( t \) are independent, that \( \alpha_t \) is independent of \( \alpha_{t-1} \), so that just like \( \epsilon_t \) the distribution of \( \alpha_t \) is unspecified to be normal. \[ [14, 15] \]

\[ \alpha_t \sim ND (0, \sigma_a^2) \]  \( \text{(3)} \)

Normal (N) distribution (D) with mean zero (0) and variance error term (\( \sigma_a^2 \)). It has been distinguished that predictable model is entirely specified only when \( \sigma_a^2 \) is given in calculation to \( \phi_1 \), \( \alpha_t \) is understood to be normal. The value of \( X_t \) may increase or decrease without bound, because \( \alpha_t \) have fixed finite variance and can not continually increase in magnitude to keep \( X_t \) within bound as depicted from Fig. 6 that explains the residual analysis specified for this model and also confirms that this model is sufficient.

**Fig. 5.** Residuals analysis of seasonal variability.
Fig. 6. Plot of Autocorrelation function for seasonal variability.

Fig. 7. Partial Autocorrelation function for the residuals of seasonal variability.
While a condition tells that if φ₁ > 1 or φ₁ < -1, then the series will be non-stationary or unsteady time series. For a stationary stable time series, Xₜ remains surrounded in the motive, it has finite variance, we would need φ₁ < 1. Fig. 6 and Fig. 7 illustrate the estimated auto-correlation function and the partial autocorrelation function from lag 1 to 15 respectively. Fig. 6 depicts the estimated correlation between the y–axis vs the lag number on the x-axis and can be used to determine the pattern (AR) in the set of data. Similarly, Fig. 7 shows partial autocorrelation plot for the residuals of the seasonal variability of ionospheric F₂ layer. Partial autocorrelation is used to measure the degree of association between Xₜ and Xₜ₋₁, when the effects of other time lags 2, 3, … up to X₋₁ are somehow partial lead out. Their singular purpose in time series analysis is identifying an appropriate AR model for forecasting. For the residuals of the virtual height of ionospheric F₂ layer, that showing an adequacy of the constructed model of seasonal variability in virtual height of F₂ for atmospheric region of Pakistan. The dashed lines in the above mentioned figures demonstrate an approximate 95% confidence interval for an individual estimated partial autocorrelation. In these cases the partial autocorrelation plots strongly recommend the presence of serial correlation in the ionospheric F₂ layer data and measure the degree of association if the process is an autoregressive order one then the partial autocorrelations can be examined to determine the order of the process. In table 1 observed value are plotted against predicted values as depicted in Fig. 8. These illustrations are verifying the results of estimates obtained from the estimating techniques.

Following parametric tests were performed for the model verification through regression and autoregressive AR (1):

1. Standard error test as computed from regression models and autoregressive AR (1) order one model.
2. MSE (Mean Square test). It is a ratio sum of square error and number of sample.
3. SSE test (Sum of the square test). It is called the total sum of the square deviations.
4. t-test (t- distribution test). It is to estimate the confidence interval for the mean of a variable, to make various assumptions about the data.
5. F-test (Fraser test). It is a ratio of the larger variance and the smaller variance.
6. P-value confidence interval (from +95% to -95% confidence level) for models [16, 17].

### 4. CONCLUSIONS

In this study, we have estimated the seasonal variation of virtual height h’ of F₂ layer and the deviation in virtual height of h’F₂ layer due to solar maximum and geomagnetic field. With the predication equation obtained from the time plot, the simple linear regression model may provide a very good fit to data. The autoregressive AR (1) model is clearer and easier to handle than the moving average from the virtual height data set
which we have analyzed. In the case of finding the appropriate model for seasonal variation in virtual height \( h'F_2 \) of ionosphere layer, we have looked into the major parametric values of model. It can be seen that the order one is suitable for making prediction and finding forecasts for the atmospheric region of Pakistan.

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6. REFERENCES