# Proceedings
OF THE PAKISTAN ACADEMY OF SCIENCES

## CONTENTS

**Volume 50, No. 2, June 2013**

### Research Articles

#### Engineering Sciences

- Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan’s Energy Future
  — Rizwan Masood
  
- Naturally Fractured Reservoirs – Yet an Unsolved Mystery
  — Muhammad Khurram Zahoor and Muhammad Haris

#### Life Sciences

- Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer’s Solutions in Experimentally-induced Hypo-volumic Shock in Dogs
  — Muhammad A. Zafar, Nemat Ullah, Imran Sarfaraz and Fahad Raza

#### Physical Sciences

- Remarks on (1,2)*-αĝ-Homeomorphisms
  — T. Salai Parkunan, S. Pious Missier, Ravi and A. Pandi

### Instructions for Authors

**Submission of Manuscripts:** Manuscripts may be submitted online at www.paspk.org or as E-mail attachment. Authors must consult the *Instructions for Authors* at the end of this issue and at the Website: www.paspk.org
Research Article

Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan’s Energy Future

Rizwan Masood*

Department of Chemical Engineering, University of Gujrat, Gujrat, Pakistan

Abstract: The utilization of solar thermal energy has got prime importance in Pakistan due to the current energy scarcity and escalating cost scenario in the country. Parabolic Trough Solar Concentrator is one of the most reliable technologies for utilization of solar thermal energy. In solar thermal power generation, Parabolic Trough Solar Concentrators are most successful as almost 96 percent of total solar thermal power is generated across the world by utilizing this technology. Its high reliability, operational compatibility, comparative low cost and high efficiency adds to its high value among other resources. Fortunately, Pakistan lies in the high Solar Insolation Zone; thus, a huge potential exists to benefit from this technology. This technology may cater to the Pakistan’s seasonal increased electricity demand. Apart from electric power generation, this technology may also have cost-effective solutions for Pakistan’s other industries, like steam generation, preheating of boiler make-up water, air-conditioning, and hot water production for food, textile, dairy and leather industries. However, economic justification of such projects would be possible only on accomplishing an indigenous technology base. Globally, this is a proven technology, but in Pakistan there is hardly any development in this field. In this study, an effort has been made by designing and fabricating an experimental Parabolic Trough Solar Water Heater by utilizing locally available materials and manufacturing capabilities. On achieving encouraging results, a solar boiler (steam generator) is proposed to be manufactured locally.

Keywords: Pakistan’s energy future, solar thermal energy utilization, solar concentrators

1. INTRODUCTION

A solar concentrator captures sunlight over a large aperture area and concentrates this energy onto a much small receiver area, multiplying intensity of the solar radiation by a concentration ratio in the range of 10–80. It is the process of concentration that allows troughs to deliver high temperature thermal energy. However, to achieve such concentration, a trough tracks the sun in one axis continually throughout the day.

A Parabolic Trough Solar Concentrator is a one-dimensional parabola that focuses solar beam (parallel) radiation onto a line. Physically, this line is a pipe (absorber) with a flowing liquid inside that absorbs the heat transmitted through the pipe wall and delivers it to the thermal load. Primarily, the heat transfer fluid (some high boiling-point liquid) is heated by passing it through the concentrator’s absorber tube and then this heated fluid exchanges absorbed heat with some thermal load, mostly with water, to make steam that can run the steam turbine to produce electricity.

2. COMPARISON WITH OTHER SOLAR ENERGY UTILIZATION TECHNOLOGIES

Parabolic Trough Solar Concentrator is one of the most reliable technologies for utilization of solar thermal energy. There are many power plants running on solar energy across the world by using parabolic trough solar concentrators. After the successful commissioning of a huge commercial power plant in 2007 (68 MW) in Nevada, USA, this technology got a exponential growth, as currently...
almost 1700 MW of electricity is being produced by utilizing this technology and by the end of 2014 it will reach 3800 MW by the completion of under-construction projects [9, 10].

2.1. Comparison with Photo-Voltaic Technology

Both Photo-Voltaic (PV) and Concentrated Solar Panel (CSP) technologies are being used to generate electricity by utilizing solar energy but CSP is more reliable as compared with photovoltaic power generation. Solar energy is available only in day time but energy demand is for 24 hours a day. So, energy storage provides an option to increase grid reliability and there are many storage options available or under development. The main difference between CSP and PV technologies is energy storage efficacy. The ability of CSP to utilize high-efficiency thermal energy storage (TES) makes its more reliable technology (Fig. 03). The addition of TES produces additional value by shifting solar energy to periods when sun is not available. This enables CSP systems to supply solar-generated electricity during periods of cloudy weather or at night.

2.2. Comparison with other Solar Thermal Technologies

Parabolic Trough Solar Concentrators are most successful in solar thermal technologies. It is because of high reliability, operational compatibility, low cost (comparatively) and high efficiency as parabolic troughs are being utilized in different applications around the world. There are many power plants running on solar thermal energy, across the world and producing almost 1750 MW of electricity out of which 1682 MW (96% of total) are being produced by using parabolic trough solar concentrators [9, 10].

3. PROSPECTS OF THE TECHNOLOGY IN PAKISTAN

Currently, the country is experiencing a severe energy crisis, and its energy requirement over time is increasing rapidly. Over the past 15 years, the primary energy consumption in Pakistan has grown by about 80%, i.e., from 34 million tons oil equivalent (MTOE) in 1994-95 to 63 MTOE in 2009-10. The energy requirement is expected to be doubled in the next few years; by 2015 it is likely to cross 120 MTOE and by 2030 the requirement would reach 360 MTOE [8]. The country’s primary energy supply currently comes mainly from fossil fuels, which is 88% of energy mix (47.5% indigenous natural gas, 0.7% LPG, 30.5% imported oil and 9.2% of energy mix). The rest 10.9% comes from hydropower and 1.2% from nuclear power [7]. Natural resources haven’t been utilized effectively due to which our natural gas reservoirs are depleting and on the other hand oil prices in international market is also increasing.

Pakistan’s current energy scenario demands extensive utilization of renewable energy resources in a cost effective manner. At present, hydropower is the only renewable energy resource, being utilized partially, and a 48 MW wind power project is in the erection phase as yet; all other renewable resources in the country are almost untapped.

Utilization of solar energy has got prime importance in current energy scarcity and cost inflating scenario. Parabolic Trough Solar Concentrator is one of the most reliable technologies in solar thermal technologies and developed countries are benefitting by utilizing this proven technology. Pakistan’s geographical position & climate favors this technology but Economical justification for utilization of this technology may only be possible by accomplishing an indigenous technology base. Manufacturing, installation and operational excellence in this technology may contribute significantly in Pakistan’s energy future.

3.1. Geographical Position and Solar Radiation Data

Pakistan’s geographical position and climate has made this technology one of the reliable energy options as Pakistan lies in high Solar Insolation Zone and because of this there is a huge potential to get benefitted from this technology. Pakistan lies in between 25°-37° latitude which is one of most favorable regions for solar thermal energy utilization. In the fig.4 the regions in dark orange color are high Direct Solar Insolation (5.0-6.5kWh/m2/day) regions and most area of Pakistan lies in this region.

If we see the average monthly solar radiation
**Fig. 1.** Schematic of a parabolic trough [2].

**Fig. 2.** Schematic of solar-thermal power plant [5].

**Fig. 3.** TES extends the power production period [4].
data and day length for Lahore (Table.1) it can be observed that for the six months (i.e. Apr, May, Jun, July, Aug and Sep) high solar flux is available for more than 12 hr daily and these are the most troubled months for power shortage in Pakistan.

3.2 Applications for Pakistan’s Local Industry

Parabolic trough solar concentrators have many cost effective solution for Pakistan’s local industries, in addition to steam and power generation [6]. This technology may be utilized in:

i. Preheating of boiler make-up water.

ii. Utility hot water production in foods industry.

iii. Utility hot water production in dairy industry, as hot water is extensively used in dairy industry for cleaning purpose.

iv. Hot water production for leather industry (tanneries)

v. Hot water production for textile industry.

vi. In air-conditioning.

3.3. Socioeconomic Benefits

Solar thermal technologies have great potential to benefit our nation. They can not only diversify our energy supply by reducing our dependence on natural gas and imported fuels but also improve the quality of the air we breathe and offset greenhouse gas emissions. It can also stimulate our economy by low cost power generation and creating new jobs.

<table>
<thead>
<tr>
<th>Month</th>
<th>Day length (hr)</th>
<th>Rad (cal/cm².day)</th>
<th>Rad (cal/m².day)</th>
<th>Rad (J/cm².day)</th>
<th>Rad (j/m².day)</th>
<th>Flux (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAN</td>
<td>10.2</td>
<td>256</td>
<td>2560000</td>
<td>1075.2</td>
<td>10752000</td>
<td>292.81</td>
</tr>
<tr>
<td>FEB</td>
<td>10.9</td>
<td>333</td>
<td>3330000</td>
<td>1398.6</td>
<td>13986000</td>
<td>356.42</td>
</tr>
<tr>
<td>MAR</td>
<td>11.8</td>
<td>435</td>
<td>4350000</td>
<td>1827</td>
<td>18270000</td>
<td>430.08</td>
</tr>
<tr>
<td>APR</td>
<td>12.8</td>
<td>499</td>
<td>4990000</td>
<td>2095.8</td>
<td>20958000</td>
<td>454.81</td>
</tr>
<tr>
<td>MAY</td>
<td>13.6</td>
<td>545</td>
<td>5450000</td>
<td>2289</td>
<td>22890000</td>
<td>467.52</td>
</tr>
<tr>
<td>JUN</td>
<td>14</td>
<td>533</td>
<td>5330000</td>
<td>2238.6</td>
<td>22386000</td>
<td>444.16</td>
</tr>
<tr>
<td>JUL</td>
<td>13.8</td>
<td>491</td>
<td>4910000</td>
<td>2062.2</td>
<td>20622000</td>
<td>415.09</td>
</tr>
<tr>
<td>AUG</td>
<td>13.1</td>
<td>448</td>
<td>4480000</td>
<td>1881.6</td>
<td>18816000</td>
<td>398.98219</td>
</tr>
<tr>
<td>SEP</td>
<td>12.2</td>
<td>436</td>
<td>4360000</td>
<td>1831.2</td>
<td>18312000</td>
<td>416.93989</td>
</tr>
<tr>
<td>OCT</td>
<td>11.2</td>
<td>361</td>
<td>3610000</td>
<td>1516.2</td>
<td>15162000</td>
<td>376.04167</td>
</tr>
<tr>
<td>NOV</td>
<td>10.4</td>
<td>297</td>
<td>2970000</td>
<td>1247.4</td>
<td>12474000</td>
<td>333.17308</td>
</tr>
<tr>
<td>DEC</td>
<td>10</td>
<td>244</td>
<td>2440000</td>
<td>1024.8</td>
<td>10248000</td>
<td>284.66667</td>
</tr>
</tbody>
</table>

Table 1. Monthly average radiation data on horizontal surface for Lahore [3].
in the manufacturing, installation and operations of solar energy systems.

4. TECHNOLOGY INCUBATION (EXPERIMENTAL RIG)

A parabolic trough concentrator system was designed and fabricated on experimental scale, for hot water production, under the research chair of SNGPL, at Chemical Engineering Department, University of Engineering & Technology, Lahore under the supervision of Drs. A.R. Saleemi and A.H. Delawari.

4.1. Parts, Construction and Materials

The experimental setup (Fig. 5) mainly consists of following components:

i. Reflecting surface
ii. Absorber
iii. Support structure
iv. Tracking system
v. Circulation system

The experimental rig was fabricated by utilizing locally available materials and local manufacturing capabilities. Brief description of components is as follow.

4.1.1 Reflecting Surface

It is the one of the most crucial parts of solar concentrator as efficiency of concentrator greatly depends on the reflectivity of reflector material.

Stainless steel sheet was used for this purpose. Although the value of reflectivity is not very good, however results obtained were encouraging (Fig. 06).

4.1.2 Absorber

Absorber is basically a metallic tube, which is usually coated black to enhance absorbance. The tube is jacketed with glass to reduce the convective heat losses. Absorbed solar radiation delivers heat energy to fluid moving inside pipe.

Copper tube has been used for the purpose. Absorber tube was fixed in glass jacket by the means of Teflon rings and mounting ends. The system was not evacuated due to cost constrains (Fig. 07).

4.1.3 Support Structure

This is the structure required to hold and support the absorber/reflector assembly at correct positions. It must be moveable in horizontal axis so that concentrator may track sun throughout the day. For this purpose trough was mounted on stand structure with the help of bearing (Fig. 08).

4.1.4 Tracking System

To achieve continuous delivery of concentrated solar energy, trough tracks the sun in one axis continually throughout the day.

Timer based tracking system has been used. It consists of two analog timers, on/off switches, electric motor and worm gear arrangement (Fig. 09). The system was calibrated with the movement of sun and the time based tracking was achieved.

Fig. 5. Three dimensional model of the experimental rig.
Fig. 6. Reflecting Surface of the experimental rig.

Fig. 7. Absorber of the experimental rig.

Fig. 8. Support Structure of the experimental rig.
4.1.5 Circulation System

Circulation system is a wide term, used here for complete circuit of heat transfer fluid including pumps hot fluid reservoir and circulation pattern.

Centrifugal pump was used along with rotameter, online thermometer and valve arrangement for in circulation system (Fig. 10).

4.2. Dimensions of the Experimental Rig

The fabricated experimental rig has the following dimensions:

- Length of trough, \( L = 2.133 \) m
- Length of aperture, \( L_a = 1.1 \) m
- Aperture area, \( A_{ap} = 2.133 \times 1.1 = 2.346 \) m²
- Outside dia of glass envelop, \( D_{og} = 0.033 \) m
- Inside dia of glass envelop, \( D_{ig} = 0.03 \) m
- Outside dia of absorber tube, \( D_{or} = 0.014 \) m
- Inside dia of absorber tube, \( D_{ir} = 0.013 \) m
- Outside surface area of absorber tube, \( A_{or} = \pi D_{or} L = 0.0938 \) m²
- Inside surface area of absorber tube, \( A_{ir} = \pi D_{ir} L = 0.087 \) m²
- Outside surface area of glass envelop, \( A_{oe} = \pi D_{oe} L = 0.221 \) m²

Now the Concentration Ratio for these dimensions may be calculated as:

\[
C.R. = \frac{Aperture \ area}{Absorber \ area} = \frac{A_{ap}}{A_{or}}
\]

\[
C.R. = 25.0106 = 25
\]

4.3. The Experiment and Its Results

An experiment was designed to study the performance of the fabricated experimental rig. As the rig have two temperature sensors, one on outlet and other on inlet of absorbing tube and no heat storage was available so the focus was on investigating the change in temperature of circulating water in one pass at different flow rates. The flow rate was optimized on the basis of heat transfer coefficient.

Volumetric flow rate of water, \( F_w \)

\[
F_w = 75 \text{ L/hr} = 2.08 \times 10^{-5} \text{ m}^3/\text{sec} \quad \text{and}
\]

Mass flow rate, \( m_0 = 0.0206 \) kg/sec

Water was circulated by means of pump and the flow rate was adjusted with the help of bypass valve on the rig. Temperatures of both points, inlet and outlet of absorber, were noted. The experiment was carried out when the direct solar radiation intensity was 480.5 W/m². The increase in temperature in one pass achieved was 9°C and in multi-pass
arrangement 85°C temperature of water reservoir was archived.

4.4. Efficiency and Performance

Overall efficiency of a parabolic trough solar concentrator is based on two types of performance: (i) optical performance; and (ii) thermal performance.

4.4.1 Optical Performance

Absorbed radiation per unit area of un-shaded aperture is given by:

\[ S = I_b \rho(\gamma \tau \alpha) K_{\gamma \tau \alpha} \]  \[1\]

Where \( I_b \) is effective incident beam radiation on the plane of aperture, \( \rho \) is the reflectance of the concentrator, \( \gamma \) is intercept factor, \( \tau \) is transmittance, and \( \alpha \) is absorptance. \( K_{\gamma \tau \alpha} \) is an incidence angle modifier that can be used to account for deviations from the normal of the angle of incidence of the radiation on the aperture.

4.4.2 Thermal Performance

\[ Q_u = A_a F_R \left( S - \frac{A_r U_L}{A_u} \left( T_i - T_a \right) \right) \]  \[1\]

where \( A_a \) is the un-shaded area of the concentrator aperture and \( A_r \) is the area of the receiver, \( S \) is the absorbed solar radiation per unit of aperture area, \( T_i \) and \( T_a \) are inlet fluid temperature and ambient temperature, \( F_R \) is the collector heat removal factor.

4. THE WAY FORWARD

To get our nation benefited by this technology, we must have to develop technical expertise in manufacturing and operations of Parabolic Trough Solar Concentrator systems. Initially, we should focus on commercialized thermal application especially for water heating and hybrid (fossil fuels-CSP) steam generation. This will make us able to run infrastructure level projects.

Development of a Parabolic Trough Solar Boiler is proposed (Fig. 11). The heating mechanism of proposed setup is indirect heating i.e. primarily the heat transfer fluid (some high boiling point liquid) will be heated, by passing it through concentrator’s absorber tube, and then this heated fluid passes through coiled heat exchanger and exchanges its heat with water and water will make steam that can run steam turbine.

5. ACKNOWLEDGEMENTS

The author is thankful to SNGPL Research Chairs, Dr. Anwar Rasheed Saleemi and Dr. Amjad Hussain Delawari, for providing me this opportunity to materialize the idea. I am also grateful to Dr. Amjad Hussain Delawari for his technical, administrative and
moral support and to Dr. Naveed Ramzan for his support and guidance.

6. REFERENCES


Naturally Fractured Reservoirs – Yet an Unsolved Mystery

Muhammad Khurram Zahoor$^1$ and Muhammad Haris$^{1,2,*}$

$^1$Petroleum and Gas Engineering Department, University of Engineering and Technology, Lahore, Pakistan
$^2$Geological Engineering Department, University of Engineering and Technology, Lahore, Pakistan

Abstract: Some of the world’s most profitable reservoirs are assumed to be naturally fractured reservoirs (NFR). Effective evaluation, prediction and planning of these reservoirs require an early recognition of the role of natural fractures and then a comprehensive study of factors which affect the flowing performance through these fractures is necessary. As NFRs are the combination of matrix and fractures mediums so their analysis varies from non-fractured reservoirs. Matrix acts as a storage medium while mostly fluid flow takes place from fracture network. Many authors adopted different approaches to understand the flow behavior in such reservoirs.

In this paper a broad review about the previous work done in naturally fractured reservoirs area is outlined and a different idea is initiated for the NFR simulation studies. The role of capillary pressure in natural fractures is always been a key factor for accurate recovery estimations. Also recovery through these reservoirs is dependent upon grid block shape while doing NFR simulation. Some authors studied above mentioned factors in combination with other rock properties to understand the flow behavior in such reservoirs but less emphasis was given for checking the effects on recovery estimations by the variations of only fracture capillary pressures and grid block shapes. So there is need to analyze the behavior of NFR for the mentioned conditions.

Keywords: Naturally fractured reservoirs, capillary pressure, grid block shape

1. INTRODUCTION

Naturally fractured reservoirs (NFR’s) have great importance and differ from conventional reservoirs as these contain fractures throughout the reservoir. These reservoirs usually have low matrix permeability and high fracture permeability having oil or water wet rock properties which may results in low recovery. So good understanding of flow behavior is required to optimize recovery process. Analysis and simulation of fractured reservoirs are done by dividing such reservoirs into matrix and fracture systems. Initially dual porosity concept was introduced but now a day’s dual porosity and dual permeability approach for performing NFR simulation studies is widely accepted.

2. LITERATURE REVIEW

From 1960’s naturally fractured reservoirs acquired great importance for recovery considerations. Barenblatt et al [1] firstly introduced the dual medium model for the simulation of naturally fractured reservoirs. Analysis was done by assuming that the total fluid flow is a combination of flow both from matrix and fracture. In 1963, Warren and Root [2] worked on the naturally fractured systems and by assuming primary porosity as a storage medium of fluid having very low flowing capacity, an idealized model was developed for analysis of fluid flow such reservoirs. During this flow, matrix blocks continuously feed the fracture system but fluid cannot move directly from one matrix to other matrix.
Fig. 1. Idealized fractured system.

Fig. 2. Time taken to achieve total recovery for different fracture widths.

Fig. 3. Recovery factor for different dimensionless fracture height.
block (Fig. 1). Unsteady state flow and pressure built up performance had also been examined and suggested that two parameters, interporosity flow parameter ($\lambda$) and fluid storativity ratio ($\omega$) are important for the NFR simulation studies.

In 1976, Reiss [4] worked to develop understanding about fractured reservoirs. He studied different fracture parameters such as fracture permeability, fracture width, fracture porosity etc, depending upon different geometrical systems in reservoirs. He also highlighted different techniques which can be used for the identification of fractures in reservoir. Van Golf Racht [6] in 1982 published a book which comprised of detailed analysis of fracture characteristics. He discussed different fracture properties like fracture permeability and capillary pressures which exists in such reservoirs and concluded that capillary forces may contribute or may oppose fluid flow during imbibitions and drainage process. He suggested that accurate knowledge of matrix and fracture properties is necessary to get truthful results while simulating naturally fractured reservoirs. Saidi [7] suggested that due to capillary effects in vertical fractures and wettability, capillary pressure in fractures could exist. He also recommended that due to blocks interactions, capillary re-infiltration phenomenon which is the fluid flow from upper to lower blocks should be considered for better simulations results.

In 2007, Jadoon et al [13] investigated the carbonate fractured reservoir in Kohat/Potwar region, Pakistan. They discussed the problems during the identification of natural and induced fractures, categorized the fractures into conductive, resistive, systematic and unsystematic based on borehole imaging tool and recognized the impact of fractures on production.

3. SIMULATION STUDIES

In 1966, Romm [3] was the first person who performed the experimentation studies to analyze fluid flow in fractured media. His findings showed that a linear relationship exists between phase relative permeability and phase saturation while considering zero fracture capillary pressure. Gilman and Kazemi [5] extended the work of Warren and Root [2] by introducing a shape factor, as given in equation (1). They worked on dual porosity simulator by giving more importance to gravity and capillary interactions. They proposed that no particular method exist which can determine fracture capillary pressure and also maximum fracture and matrix capillary pressure must be equal for balanced gravity and capillary forces.

$$\sigma = \frac{1}{4} \left( \frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z} \right)$$

In 1990, Firoozabadi and Hauge [8] analyzed capillary pressure in naturally fractured reservoirs. They proposed a model which relates the existence of capillary pressure in fractures and phase saturation by considering two fracture parameters, aperture and roughness. Through experimental work they indicate the presence of non-zero capillary pressure in fractures. In 2001, Akin [9] analyzed fracture network system with the help of history matching experimental data and numerical model to get the fracture relative permeability. During his analysis, he used different matrix and fracture capillary pressures combinations to obtain the best production and pressure history match. De la Porte et al [10] in 2005 did their analysis on capillary pressures in fracture network system. Two different fluid injection scenarios were analyzed by considering different fracture properties like fracture width and dimensionless fracture height and examined the variations on recovery. Results shows that total recovery is achieved in early times when fracture width is increased and maximum recovery took place for greatest dimensionless fracture height (Fig. 2, 3).

Qasem et al [11] investigated capillary imbibition effects on recovery in partially fractured reservoirs. During analysis, they implemented low and high water injection scenarios and concluded that generally fracture intensity (FI) has a direct impact on recovery as it decreases with low FI for a particular high water injection rate and acts inversely for reverse conditions. They also proposed that counter-current capillary imbibition phenomena occur for high fracture intensity reservoirs during low water injection rates which cause an increase in recovery due to higher areal sweep efficiency and later breakthrough and co-current capillary
Fig. 4. Ultimate recovery achieved for different block height.

Fig. 5. Inverse relation existing between recovery and capillary pressure.

Fig. 6. Comparison of field oil recovery for different fracture widths.
imbibition phenomena occur on low fracture intensity reservoirs thus decreases recovery. Sharait et al [12] studied NFR for oil/gas system having immobile water. They accepted that re-infiltration term effects on production rate but it could not change ultimate recovery. They also suggested that because of matrix block interactions recovery acts as independent from vertical fracture permeability and an early ultimate recovery is achieved when fracture capillary pressure is increased. By increasing the block height oil recovery is increased and an inverse trend exists between matrix capillary pressure and oil recovery (Fig. 4, 5). The reason is that when capillary force increases in matrix less oil will drain to fracture.

In 2007, Tajdar et al [14] in their work highlighted the importance and effects of matrix sub-gridding on recovery in naturally fractured reservoirs. They utilized the technique of dividing a single matrix block into several subgrids so that a more accurate transmissibility of fluid between matrix and fractures could occur. They analyzed different scenarios and concluded that in some cases there could be an error in recovery estimation if improper sub-gridding of matrix block is done. In their analysis variation of fracture properties like fracture capillary pressures are not being discussed.

Uleberg and Kleppe [15] in 2010 did their analysis for the simulation of fractured reservoirs based on physical characteristics of reservoir and fluid flow mechanism in them. They showed that capillary continuity of blocks increase the ultimate recovery. They emphasized to model fractured reservoirs on the source of multiple grid concepts for better results. From this study it can be concluded that the existence of capillary pressure in the fracture system will also affect the recovery. In 2010, Noroozi et al [16] investigated the similar sort of work done by De la porte [10] by considering instead of single porosity single permeability model to dual porosity dual permeability concepts. He focused his work on water and gas injection scenarios while simulating the fractured reservoirs by changing the capillary pressures, fracture width and fracture height. (Fig. 6) shows that when fracture aperture is increased from 5 to 10 microns, recovery is almost doubled.

In 2010, Kiasari et al [17] studied the effects of rock fluid properties during SAGD production and showed that capillary pressure helps in steam penetration from fractures into matrix blocks under water wet condition and act vice versa in oil wet systems. Also an increased early production occurred when capillary pressure is increased from zero. Famian and Masihi [18] in 2010 studied the influence of different properties on recovery while applying immiscible gas injection and during natural depletion. Results indicated that recovery is much affected by matrix threshold capillary pressure and matrix block height.

In 2010, Lemonnier and Bourbiaux [19] described the brief history about the simulation of naturally fractured reservoirs. They discussed different driving mechanism which effect on recovery in fractured reservoirs. In this paper, the use of dual porosity simulator is also been discussed in detail. Jabbari and Zeng [20] in 2011 analyzed the behavior of naturally fractured reservoirs under stress sensitive conditions. Production creates the pressure depletion which may results in changing the characteristics of fractures. So they proposed a model which includes the use of geo-mechanical and fluid flow factors while simulating NFR. The effect of stresses on pressure build up and drawdown curves is also discussed.

4. CONCLUSION AND RECOMMENDATIONS

This study signifies the importance of fracture capillary pressure and grid block shape in recovery estimations. As a conventional practice, the recovery estimations have been made by changing fracture properties such as fracture width, dimensional fracture height, etc while keeping the same grid block shape (mostly cubical). Furthermore, fracture capillary pressure effects are not incorporated while conducting simulation studies, which may lead to inaccurate estimations. So, there is an immense need of study to analyze the effects of varying fracture capillary pressure from zero to other numeric values while keeping the same other fracture properties. In addition, the variations in the grid block shape other than cubic must also be analyzed for better representation of reservoirs while numerical modeling for recovery estimations. Consequently,
the conducted numerical or simulation studies can lead to improved recovery estimations and, thus, result in better field development planning and economical analysis.

5. NOMENCLATURE AND ABBREVIATIONS

NFR  Naturally fracture reservoir
FI  Fracture intensity
SAGD  Steam assisted gravity drainage
L_x  Length of matrix block in x-direction
L_y  Length of matrix block in y-direction
L_z  Length of matrix block in z-direction
σ  Shape factor

6. REFERENCES

Classification of Scientific Publications using Swarm Intelligence

Tariq Ali *, Sohail Asghar, Naseer Ahmed Sajid and Munir Ahmad

Department of Computer Science,
Mohammad Ali Jinnah University, Islamabad, Pakistan

Abstract: Document classification is an important task in data mining. Currently, identifying category (i.e., topic) of a scientific publication is a manual task. The Association for Computing Machinery Computing Classification System (ACM CCS) is most wildly used multi-level taxonomy for scientific document classification. Correct classification becomes difficult with an increase in number of levels as well as in number of categories. Domain overlapping aggravates this problem as a publication may belong to multiple domains. Thus manual classification to taxonomy becomes more difficult. Most of the existing text classification schemes are based on the Term Frequency and Inverse Document Frequency (TF-IDF) technique. Similar approaches become computationally inefficient for large datasets. Most of the techniques for text classification are not experimentally validated on scientific publication datasets. Also, multi-level and multi-class classification is missing in most of the existing schemes for document classification. The proposed approach is based on metadata (i.e., structural representation), in which only the title and keywords are considered. We reduced the features set by dropping some of the metadata, like abstract section of the scientific publication that diversifies the result accuracy. The proposed solution was inspired from the well-known evolutionary Particle Swarm Optimization (PSO). The proposed technique results in overall 84.71% accuracy on Journal of Universal Computer Science (J.UCS) dataset.

Keywords: Topic identification, category identification, document classification, multi-class, multi-level

1. INTRODUCTION

Classification is an important task in data mining [1]. Automated text categorization is becoming more important with the advent of digital libraries and with a rapid increase in the number of documents on the web. The research community is producing a large number of scientific documents. These documents are then searchable over the internet using search engines, digital libraries and citation indexes. There is a need to classify this huge amount of documents into a hierarchy or taxonomy [2]. Similarly, the document’s relatedness to a node in an existing taxonomy will assist in searching the user-relevant information in an efficient way. Accuracy of information retrieval basically depends on accurate classification of the documents [3]. Besides information retrieval, accurate classification helps in analysing trends, finding expertise, and the relevant document recommender system.

Classification is a two level approach in which first level generates a model from the training set of instances and the second level checks accuracy of the classifier [4]. There are a number of approaches for document classification, such as Decision Tree [5], Naive Bays Classifier [6], Particle Swarm Optimization (PSO) [7], Support Vector Machine (SVM) [8], and Term Frequency and Inverse Document Frequency (TF-IDF) [9]. We have already reported a detailed survey [10] towards automated text classification in the context of supervised learning.

One important step towards the document classification is the category identification. Currently, authors of scientific publications identify the relevant category or categories (from
onward written as category/ies) to their papers manually. Common categorization used in research community is the Association for Computing Machinery Computing Classification System (ACM CCS) [11].

Manually, category identification for a document is difficult task for new researcher, especially if the work belongs to multiple domains. Due to the diversity in domains and mapping of one domain to another domain, the manual classification task is becoming extremely difficult. This research is an effort to bridge the gap between users towards identifying correct document category, and suggest possible categorization to the author’s work automatically.

Accurate categorization can be helpful in relevant information (i.e., document) retrieval. Traditional text classification is usually attained by assigning a document to one class, but in scientific community document can belong to multiple categories. We propose that initially the document may be categorized in the top category, and after matching with the top category it may be further classified to its sub-levels as depicted in Fig. 1. The search space for new document classification is reduced by considering the sub-levels of the parent category/ies selected at the first level. Similar approach may be adopted for the third level, if it exists for any category selected at first and second level.

Scientific document classification has structural advantage over the unstructured document classification, as structural representation increases accuracy of the classification [10]. Most of the existing text classification schemes are based on the TF-IDF technique [7, 12]. Similar approaches become computationally inefficient for large datasets. Detail survey towards document classification is given by Sebastiani [12]. Similarly, most of the techniques for text classification are not experimentally validated on scientific publication datasets. Also, multi-level and multi-class classification is missing in most of the existing schemes for document classification [7, 13-18].

The proposed technique is based on metadata (i.e., structural representation), in which we considered the title and the keywords. The title of a scientific publication normally reflect theme of the work and and the keywords are the representative features of the paper to their category/ies.

Every year large number of documents are added to the web. These documents contain a large number of attributes, on the basis of which accurate classification is becoming extremely difficult. Self-adaptability of evolutionary approaches makes it possible to use it for such a dynamic problem having many features for huge number of documents. A document can act as a particle with regard to its own position and the position of other documents in the taxonomy. Thus, based on their position, we can find the similarity between any two documents.

The proposed solution is inspired from the well known PSO algorithm [19, 20]. Documents in the taxonomy are represented with its local position in a category along with global position with neighbourhood categories documents. Classifying a new Test Document (TD) depends with the similarity measure of all particles in each individual category. At second and third level, the movement of new document in the taxonomy depends on the selection of category/ies at the first level. The movement of TD is inspired with the document’s similarity in each individual category. Classification of a scientific publication to a taxonomy is a multi-level classification. The number of documents and the nature of classification (e.g., multi-class and multi-level) makes this problem more complex. Due to these two reasons, PSO stands out as one of the optimum solution. PSO is simple, easy to implement and computationally efficient.

We have implemented our proposed solution and tested it on the Journal of Universal Computer Science (J.UCS) dataset [21], in which 2/3 instances were used for the training set and 1/3 for the test dataset. Furthermore, we assigned some heuristics for the selection of system generated category. We implemented our proposed
techniques and concluded overall 84.71% accuracy on the J.UCS dataset.

Rest of the paper is organized in a way that Section 2 presents the problem statement and Section 3 contains related approaches with critical analysis, Section 4 presents the representation of scientific documents, Section 5 contains the proposed technique for the scientific publication classification, Section 6 contains the experimental results of the proposed approach with detailed discussion and analysis and Section 7 concludes the paper and provides future directions.

2. PROBLEM STATEMENT

Document classification assigns a new document (e.g., a research paper) to a set of previously defined taxonomy. The pre-defined categories can be of any type. Normally, in computer science the most common categorization is the ACM Computing Classification System (ACM CCS 1998) on which we have tested our proposed approach. Formally, the problem can be defined as given in Eq. 1.

\[ d_i : D \in C_{j..k} : C = d_i : C \quad \ldots \quad Eq: 1 \]

Where \( D \) is a set of documents. \( C \) is a set of predefined categories. The problem is to classify a document \( d_i \) of type \( D \) to category/ies \( C_{j..k} \) belonging to category set \( C \). The problem is also depicted in Fig. 1. TD is the user provided publication, whereas \( A to K \) are the main categories of the ACM CCS. Each category is divided into sub-categories which have further sub-categories at third level. Relevant category for new document has to be calculated in the ACM CCS.

3. RELATED WORK

Different approaches for document classification based on Particle Swarm Optimization (PSO) [19, 20], Support Vector Machine (SVM) [8], Bayesian network [6] etc exist in literature. Some of the document classification techniques work on metadata, while others work on the complete text available in the document. Very few Classification techniques are available for the scientific publication category identification as compare to other document classification approaches. Scientific publication contains both metadata and text, which increases its classification accuracy. Some of the approaches towards document classification with reference to the multi-class and multi-level classification are analysed in detail.

Our survey towards the document classification in the context of supervised learning technique is given in [10]. Classification in both structured and unstructured context is analysed. With published literature [13] it is strongly argued that structured documents give more accuracy in classification over unstructured documents. This survey provides theoretical comparison of different techniques, while accuracy and efficiency of classifiers is missing [10].

A PSO based document classification for web documents is given in [7]. In this approach the documents are pre-processed by removing stopwords [22] and words stemming by using porter stemmer [23] algorithm. After preprocessing, the documents were represented as document term frequency matrix. Documents were finally represented as term vectors, using TF-IDF weighting approach [7]. Feature selection was done through entropy weighting scheme [25]. Entropy weighting scheme is done using local weighting of term \( k \) and global weighting of term \( k \) as \( Ljk \times Gk \). after feature selection, particle swarm optimization (PSO) was used as a classifier.

Initialization of individual particles is done randomly; the structure for each particle at given iteration is represented [7] as

\[ X_i^0 = (x_{i1}^0, \ldots, x_{in}^0) \]

Where \( \theta \) represents the iteration and \( n \) represents the term numbers in document set. The velocity of individual particles [7] is given as, which corresponds to the update quantity of all weighting values.

\[ V_i^0 = (v_{i1}^0, \ldots, v_{in}^0) \]

Finally, the effectiveness is measured in terms of precision and recall, as:
\[ F1 = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \]

Experimental evaluations were performed on two standard text dataset reuter-21578 and TREC-AP. In this approach [7] no weighting mechanisms for structural contents are used. Similarly, classifying a document to multiple categories is missing. Strategies for multileveled classification are also not discussed.

Effectiveness of PSO with respect to different dataset towards classification problem is given in [15]. Ten different datasets with multiple instances, classes and number of parameters composing each instance are taken. PSO accuracy in terms of error rate is compared with other nine classifiers. On three data sets PSO outperformed than all other classifiers. PSO efficiency with increasing number of classes is highlighted, which may be due to implementation or similarity measure used for evaluating fitness function.

Improving the document classification with structural contents and citation based evidence is given in [16]. For classification, both structural (i.e., title and abstract) and citation based information is considered. Different similarity measures for both structural (i.e., bag of words, cosine, and Okapi) and citation based (e.g., Bibliographic coupling, Co-citation, Amsler, and Companion) similarity are used. Genetic programming is used for classification of new document. For prediction of new document, best similarity tree for each class is maintained. Class for the new document, is decided on the based of majority voting from each classifier.

A new approach based on the neighbourhood preserving embedding (NPE) with PSO is given in [17]. NPE preserves the local manifold structure and preserve the most discriminating features for classification. Documents features in the higher domain \( X \) are reduced to the lower domain \( Y \) by using the NPE. PSO is used similar to the approach presented in [7] Discriminative features extraction plays an important role in increasing the document classification accuracy. Results of the NPE with PSO has shown better results than LDA PSO, LSI PSO, and LSI-KNN [17].

Bayesian based approach for the classification of conference paper is given in [13]. 400 educational conference papers in four categories (e-learning, Teacher Education, Intelligent Tutoring System, and Cognition issues) are used for constructing the Bayesian network. Only keywords are used for conference paper classification. Compound keywords are parsed into Single keywords which are ranked with respect to frequency and top 7 keywords for each category are considered as input for Bayesian network. Each category shares some common keywords along with some individual keywords. The network has trained with 100 papers for each category. This technique is efficient due to the reason that only keywords are used for classification. Conversely, the misclassification error can increase due to non availability of keywords in some documents or due to wrong keywords assigned by the authors.

Text classification using swarm intelligence in terms of automated grouping of PDF documents is given in [18]. The algorithm presented is inspired from the ant colony optimization. Basically classification is used in terms of clustering; PDF documents are converted into text files. Relative frequency of words in a document is calculated which is normalized with the word frequency in all documents to lower the importance of words occurring in all documents. Cosine similarity is used as a measure between the two objects.. For convergence the picking parameter of ant for picking an object was reduced.

Association rule mining approach towards document classification is given in [26]. Associative rule mining discovers relationships among items in a dataset. Documents are represented as transactions. Stopwords removal and Stemming is used to reduce the transaction size. Initially, rules are generated using the apriori algorithm. Two methodologies are used for the rules generation: one is, rule generation for each category; and the other one is association rule mining for the categories collectively. On the basis of these rules, classifier is developed. Experimental results are presented on the Reuters-21578 text collection [27].
Fig. 1. Classifying new document to ACM CCS.

Fig. 2. Relevant information of documents.
Linear text classification using category relevance factors (CRF) is given in [14]. CRF is maintained for all documents that belongs to a category. Profile vector for each category is maintained from CRF feature vector. Based on the cosine similarity between papers and categories, the document is classified to the category having maximum similarity.

Structured document representation increases the classification accuracy, as scientific publications are well structured documents; therefore, it is necessary to classify them in taxonomy with high percentage of accuracy. Existing approaches lack in use of relevant information of both metadata data and text available in the document. Some approaches towards classification rely on either keywords, or abstract or fulltext of the document. Most of the existing schemes are not evaluated for scientific publications datasets. Only two approaches [13, 16] focus on the classification of scientific publications. Similarly, most of the schemes do not consider multi class classification and multi level classification. To overcome these limitations, we have devised an approach that not only uses the relevant information for classification but also deals with multi-class and multi-level classification. Detailed discussion on relevant data selection from scientific publication is presented in the following sections.

4. DOCUMENT REPRESENTATION
For efficient document classification metadata and contents can be used. The common features are title, abstract, authors, keywords, and references. In most of the cases, title contains the theme of the work presented in the paper. Author’s information helps in identifying previous papers of the authors in the database. Abstract summarizes the paper and almost contains the important terms and theme of the paper. Keywords are the most weighted terms in assigning a document. This part mostly shows the domain of the paper. References can help in finding the cited paper’s category. In case of the most papers category of references, it is highly likely to assign the paper to that category [28]. Among these features, we selected title and keywords. The selection of these is discussed in detail in the discussion section. The relevant information required for document classification is depicted in the Fig. 2.

The information about a document is first pre-processed. Pre-processing prepares the data for accurate classification. First step in the pre-processing is the removal of unnecessary words from the TD. From this relevant feature (Title and Keywords) stopwords are removed using a list containing 548 stopwords [29]. After removing the stopwords, these features are stemmed using the well-known porter stemmer algorithm [23]. Based on the pre-processed data, we apply our classification algorithm.

5. PROPOSED TECHNIQUE FOR SCIENTIFIC DOCUMENT CLASSIFICATION
Document classification process is depicted in the proposed framework (Fig. 3). Initially, the dataset is populated with the similar approach. Title and keywords are extracted from each of the documents in the training dataset. Extracted title and keywords are pre-processed using stopwords removal and stemmer module in the framework. Representation of the dataset with respect to the documents is given in Fig. 2 and Fig. 3.

In Fig. 3, the user issues a TD for category identification. The TD is parsed in the dataextractor module, in which relevant data (Title and Keywords) are retrieved. The data extracted is passed on to the stopwordremoval module, which removes the unnecessary words using the list provided in [29]. The remaining text is passed on to the stemmer module, which returns the stemmed result to the matcher module. For stemming, we used the well-known porter stemmer algorithm [23]. The matcher module then predicts the category of the TD using the existing dataset. The category result is returned to the user and dataset is updated with the relevant category/ies information of the TD.

Matcher module is the main module of the
proposed framework. The matcher classifies an input TD into their relevant category/ies. Proposed solution towards automated category identification is inspired from the well known evolutionary particle swarm optimization. Proposed solution overcomes the two well-known problems (Multi-class and Multi-level) in the solution towards automated category identification.

Initially, the TD is matched with all the documents in each category. Average similarity of each category is computed, among which highly similar (using Eq. 3) categories are selected. At each level, besides identifying the category, we find similarity among the similarity score of each category. The search space is reduced at the second level by assigning the TD to selected categories at first level. Our algorithm recursively reaches to the bottom level to assign the paper to its correct category.

Proposed solution towards the classification of new document is a recursive approach, as depicted in Fig. 4. Initially, the TD will be checked with the same number of documents from each category. Global best gBest among all the local best pBest available categories will be selected. In our case, more than one gBest can be selected based on the Eq. 3 among average similarity measures with each category. If the difference is higher than a certain threshold then more than one category can be selected for the new document. After selection at first level, the new document will be matched with all the subcategories of the selected category/ies. The process can be continued till the leaf level of the ACM CCS is achieved or the fitness function is achieved.

Formally, the solution can be formulated as:

\[ C = \{A, B, \ldots, K\} \]

where \( C \) is the set of categories.

Each category contains sub-categories. For example

\[ A = \{A_0, A_1, \ldots, A_m\} \]

Each sub-category may itself contain third level sub-categories. Each category contains a set of documents as for example

\[ A = \{d_{a1}, d_{a2}, \ldots, d_{am}\} \]
\[ B = \{d_{b1}, d_{b2}, \ldots, d_{bm}\} \]
\[ \ldots \]
\[ K = \{d_{k1}, d_{k2}, \ldots, d_{km}\} \]

Each document contains a set of words among which \( t_1, t_2, \ldots, t_k \) are terms belonging to \( \text{title} \) and \( \text{keywords} \), as a feature vector, as

\[ D_{ci} = \{t_1, t_3, \ldots, t_k\} \]

Similarly, the new document is to classify contains terms of \( \text{title} \) and \( \text{keywords} \) as

\[ TD = \{t_1, t_3, \ldots, t_k\} \]

Membership of the TD (similarity) with each category is calculated as

\[ \mu_{ci}(TD) = \frac{\text{AVG}(\sum_{i=1}^{n}\text{similarity}(TD, d_{ci}))}{n} = x_{ci} \text{ where n the minimum number of documents in Ci} \] Eq:2

For multi-topic classification, Eq. 3 is used. In Eq. 3, \( \text{max}(x_{ci}) \) is the maximum average similarity selected at any level in the hierarchy; whereas \( x_{ci} \) is the membership of the document in each category using Eq. 2, \( \Psi \) is the threshold defined by domain expert which can be the maximum similarity difference between any two categories. We used Levenshtein distance [30] as a similarity measure. Classification at the next level (lower level) will be performed only for the categories selected using equation 3. The document movement in the taxonomy is identified using the Eq 3.

\[ D_t \in C_i \text{ where difference } (\text{max}(x_{ci}) - x_{ci}) > \Psi \text{ ...Eq:3} \]

Concept of social interaction is applied in using the PSO. Each particle (category) takes part in classification of the TD category identification in the taxonomy. Position and velocity of TD using PSO is given by the Eq. 2 and Eq. 3. In our velocity equation, we are not using cognitive and
Fig. 3. Framework for classification of test document.

Fig. 4. Classification of test document.
social parameters of PSO, as scientific document classification is hierarchical in nature. And at each level, the document is classified to a category or some of the categories. At lower level, PSO is again applied on the selected categories using Eq. 3.

Fig. 5 explains the proposed algorithm inspired from PSO. This algorithm works as a central module (Matcher) of the framework shown in Fig. 4. Initially the category with minimum numbers of documents is selected from the available categories are selected. $pBest$ and $gBest$ for each category are calculated using Eq. 2 and Eq. 3 respectively. The resultant category/ies information of the TD is updated in the database.

6. EXPERIMENTAL RESULTS

We have implemented the proposed scheme on the Journal of Universal Computer Science (J.UCS) dataset. Our proposed dataset contains 1460 research publication. We have taken 2/3 documents for training dataset and 1/3 for test dataset. J.UCS has extended the ACM CCS with two more categories with $L$ and $M$. Test accuracy result for 30 test documents from each category are shown in Fig. 7. The test sample selected was random in each category as shown in Fig. 7; each column shows the selected input documents in each category. The result of our test documents with each category is given in Fig. 6. The blue bar shows the correctly classified categories whereas the red bar shows the number of incorrect classified documents in each category. Overall accuracy of our approach is more than 84.71%. We have implemented our approach using MySQL as database with PHP.

We performed a set of experiment for the automation of topic identification in ACM CCS. In our first experiment, we used the features provided in the ACM CCS. For top level, we aggregated child features for a parent category. At each level, we stored all of the decedent’s features for each category. We used this database for the classification of TD. We matched the extracted TD features with our stored features for each category in the database. After several tests, the classification was not accurate. We changed the similarity measure used for finding the relatedness between the TD features with the features for each category, but the results were not promising. After manual expectation of the extracted features, similarity with the stored features in the database we conclude that the features provided in the ACM CCS are not suitable for the automation of topic identification.

Our second experiment was the selection of relevant information from test document to find similarity with individual documents in each category. Initially, we selected title, keywords and abstract, after a set of experiments we concluded that abstract diversify our classification results. When we tested the similar approach by excluding the abstract, the results were satisfying. Abstract contains a lot of text, which diversify the classification results. Another reason to exclude the abstract from text classification is the similarity measure which we used for our experimentations. After analyzing the result from our second experimentation for each individual category we observed that the categories having larger number of documents as compared to other categories, their results were comparably poor with the categories having less number of documents. The classification of a category having less number of documents was remarkable.

In our third set of experimentation, we selected the same number of document from each category. This time the classification for each category was relevant, closer to the results with other categories. Detail result for each category is shown in Fig. 7. The instance $(x,y,---,value)$ in a cell represents $x$ for original category, $y$ for the system identified category and $value$ representing the maximum average similarity. In some categories ($D$, $G$, and $K$) the misclassification error is relatively high as compared to the remaining categories.

One major problem in this experimentation is the error rate in training dataset. As previously used techniques for category identification were manual, in some of the cases in training documents we noticed that the assigned categories
For each category
   Initialize each category with minimum number of documents in a category among all the categories $C$
End
Do
   For each category
      Calculate similarity (Levenshtein distance) of TD with each document
      calculate the average similarity for each category as new pBest using Eq. 2
   End
Choose the category with the best average similarity value as gBest using Eq. 3
   For each category
      Calculate TD velocity (relevant categories for next level) using Eq.3
      Update category information of the new TD in database
   End
While leaf level is not attained

Fig. 5. Classification of new document algorithm.

Fig. 6. Result of test documents classification with each category.

Fig. 7. Detail result of random selected document in each category along with their results.
Classification of Scientific Publications

...to documents are not relevant, while in some cases it was observed that a document was assigned to some extra categories, beside their main categories. Result of proposed technique can be improved by removing the error rate from the training dataset.

Our experimental results are better than the Bayesian approach presented in [13] with 83.75% classification accuracy tested on four categories. Similarly, the proposed approach accuracy 84.71% is better than Bayesian network learned from data with 76.25% accuracy and naïve Bayesian classifier with 82.5% accuracy respectively. Majority best evidence, majority Genetic programming approach and SVM results are compared with the proposed approach. Majority best evidence, Majority GP and SVM [26, 31] having performance accuracy of 53.60%, 57.74% and 57.74% respectively. The compared results with the approaches for scientific publication classification are given in Table 1. In our experiments, we have included all of the categories provided in J.UCS and provided results for each category. The other advantage of the proposed approach is to overcome the multi-class and multi-level classification of scientific publication to the taxonomy.

Table 1. Comparison of different approaches.

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Number of Categories</th>
<th>Average Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bayesian Approach</td>
<td>4</td>
<td>83.75%</td>
</tr>
<tr>
<td>Bayesian Network learned from Data</td>
<td>4</td>
<td>76.25%</td>
</tr>
<tr>
<td>Naïve Bayesian</td>
<td>4</td>
<td>82.50%</td>
</tr>
<tr>
<td>Majority Best Evidence</td>
<td>11</td>
<td>53.60%</td>
</tr>
<tr>
<td>Majority GP</td>
<td>11</td>
<td>60.81%</td>
</tr>
<tr>
<td>SVM</td>
<td>11</td>
<td>57.74%</td>
</tr>
<tr>
<td>Proposed Approach</td>
<td>13</td>
<td>84.71%</td>
</tr>
</tbody>
</table>

7. SUMMARY

Classification is, in general, a central problem in different domains. The multi-class classification is different to the ordinary classification problem. Similarly, classifying the document at different levels is also an important issue to many classification problems. Therefore, in this paper we have proposed a solution for both multi-class classification and multi-level classification. Our classification technique is an enhanced from of PSO in the context of document classification. Efficiency is achieved by reducing the size of features set and by considering the minimum number of documents in all categories. This solution can be applied for different domain where an instance can belong to multiple categories along with multi-level classification. We have implemented and tested our technique which provides better results as compared to existing techniques. This work will help authors in selecting the correct category for papers. Correct classification can, in terms, be quite useful for document retrieval and analysis.

8. REFERENCES

Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer’s Solutions in Experimentally-induced Hypovolumic Shock in Dogs

Muhammad A. Zafar1*, Nemat Ullah1, Imran Sarfaraz2 and Fahad Raza1

1Department of Clinical Sciences, Faculty of Veterinary and Animal Sciences, PMAS Arid Agriculture University, 46300, Rawalpindi
2Poultry Research Institute, Murree Road, Rawalpindi

Abstract: This study was conducted to evaluate the relative efficacy of bolus administration of lactated Ringer’s (LRS) and hypertonic saline (HSS) solutions at the same dose rate in hypovolaemic shock (HS). Twelve healthy adult Mongrel dogs were acclimatized and baseline data were obtained. The animals were then subjected to HS through shedding blood and mean arterial pressure (MAP) of < 40 mm of Hg was considered as cut-off point for the development of HS. After 30 minutes of HS, the dogs were randomly divided into two equal groups (A & B) before initiation of treatment. The dogs of group A were infused with LRS @ 4 mL kg-1 body weight (BW) followed by slow administration of LRS @ 10 mL kg-1 BW, while animals of group B were administered with HSS @ 4 mL kg-1 BW followed by slow administration of normal saline @ 10 mL kg-1 BW. Relative efficacy was measured through MAP, haematological parameters, serum electrolytes and blood pH. All parameters were measured during shock, and 30, 60 minutes, 3, 6, 12, and 24 hours of post-infusion. The animals of group B showed better recovery from shock, while 50% mortality was observed in the group A animals at the monitoring time. In group B, the mean ±SD of MAP was 116±8.0, equivalent to baseline within 60 minutes of infusion but in group A, it was far less than the baseline value. Sodium elevated above baseline in the group B animals but within observing time it became close to the baseline. Blood pH was restored towards normal more rapidly in group B animals which was significantly different from the group A animals (P < 0.05). Recovery towards normal baseline values of measured variables was more rapid and sustained for the HSS treated group. It was concluded that HSS has a rapid effect in resuscitating the dogs from HS than LRS.

Keywords: Resuscitation, hypovolaemia, 7.5% NaCl solution, dogs

1. INTRODUCTION

Hypovolaemia is the principal cause of death in many of the traumatized dogs and diarrheic calves etc. Survival from hypovolaemic shock is dependent upon the restoration of cardiac output, oxygen transport and blood volume. So, for rapid recompensation, isotonic fluids have been widely used and considered key to success in rectification of this alarming condition [1].

Sodium is expected to elevate extracellular volume more than the transfused amount because hypertonic Na+ solution attracts water out of the cells, thus expands plasma volume and increases the venous preload [2, 3]. It has been a matter of debate about the most suitable solution for the resuscitation of the animals from hypovolaemic shock and in recent years, resuscitation with hypertonic saline solution (HSS) has gained widespread acceptance because hypertonic salt is a potent volume expander. Hypertonic saline solution induces a rapid plasma volume, increase cardiac output, mean arterial pressure (MAP) and may improve survival [4, 5]. So, HSS seems likely to produce a more rapid resuscitation and beneficial effects than isotonic solutions.
In this study, relative resuscitative efficacy of hypertonic saline solution and lactated Ringer’s solution (LRS) was evaluated in experimentally induced hypovolaemic shock in the dogs at the same dose and infusion rates.

2. MATERIALS AND METHODS

Twelve adults (1±0.15 years old), healthy mongrel dogs of either sex were used in the study. During the acclimatization period of one week, all the animals were fed on the same feed and fresh water was available ad libitum. Blood, faecal and urine examination during this period indicated that these dogs were healthy.

Each dog was sedated with intramuscular administering Acepromazine + Ketamine hydrochloride @ 0.1 mg/kg + 4 mg/kg, respectively, after the subcutaneous administration of atropine sulphate (0.05 mg/kg of body weight). Medial side of the right limb was shaved for exposure of femoral artery and was cannulated with 22 gauge (Vasocan Brannula®) IV cannula for measurement of mean arterial blood pressure (MAP) by using saline sphygmomanometer. For infusion of solutions, cannula (Vasocan Brannula®) of 18 gauge was used percutaneous into the left cephalic vein for administration of allotted solutions. Dogs were bled through jugular vein for the induction of hypovolaemic shock and blood was collected in blood bags @ 40±5 ml/kg BW [6]. For the assessment of shock state, mean arterial pressure of 50 mm Hg was considered cut-off point to stop bleeding [6].

After 30 minutes of inducing shock, all animals were randomized into two groups (n=6). Group A was infused with lactated Ringer’s solution @ 4 ml/kg, BW, and was maintained with the same solution @ 10 ml/kg, BW. Animals of group B were infused with hypertonic saline (7.5% sodium chloride) solution @ 4 ml/kg, BW, and maintained with isotonic saline solution (ISS) @ 10 ml/kg, BW. Both lactated Ringer’s and hypertonic saline solutions were administered at the infusion rate of 20 ml/min.

Blood samples of all the animals were collected at baseline (before shock), during shock, 30, 60 minutes, 3, 6, 12, and 24 hours post infusion of respective solutions. The venous blood samples with anticoagulant were used to determine haemoglobin concentration (Hb conc.), haematocrit (Hct) and pH values, while the samples without anticoagulant were used to harvest serum to determine serum electrolytes (Na+, K+ and Cl-) and bicarbonates (HCO₃⁻). The Hb concentration was determined by cyanmethemoglobin method and Hct values were determined with microhematocrit method as described by Benjamin [7]. Serum Na+, K+ and Cl- were determined with the help of EasyLyte Analyzers (Medica Corporation, Bedford) and HCO₃⁻ with the help of Microlab-200 (Vital Scientific, Merck, Netherlands).

The data are presented as mean ± SD (Table 1). Statistical analyses were performed using t-test. All analysis was performed using the Statistical Software Package (SPSS Version 11.5). Statistical significance was assigned at P < 0.05.

3. RESULTS

Baseline values for the blood parameters of both the groups were not significantly different (Table 1). After induction of shock and infusion treatment 50% mortality was recorded within monitoring time in group A, while no mortality was observed in group B infused with hypertonic saline solution.

3.1 Haematological Parameters

After infusion of respective treatments, group B showed significant difference (P < 0.05) over group A by increasing their values from 3rd hour to onwards for Hb concentration and haematocrit (Table 1). After 24 hours, HSS recovered the haematocrit values near to baseline in animals of group B while LRS (group A) was unable to recover these values.

3.2. Mean Arterial Pressure

There were no differences regarding MAP between two groups before or during shock. However, MAP increased dramatically in group B after 30 minutes of treatment and this value was significantly higher (P < 0.05) than group A (Table 1). Mean arterial pressure in group B was near to baseline within 60 minutes. In group A, MAP did not recover even after 24 hours.

3.3. Serum Electrolytes and Biochemistry

There was no significant difference between the groups during shock in all the serum electrolytes
i.e. Na⁺, K⁺, & Cl⁻. After treatment, sodium was near to baseline within 60 minutes in group B and then it increased to 156 ± 9.0 mEq/L which was much higher than the normal values and it was significantly higher (P < 0.05) than group A. Group A showed decreased level of sodium even after 24 hours. In case of potassium, there was no difference between both groups (Table 1).

After fluid resuscitation, there was no significant difference for bicarbonate (HCO₃⁻) in two treatment groups. Blood pH was near to baseline within 3 h after administration of HSS to the animals in group B and it was significantly higher (P < 0.05) in this group as compared to group A. But after 6 h, there were no significant difference between both the groups (Table 1).

4. DISCUSSION

The objective of resuscitation of a hypovolaemic shocked patient is the restoration of the lost intravascular fluid volume [3]. The increase in fluid volume as well as accompanying increase in mean arterial pressure and oxygen transport has been assumed to contribute to an increased opportunity for treatment strategies. Haematocrit (Hct) and haemoglobin concentration indicate the presence of total cells including erythrocytes, leukocytes and platelets [8]. Any increase or decrease in values of both parameters showed the signs of dehydration and hypovolaemia, respectively. In the present study, these values decreased which clearly indicates reduction in the number of cells and ultimately plasma volume. Hypertonic saline solution generates plasma volume expansion rapidly by increasing Hct and Hb concentration values and also increased osmosis and promoted movement of endogenous fluid from the extravascular to intravascular space. While lactated Ringer’s solution administration at low dose was unable to cause plasma volume

### Table 1. The results of haematological parameters, mean arterial pressure, serum electrolytes and blood gases in both groups

<table>
<thead>
<tr>
<th>Variable</th>
<th>Group A</th>
<th>Baseline</th>
<th>Shock</th>
<th>30 min</th>
<th>60 min</th>
<th>3 hr</th>
<th>6 hr</th>
<th>12 hr</th>
<th>24 hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Dogs</td>
<td>Group A</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Hct (%)</td>
<td>Group A</td>
<td>37.0±3.7</td>
<td>14.7±0.5</td>
<td>14.7±0.7</td>
<td>15.5±0.5</td>
<td>16.3±0.5</td>
<td>17.3±0.6</td>
<td>19.2±0.8</td>
<td>20.0±0.6</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>38.0±2.8</td>
<td>15.7±2.6</td>
<td>16.3±2.3</td>
<td>18.2±2.6</td>
<td>22.5±1.9⁺</td>
<td>24.7±2.3⁺</td>
<td>30.0±1.1⁺</td>
<td>32.7±2.0⁺</td>
</tr>
<tr>
<td>Hb conc. (g/dL)</td>
<td>Group A</td>
<td>14.3±2.3</td>
<td>8.9±1.8</td>
<td>9.0±1.8</td>
<td>9.2±1.9</td>
<td>9.4±2.0</td>
<td>9.8±1.6</td>
<td>10.2±1.6</td>
<td>10.7±1.5</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>12.7±2.0</td>
<td>8.5±1.2</td>
<td>8.8±1.2</td>
<td>9.5±1.2</td>
<td>10.3±1.6⁺</td>
<td>10.8±1.1⁺</td>
<td>11.3±1.8⁺</td>
<td>11.9±1.9⁺</td>
</tr>
<tr>
<td>MAP (mm Hg)</td>
<td>Group A</td>
<td>126±10.6</td>
<td>48±4.0</td>
<td>57±6.1</td>
<td>65±4.9</td>
<td>79±5.1</td>
<td>88±4.9</td>
<td>90±6.8</td>
<td>90±6.8</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>120±8.9</td>
<td>49±2.0</td>
<td>94±5.4⁺</td>
<td>116±8.0⁺</td>
<td>114±10.2⁺</td>
<td>117±13.3⁺</td>
<td>120±12.7⁺</td>
<td>118±9.8⁺</td>
</tr>
<tr>
<td>Sodium (mEq/L)</td>
<td>Group A</td>
<td>140±3.7</td>
<td>121±1.9</td>
<td>121±1.9</td>
<td>123±2.4</td>
<td>123±2.4</td>
<td>124±1.5</td>
<td>126±1.4</td>
<td>126±1.4</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>141±3.4</td>
<td>120±4.2</td>
<td>135±4.1⁺</td>
<td>145±4.2⁺</td>
<td>153±4.8⁺</td>
<td>156±9.0⁺</td>
<td>147±6.1⁺</td>
<td>145±3.7⁺</td>
</tr>
<tr>
<td>Potassium (mEq/L)</td>
<td>Group A</td>
<td>4.3±0.4</td>
<td>3.2±0.3</td>
<td>3.2±0.3</td>
<td>3.3±0.2</td>
<td>3.3±0.3</td>
<td>3.4±0.2</td>
<td>3.5±0.2</td>
<td>3.5±0.5</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>4.3±0.3</td>
<td>3.5±0.2</td>
<td>3.7±0.2</td>
<td>3.8±0.3</td>
<td>3.9±0.3</td>
<td>4.1±0.4</td>
<td>4.1±0.3</td>
<td>4.1±0.3</td>
</tr>
<tr>
<td>Chloride (mEq/L)</td>
<td>Group A</td>
<td>119±4.7</td>
<td>109±5.4</td>
<td>109±5.3</td>
<td>109±5.3</td>
<td>107±6.7</td>
<td>111±5.1</td>
<td>112±5.3</td>
<td>112±5.4</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>116±6.5</td>
<td>105±6.8</td>
<td>112±7.3</td>
<td>118±7.0</td>
<td>123±6.9⁺</td>
<td>122±3.9⁺</td>
<td>119±6.2</td>
<td>121±3.4⁺</td>
</tr>
<tr>
<td>HCO₃⁻ (meq/L)</td>
<td>Group A</td>
<td>19.5±2.6</td>
<td>13.5±1.5</td>
<td>12.8±0.7</td>
<td>13.6±1.3</td>
<td>14.1±1.1</td>
<td>15.3±1.2</td>
<td>15.0±0.8</td>
<td>15.8±0.9</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>17.8±1.3</td>
<td>12.6±0.5</td>
<td>13.0±0.8</td>
<td>13.5±0.5</td>
<td>14.1±1.3</td>
<td>15.8±1.9</td>
<td>16.3±1.5</td>
<td>16.1±1.7</td>
</tr>
<tr>
<td>Blood pH</td>
<td>Group A</td>
<td>7.3±0.12</td>
<td>6.7±0.27</td>
<td>6.7±0.21</td>
<td>6.9±0.11</td>
<td>7.0±0.17</td>
<td>7.1±0.10</td>
<td>7.1±0.09</td>
<td>7.2±0.04</td>
</tr>
<tr>
<td></td>
<td>Group B</td>
<td>7.3±0.20</td>
<td>7.1±0.52</td>
<td>7.0±0.55</td>
<td>7.2±0.44⁺</td>
<td>7.3±0.62</td>
<td>7.5±0.55⁺</td>
<td>7.4±0.31</td>
<td>7.3±0.21</td>
</tr>
</tbody>
</table>

*⁺= Significant difference to other group; Group A= Lactated Ringer’s solution; Group B= Hypertonic saline solution; Hb conc.= Haemoglobin concentration; Hct= Haematocrit; MAP= Mean Arterial pressure; HCO₃⁻ = Bicarbonates.
expansion. It is widely accepted that HSS increases MAP more rapidly, principally, the rise in serum osmolality is thought to cause the major effect, by osmotic extraction of water from the interstitium and expansion of the intravascular volume [3]. In this study, MAP was rapidly restored in animals treated with HSS [5, 9, 10, 11]. The higher concentration of Na+ ions is considered the main reason of MAP restoration because sodium is the main source which extracts water from intracellular space to the extracellular and increase serum osmolality.

Since sodium is largely confined to the extracellular compartments, HSS expands extracellular fluid space by extracting water from the cell. However, a rapid increase in serum sodium concentration could cause salt poisoning [2, 10]. The increase in serum sodium may also cause hypernatraemia, but at risk. Here, the administration of 4 ml/kg of HSS produced transient high sodium level (Table 1) but it remained under 160 mEq/L level which is the cut-off point of hypernatraemia [10] and no adverse effect was observed during the whole study and these results are inline with the results reported by other scientists [5, 9, 10, 11, 12]. On the other hand, LRS is hypotonic regarding sodium ions, so, sodium remained suppressed throughout the observing time. This low level of sodium didn’t cause any transient change in extracting water from the cells.

Blood pH is an indicator of acidosis. During hypovolaemia, blood pH decreased which revealed that animals were suffering with acidemia. In our study, after HSS infusion, it increased satisfactory which showed that hypertonic saline could overcome acidosis as reported in previous studies [4, 12, 13]. Hence, HSS had shown well affectivity against acidosis.

5. CONCLUSIONS
Hypertonic saline solution (HSS), immediately followed by normal saline solution, seems to be a promising and practical method for the initial treatment of hypovolaemic shock. Use of Lactated Ringer’s solution in similar situations will not help to resuscitate the hypovolaemic animals.

6. REFERENCES
Comparative Phycoremediation of Sewage Water by Various Species of Algae

Farooq Ahmad*, Amin U. Khan and Abdullah Yasar
Sustainable Development Study Centre, GC University, Lahore, Pakistan

Abstract: In this study sewage water treatment efficiency of Chlorella vulgaris, Rhizoclonium hieroglyphicum and mixed algae culture (Microspora sp., Navicula sp., Lyngbya sp., Cladophora sp., Spirogyra sp. and Rhizoclonium sp.) was compared. Sampled wastewater was analyzed for various parameters (i.e., COD, BOD, TS, TSS, TDS, TC, FC, TKN, TP, NO₃-N, PO₄, SO₄ and Cl⁻) and concentrations of all these parameters in the untreated water were above the permissible limits of National Environmental Quality Standards of Pakistan (2000). Various algal species were used to treat sewage water by varying pond size, treatment duration, seasonal variation and growth rate of algae to arrive at the optimum outcome. Maximum percent reductions of various parameters, attained with C. vulgaris, were: chemical oxygen demand (98.3%), biochemical oxygen demand (98.7%), total Kjeldahl nitrogen (93.1%), total phosphorus (98.0%), nitrate (98.3%), phosphate (98.6%), chloride (94.2%), total coliforms (99.0%), faecal coliforms (99.0%) and total dissolved solids (98.2%) while maximum reduction in total suspended solids (92.0%) was obtained with a mixed algae culture and maximum increase in biomass by R. hieroglyphicum (0.75 g L⁻¹ day⁻¹). Reduction in the concentration of pollutants in sewage water was to such a low level that it can be thrown in waterbodies without any further treatment.

Keywords: Algae, Chlorella, phycoremediation, pollution, pond, rhizoclonium, wastewater

1. INTRODUCTION

Phycoremediation is the use of micro- or macro-algae for the removal or biotransformation of pollutants, including nutrients and toxic chemicals from wastewater [1, 2]. The term phycoremediation was introduced by John [3] to refer to the remediation carried out by algae. Phycoremediation is comprised of several applications: (i) nutrient removal from municipal wastewater and effluents rich in organic matter; (ii) nutrient and xenobiotic compounds removal with the aid of algae-based biosorbents; (iii) treatment of acidic and metal wastewaters; (iv) CO₂ sequestration; (v) transformation and degradation of xenobiotics; and (vi) detection of toxic compounds with the aid of algae-based biosensors. A distinct comparison can be made between microalgae and other conventional technologies on the basis of nutrient removal [4].

The prevalent sewage treatment strategies are costly and not affordable in the developing countries of the world due to high consumption of energy. Sewage irrigation is another big problem which affects human health directly and indirectly. Pakistan is also facing problems related to contamination of water, where excessive and untreated sewage is discharged directly into water bodies as a result this water can not even be used for irrigation purposes. Sewage is also loaded with diseased pathogens, where they get their direct entry into the drinking water by leakage and cause serious intestinal infections especially in children. Prolong exposure to these hazards can cause several diseases including cancer and birth defects in young ones [5].

Microalgae can be used to treat municipal wastewater, industrial effluents and solid wastes aerobically as well as anaerobically. The use of algae to treat wastewater has been in practice for over 40
Farooq Ahmad et al

years and first descriptions of this application was reported by Oswald [6]. So the use of microalgae for the treatment of municipal wastewater has been a subject of research and development for several decades. Extensive work has been conducted to explore the feasibility of using microalgae for wastewater treatment, especially for the removal of nitrogen and phosphorus from effluents [7, 8] which would otherwise result in eutrophication if dumped into water bodies.

Biological treatment enhances the removal of nutrients, heavy metals, pathogens and provides O₂ to heterotrophic aerobic bacteria for mineralization of organic pollutants, as a result CO₂ is released from bacterial respiration [9]. Photosynthetic aeration is important to reduce operation costs and limit the risks for pollutant volatilization under mechanical aeration. Recent studies have shown that microalgae can indeed support the aerobic degradation of various hazardous contaminants [10]. The hyper concentrated algal cultures, called ‘activated algae’ can be used to decrease the land and space requirements for wastewater treatment. This process removed nitrogen and phosphorus in a very short period of time, i.e., less than 1 hour [4]. Microalgae can be efficiently used to remove significant amount of nutrients because they require high amounts of nitrogen and phosphorus for the synthesis of proteins (45–60% of microalgae dry weight), nucleic acids and phospholipids. Nutrient removal can further be increased by NH₂ stripping or NH₃ precipitation due to the raise in pH [11]. This method is not appropriate for large scale wastewater treatment, therefore there is need to improve the technology.

The advantages of using algae for sewage treatment include low cost of the operation, possibility for assimilation of nitrogen and phosphorus into algal biomass, which consequently can be used as fertilizer thus avoiding sludge handling problem and the discharge of oxygenated effluent into the water body. In addition, the process has no carbon requirement for nitrogen and phosphorus removal, which is sustainable for the treatment of sewage effluents [2].

The current study was conducted to assess the pollution load carried by municipal drains into the water bodies and its treatment with environment friendly and naturally available fresh water algae. A comparison was made to find out the algal species with best reduction efficiency. Treatment efficiency was also measured in different months of the year to assess the effect of seasonal variations.

2. MATERIALS AND METHODS

2.1 Experimental Location and Layout

The experimental outdoor work was carried out in Botanic Garden and roof of Sustainable Development Study Centre, GC University, Lahore. The locations were selected in consideration of availability of enough light, space and ponds facility. The study was conducted to test the feasibility of growing algae as a sewage nutrients reduction agent. These experiments were carried out in batch mode and cultivation time for each condition was 8 days with three replications. The effect was studied in terms of algal growth and wastewater nutrients reduction. Aquatic cultures were conducted in synthetic ponds with different sized and having dimensions of 0.15 x 0.15 x 0.3 m (P1), 0.3 x 0.3 x 0.15 m (P2) and 0.9 x 0.15 x 0.45 m (P3) with final capacity 6.75 L, 13.5 L and 60.75 L respectively. In addition a pilot scale large pond with dimensions of 8 x 2 x 1 m (P4) and final capacity of 16000 L was used for sewage treatment [12]. Initially the algal biomass was kept constant in each pond with equal quantity of wastewater except in pond P4 where biomass was added on per litre bases deducted from other ponds.

2.2 Wastewater Sampling and Analysis

Waste water samples were collected from various sewage drains of Lahore city by grab sampling techniques; then these samples were mixed to form composit sample [13]. The composite sample was filtered to remove coarse particles and divided into three replicates. The analysis for various physicochemical parameters like biochemical oxygen demand (BOD), chemical oxygen demand (COD), total solids (TS), total dissolved solid (TDS), total suspended solids (TSS), total phosphorus (TP), total Kjeldhal nitrogen (TKN), Nitrate-nitrogen (NO₃-N), phosphate (PO₄) and chloride (Cl⁻) was carried out by using standards methods
of American Public Health Association [14]. The sampled untreated wastewater was stored at 4°C until used for algae cultivation.

2.3. Algal Sampling, Identification and Incubation

*R. hieroglyphicum* was collected from the fish forms of the Department of Fisheries near Manawa police station, Lahore. *C. vulgaris* sample was obtained from Mr. Tariq Rashid, working in the same department on utilization of *Chlorella species* as fish feed. Mixed algae culture was collected from Baradari in River Ravi, near Lahore and Botanic Garden of GC University, Lahore. These algal species were identified by adopting a standard methods described by Zarina et al [15, 16].

Collected algae were kept under optimum conditions as local outdoor cultures and inocula of algae were transferred in previously mentioned ponds for sewage treatment. All the experimental ponds with same quantity of selected algal species were kept under similar light intensity and photoperiod to get comparative results. Before analysis samples were filtered to remove algal biomass present in the sample after treatment, it was done to avoid the interference caused by algal biomass in final results. The rate of growth of these algal species was measured by the estimation of fresh weight. These algal species were harvested every second day to measure its fresh weight.

3. RESULTS AND DISCUSSION

3.1. Sewage Treatment with Algae

The collected sample was characterized for various parameters showing the results which were above National Environmental Quality Standards (NEQS) [17] values. It was observed from the study that in comparison to *R. hieroglyphicum* and mixed algae culture, the *C. vulgaris* was more efficient in treating wastewater (Table 1). Przytocka-Jusiak et al [18] reported that the nutrient reduction depends on the quantity of nutrients present in wastewater and extent of these nutrients absorbed by the algae for incorporation into algal tissues. *Chlorella* has the ability to uptake maximum phosphorous for the synthesis and accumulation of polyphosphates in their bodies.

3.2. Effect of Algal Species on Pollution Reduction

Various parameters of wastewater were analyzed after treatment with *R. hieroglyphicum*, mixed algae culture and *C. vulgaris*. Before analysis samples were filtered to remove algal biomass which can interfere with the results. The reduction percentage of COD (98.27%), BOD (98.69%), TKN (93.14%), TP (98%), NO₃-N (98.33%), PO₄ (98.63%), Cl⁻ (94.16%), TC (99%), FC (99%) and TDS (98.21%) was noted to be maximum with *C. vulgaris*. The treatment efficiency of *R. hieroglyphicum* was

| Table 1. Characterization of untreated wastewater and quantity of pollutants reduced by *C. vulgaris*, *R. hieroglyphicum* and mixed algae culture. |
|-----------------|----------------|----------------|----------------|----------------|
| Parameters      | Untreated      | Reduction with *C. vulgaris* | Reduction with *R. hieroglyphicum* | Reduction with Mixed algae culture | NEQS 2000 |
| COD (mg/L)      | 721            | 708.5            | 705.8          | 683.6          | 150        |
| BOD (mg/L)      | 407            | 401.7            | 396.4          | 389.8          | 80         |
| TSS (mg/L)      | 970            | 836.1            | 869.6          | 891.9          | 200        |
| TDS (mg/L)      | 4650           | 4567.2           | 4546.2         | 4546.2         | 3500       |
| TC (MPN)        | 1.6×10³        | 1.58×10³         | 1.58×10³       | 1.57×10³       | ---        |
| FC (MPN)        | 1.6×10³        | 1.58×10³         | 1.58×10³       | 1.57×10³       | ---        |
| TKN (mg/L)      | 35             | 32.6             | 31.8           | 30.5           | ---        |
| TP (mg/L)       | 22             | 21.6             | 21.1           | 20.9           | ---        |
| NO₃-N (mg/L)    | 12             | 11.8             | 11.8           | 11.7           | ---        |
| PO₄ (mg/L)      | 10             | 9.86             | 9.86           | 9.4            | ---        |
| SO₄ (mg/L)      | 55             | 53.5             | 50.2           | 53.5           | 600        |
| Cl⁻ (mg/L)      | 60             | 56.5             | 55.5           | 51.5           | 1000       |
Fig. 1. Comparative efficiency of algal species in removing wastewater pollutants.

Fig. 2. Comparative percentage reduction of various parameters with different species of algae in time.
less than *C. vulgaris* but it was found to be higher than mixed algae culture. However the reduction percentage of TSS (91.3%) with mixed algae culture was higher than both *R. hieroglyphicum* and *C. vulgaris* (Figure 1). Microalgae showed efficient results in removing N and P from sewage-based wastewater either applied in free-swimming suspension or in an immobilized form. Increase in pH to 8 or above causes precipitation of phosphorous from the wastewater so it will not be wrong to say that nutrient reduction in wastewater occurs due to algal uptake as well as other reasons. Algal biomass contain phosphorous concentration ranging from 0.5-3.3 % of its dry mass which means 3-23 % of phosphorous was reduced due to algal uptake. In secondary treated wastewater more than 80% reduction of nitrate and total phosphorous was shown by various species of *Chlorella* and *Scenedesmus* [19, 20, 21].

### 3.3. Effect of Treatment Time on Pollution Reduction

Treatment duration has profound effect on treatment efficiency as the treatment time increases, algal biomass increases and it absorbs more nutrients from wastewater which results in the reduction of those nutrients. In the current study treatment time was taken in days (0-8) and wastewater was analyzed after every two days. It was observed that reduction became constant on 8th day with all the three types of algae however minimum reduction was achieved with mixed algae culture (Figure 2). Algae species *Chlorella* was widely applied for wastewater treatment and had proven abilities of removing nitrogen, phosphorus, and chemical oxygen demand (COD) with different retention times ranging from 10 hrs to 42 days [22].

### 3.4. Increase in Biomass of Algae in Wastewater

Increase in biomass of algae with the passage of time was measured by taking into account its fresh weight. Increase in biomass was observed from day 2 to 8 in all three types of algae. As five litre of wastewater was used to grow algal species for measurement of growth so increase in biomass on 8th day was divided by five for per litre and 8 for per day calculation. Maximum growth rate (0.75g.L⁻¹.day⁻¹) was shown by *R. hieroglyphicum*

<table>
<thead>
<tr>
<th>Algal species</th>
<th>Algal biomass (g)</th>
<th>Days</th>
<th>Increase in weight <em>(g. L⁻¹ .day⁻¹)</em></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>C. vulgaris</em></td>
<td>Fresh weight</td>
<td>0  2  4  6  8</td>
<td>100 102.7±2 115±0.6 121.8±2.3 122±1.5</td>
</tr>
<tr>
<td><em>R. hieroglyphicum</em></td>
<td>Fresh weight</td>
<td>0  2  4  6  8</td>
<td>100 109±1.1 120±2.1 128.5±1.6 130.01±1.8</td>
</tr>
<tr>
<td>Mixed algae culture</td>
<td>Fresh weight</td>
<td>0  2  4  6  8</td>
<td>100 105.5±0.8 118.4±0.9 122.3±1.5 124.0±1</td>
</tr>
</tbody>
</table>

* 8th day increase in biomass was divided by 5 (for L⁻¹) and 8 (for day⁻¹) measurement

### Table 3. Percent reduction in various parameters with algal species in different sized ponds.

<table>
<thead>
<tr>
<th>Parameters</th>
<th><em>C. vulgaris</em> P1</th>
<th><em>R. hieroglyphicum</em> P1</th>
<th><em>R. hieroglyphicum</em> P2</th>
<th><em>C. vulgaris</em> P3</th>
<th><em>R. hieroglyphicum</em> P3</th>
<th><em>C. vulgaris</em> P4</th>
<th><em>R. hieroglyphicum</em> P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>COD</td>
<td>89.4</td>
<td>98.6</td>
<td>88.1</td>
<td>98.7</td>
<td>86.6</td>
<td>98.8</td>
<td>89.9</td>
</tr>
<tr>
<td>BOD</td>
<td>93.7</td>
<td>98.8</td>
<td>92.9</td>
<td>98.9</td>
<td>89.5</td>
<td>98.9</td>
<td>93.7</td>
</tr>
<tr>
<td>TS</td>
<td>86.2</td>
<td>95.3</td>
<td>93.1</td>
<td>95.1</td>
<td>84.2</td>
<td>97.6</td>
<td>87.5</td>
</tr>
<tr>
<td>TC</td>
<td>94.1</td>
<td>99</td>
<td>95.2</td>
<td>99</td>
<td>92.4</td>
<td>99</td>
<td>93.8</td>
</tr>
<tr>
<td>TKN</td>
<td>89.4</td>
<td>97.7</td>
<td>87.7</td>
<td>97.2</td>
<td>86.3</td>
<td>97.7</td>
<td>90.9</td>
</tr>
<tr>
<td>TP</td>
<td>89.7</td>
<td>93.8</td>
<td>92.6</td>
<td>94</td>
<td>86.9</td>
<td>98.4</td>
<td>91</td>
</tr>
<tr>
<td>NO₃-N</td>
<td>91.2</td>
<td>98.1</td>
<td>94.7</td>
<td>98.5</td>
<td>89.1</td>
<td>98.9</td>
<td>92.8</td>
</tr>
<tr>
<td>PO₄</td>
<td>88.4</td>
<td>94.3</td>
<td>88.9</td>
<td>95</td>
<td>86</td>
<td>98.7</td>
<td>89.9</td>
</tr>
</tbody>
</table>
**Fig. 3a.** Percent reduction in wastewater characteristics in different months of the year with *C. vulgaris* species.

**Fig. 3b.** Percent reduction in wastewater characteristics in different months of the year with *R. hieroglyphicum*. 
while the increase in fresh weight resulted by C. vulgaris and mixed algae culture was observed to be 0.55 g L\(^{-1}\) day\(^{-1}\) and 0.6 g L\(^{-1}\) day\(^{-1}\), respectively. It was also observed that increase in growth became almost constant after 6\(^{th}\) day due to deficiency in the availability of the nutrients in the medium (Table 2). Ruiz-Marin et al [20] performed many experiments under batch culture condition in which microalgae showed high growth rates in initial days but growth and chlorophyll contents were decreased after four cycles of culture indicating collapse of the culture due to nutrient deficiency.

3.5. Effect of Pond Size in Pollution Reduction

Ponds of different shapes and sizes (P1, P2, P3 & P4) were selected to find out the relationship of exposed surface to pollution reduction. Results showed that maximum reduction was carried out in P2 and P4 with all three types of algae. It was because in these pond there was more surface area exposed for algae to be in contact with the wastewater and more light was available for the growth of the algal species. C. vulgaris also showed more reduction in P1 and P3 (Table 3) as compared to R. hieroglyphicum and mixed algae culture because microalgae remained suspended in the wastewater while macroalgae come at the surface mostly therefore the wastewater at the depth was not in contact with the biomass which can be the reason of least reduction so in order to get maximum reduction a pond should be designed in such a way that it should have more surface area exposed to light and shallow water for maximum penetration of light.

3.6. Effect of Seasonal Variations on Pollution Reduction

In order to evaluate the seasonal variation on pollution reduction all three types of algal cultures were grown under optimum conditions in alternative months of the year. Experiments were conducted for 6 alternative months by renewal of sewage and algae but the initial concentration of pollutants in experimental wastewater was kept similar in all the selected months with same initial quantity of biomass of three selected algae. It was concluded from these experiments that maximum pollution reduction was observed in the month of November and March with all three types of algal cultures (Figure 3a,3b & 3c). It was due to moderate temperature found in these months which promote algal growth while minimum reduction was found in January, May and July due to low temperature in January and very high temperature in May and July. It was observed that the reduction percentage of

**Fig. 3c.** Percent reduction in wastewater characteristics in different months of the year with mixed algae culture.
pollutants in sewage in all the months was enough to be used for irrigation purposes. Green algae demands more nitrogen and phosphorous than do many other species, and they can take up generous nitrogen when the phosphorous content is relatively high [23].

4. CONCLUSIONS

The study revealed that algae are an effective organism for the reduction of biological and chemical pollutants from sewage water. Growth rate indicated that algae can luxuriously grow in wastewater medium due to availability of all necessary nutrients. Wastewater treated by algae can be used for irrigation purpose or released into water bodies as all the measuring parameters were in the permissible limits of National Environmental Quality Standards (2000) with selected species of algae. The study showed that phycoremediation process can be inversely affected by high temperature (May and July) and low temperature (January) but the reduction of pollutants in these months was enough to bring the remaining concentration into permissible limits. Consequently, this algal based treatment can be a sustainable technique for wastewater treatment.

5. ACKNOWLEDGEMENTS

The authors acknowledge Government College University, Lahore for providing funding for the current study. The research work was done in laboratories of Sustainable Development Study Centre of the University. The authors also acknowledge Mr. Tariq Rashid and Director, Punjab Fisheries Department for providing algal samples.

6. REFERENCES

19. Martinez, M. E., S. Sanchez, J. M. Jimenez, F. El


Remarks on \((1,2)\)^{*}-\(a\hat{g}\)-Homeomorphisms

T. Salai Parkunan\(^1\)*, S. Pious Missier\(^2\), Ravi\(^3\) and A. Pandi\(^3\)

\(^1\)Department of Mathematics, Arul Anandar College, Karumathur, Madurai-Dt, Tamil Nadu, India
\(^2\)Department of Mathematics, V. O. Chidambaram College, Thoothukudi, Tamil Nadu, India
\(^3\)Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai-Dt, Tamil Nadu, India

Abstract: The aim of this paper is to introduce two new class of functions called \((1,2)^{*}-a\hat{g}\)-homeomorphisms and strongly \((1,2)^{*}-a\hat{g}\)-homeomorphisms using \((1,2)^{*}-a\hat{g}\)-closed sets and study their basic properties in bi-topological spaces.

2000 Mathematics Subject Classification: 54E55

Keywords and Phrases: \((1,2)^{*}-a\hat{g}\)-closed set, \((1,2)^{*}-a\hat{g}\)-open set, \((1,2)^{*}-a\hat{g}\)-continuous function, \((1,2)^{*}-ag\)-continuous function, \((1,2)^{*}-a\hat{g}\)-irresolute function, pre-(1,2)^{*}-a-closed function, (1,2)^{*}-gc-irresolute function.

1. INTRODUCTION

Njastad [16] introduced \(\alpha\)-open sets. Maki et al. [14] generalized the concepts of closed sets to \(\alpha\)-generalized closed (briefly \(ag\)-closed) sets which are strictly weaker than \(\alpha\)-closed sets. Veera Kumar [30] defined \(g\)-closed sets in topological spaces. El Monsef et al. [1] introduced \(a\hat{g}\)-closed sets which lie between \(\alpha\)-closed sets and \(ag\)-closed sets in topological spaces.

Maki et al [15] introduced the notion of generalized homeomorphisms (briefly \(g\)-homeomorphism) which are generalizations of homeomorphisms in topological spaces. Subsequently, Devi et al [6] introduced two class of functions called generalized semi-homeomorphisms (briefly \(gs\)-homeomorphism) and semi-generalized homeomorphisms (briefly \(sg\)-homeomorphism). Quite recently, Zbigniew Duszyński [32] has introduced \(a\hat{g}\)-homeomorphisms in topological spaces.

It is well-known that the above mentioned topological sets and functions have been generalized to bitopological settings due to the efforts of many modern topologists [see 7, 9, 10, 17-26]. In this present paper, we introduce two new class of bitopological functions called \((1,2)^{*}-\alpha\hat{g}\)-homeomorphisms and strongly \((1,2)^{*}-\alpha\hat{g}\)-homeomorphisms by using \((1,2)^{*}-a\hat{g}\)-closed sets. Basic properties of these two functions are studied and the relation between these types and other existing ones are established.

2. PRELIMINARIES

Throughout this paper, \((X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2)\) and \((Z, \eta_1, \eta_2)\) (briefly \(X, Y\) and \(Z\)) will denote bitopological spaces.

2.1. Definition

Let \(S\) be a subset of a bitopological space \(X\). Then \(S\) is said to be \(\tau_{1,2}\)-open [9] if \(S = A \cup B\), where \(A \in \tau_1\) and \(B \in \tau_2\).

The complement of \(\tau_{1,2}\)-open set is called \(\tau_{1,2}\)-
closed.

Notice that \( \tau_{1,2} \)-open sets need not necessarily form a topology.

2.2. Definition [9]

Let \( S \) be a subset of a bitopological space \( X \). Then

1. the \( \tau_{1,2} \)-closure of \( S \), denoted by \( \tau_{1,2}\text{-cl}(S) \), is defined as \( \bigcap \{ F : S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed} \} \).

2. the \( \tau_{1,2} \)-interior of \( S \), denoted by \( \tau_{1,2}\text{-int}(S) \), is defined as \( \bigcup \{ F : F \subseteq S \text{ and } F \text{ is } \tau_{1,2}\text{-open} \} \).

2.3. Definition

A subset \( A \) of a bitopological space \( X \) is called

1. \((1,2)^*\)-semi-open set [10] if \( A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \).

2. \((1,2)^*\)-\(\alpha\)-open set [10] if \( A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) \).

3. regular \((1,2)^*\)-open set [17] if \( A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \).

The complements of the above mentioned open sets are called their respective closed sets.

The \((1,2)^*\)-semi-closure (resp. \((1,2)^*\)-\(\alpha\)-closure) of a subset \( A \) of a bitopological space \( X \), denoted by \((1,2)^*\)-scl(A) (resp. \((1,2)^*\)-acl(A)), is the intersection of all \((1,2)^*\)-semi-closed (resp. \((1,2)^*\)-\(\alpha\)-closed) sets of \( X \) containing \( A \).

2.4. Definition

A subset \( A \) of a bitopological space \( X \) is called

1. \((1,2)^*\)-generalized closed (briefly, \((1,2)^*\)-g-closed) [19] if \( \tau_{1,2}\text{-cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \tau_{1,2}\)-open in \( X \).

2. \((1,2)^*\)-semi-generalized closed (briefly, \((1,2)^*\)-sg-closed) [21] if \( \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((1,2)^*\)-semi-open in \( X \).

3. \((1,2)^*\)-generalized semi-closed (briefly, \((1,2)^*\)-gs-closed) [22] if \( \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((1,2)^*\)-gs-closed in \( X \).

4. \((1,2)^*\)-\(\bar{g}\)-closed [7] if \( \tau_{1,2}\text{-cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((1,2)^*\)-semi-open in \( X \).

5. \((1,2)^*\)-\(ag\)-closed [18] if \( \tau_{1,2}\text{-cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \tau_{1,2}\)-open in \( X \).

The complements of the above mentioned closed sets are called their respective open sets.

6. \((1,2)^*\)-\(ag\)-closed [7] if \( \tau_{1,2}\text{-cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \((1,2)^*\)-\(\bar{g}\)-open in \( X \).

2.5. Definition

A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called \((1,2)^*\)-g-open [22] (resp. \((1,2)^*\)-\(\bar{g}\)-open [26], \((1,2)^*\)-open [20], \((1,2)^*\)-sg-open [22], \((1,2)^*\)-gs-open [22], \((1,2)^*\)-\(\alpha\)-open [26], \((1,2)^*\)-ag-open [23], \((1,2)^*\)-\(ag\)-open [26]) if the image of every \( \tau_{1,2} \)-open set in \( X \) is \((1,2)^*\)-g-open (resp. \((1,2)^*\)-\(\bar{g}\)-open, \( \sigma_1, \sigma_2 \)-open, \((1,2)^*\)-sg-open, \((1,2)^*\)-gs-open, \((1,2)^*\)-\(\alpha\)-open, \((1,2)^*\)-ag-open, \((1,2)^*\)-\(ag\)-open) in \( Y \).

2.6. Definition

A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called

1. \((1,2)^*\)-g-continuous [21] if \( f^{-1}(V) \) is \((1,2)^*\)-g-closed in \( X \), for every \( \sigma_1, \sigma_2 \)-closed set \( V \) of \( Y \).

2. \((1,2)^*\)-sg-continuous [21] if \( f^{-1}(V) \) is \((1,2)^*\)-sg-closed in \( X \), for every \( \sigma_1, \sigma_2 \)-closed set \( V \) of \( Y \).

3. \((1,2)^*\)-gs-continuous [21] if \( f^{-1}(V) \) is \((1,2)^*\)-gs-closed in \( X \), for every \( \sigma_1, \sigma_2 \)-closed set \( V \) of \( Y \).

4. \((1,2)^*\)-\(\bar{g}\)-continuous [23] if \( f^{-1}(V) \) is \((1,2)^*\)-\(\bar{g}\)-closed in \( X \), for every \( \sigma_1, \sigma_2 \)-closed set \( V \) of \( Y \).

5. \((1,2)^*\)-continuous [17] if \( f^{-1}(V) \) is \( \tau_{1,2}\)-closed in \( X \), for every \( \sigma_1, \sigma_2 \)-closed set \( V \) of \( Y \).

2.7. Definition [22]

A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called

1. \((1,2)^*\)-g-homeomorphism if \( f \) is bijection, \((1,2)^*\)-g-open and \((1,2)^*\)-g-continuous.

2. \((1,2)^*\)-sg-homeomorphism if \( f \) is bijection, \((1,2)^*\)-sg-open and \((1,2)^*\)-sg-continuous.

3. \((1,2)^*\)-gs-homeomorphism if \( f \) is bijection, \((1,2)^*\)-gs-open and \((1,2)^*\)-gs-continuous.

4. \((1,2)^*\)-homeomorphism if \( f \) is bijection, \((1,2)^*\)-open and \((1,2)^*\)-continuous.

2.8. Definition [26]

A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called
(1) (1,2)*-α-continuous if \( f^{-1}(V) \) is (1,2)*-α-open in \( X \), for every \( \sigma_{1,2} \)-open set \( V \) of \( Y \).

(2) (1,2)*-αĝ-continuous if \( f^{-1}(V) \) is (1,2)*-αĝ-closed in \( X \), for every \( \sigma_{1,2} \)-closed set \( V \) of \( Y \).

(3) (1,2)*-αĝ-irresolute if \( f^{-1}(V) \) is (1,2)*-αĝ-closed in \( X \), for every (1,2)*-αĝ-closed set \( V \) of \( Y \).

2.9. Definition [25]
A function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called

(1) pre-(1,2)*-α-closed (resp. pre (1,2)*-α-open) if the image of every (1,2)*-α-closed (resp. (1,2)*-α-open) in \( X \) is (1,2)*-α-closed (resp. (1,2)*-α-open) in \( Y \).

(2) (1,2)*-α-irresolute if \( f^{-1}(V) \) is (1,2)*-α-open in \( X \), for every (1,2)*-α-open set \( V \) of \( Y \).

(3) (1,2)*-αĝ-irresolute if \( f^{-1}(V) \) is (1,2)*-α-open in \( X \), for every (1,2)*-αĝ-open set \( V \) of \( Y \).

(4) (1,2)*-α-homeomorphism if \( f \) is bijection, (1,2)*-α-irresolute and pre-(1,2)*-α-closed.

2.10. Remark [7]

(1) Every (1,2)*-α-closed set is (1,2)*-αĝ-closed but not conversely.

(2) Every (1,2)*-αĝ-open set is (1,2)*-gs-open but not conversely.

3. (1,2)*-ΑΓ-HOMEO MORPHISMS

3.1. Definition

(1) A bijective function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called a strongly (1,2)*-αĝ-closed (resp. strongly (1,2)*-αĝ-open ) if the image of every (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) set in \( X \) is (1,2)*-αĝ-closed (resp. (1,2)*-αĝ-open) of \( Y \).

(2) A bijective function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) is called an (1,2)*-αĝ-homeomorphism if \( f \) is both (1,2)*-αĝ-open and (1,2)*-αĝ-continuous.

3.2. Theorem
Every (1,2)*-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof
Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be (1,2)*-homeomorphism. Then \( f \) is bijective, (1,2)*-open and (1,2)*-continuous function. Let \( U \) be an \( \tau_{1,2} \)-open set in \( X \). Since \( f \) is (1,2)*-open function, \( f(U) \) is an \( \sigma_{1,2} \)-open set in \( Y \). Every \( \tau_{1,2} \)-open set is (1,2)*-αĝ-open and hence \( f(U) \) is (1,2)*-αĝ-open in \( Y \). This implies \( f \) is (1,2)*-αĝ-open. Let \( V \) be a \( \sigma_{1,2} \)-closed set in \( Y \). Since \( f \) is (1,2)*-continuous, \( f^{-1}(V) \) is (1,2)*-αĝ-closed in \( X \) and therefore, \( f \) is (1,2)*-αĝ-continuous. Hence, \( f \) is an (1,2)*-αĝ-homeomorphism.

3.3. Remark
The converse of Theorem 3.2 need not be true as shown in the following example.

3.4. Example
Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, X\} \) and \( \tau_2 = \{\phi, X, \{a, b\}\} \).
Then the sets in \( \{\phi, X, \{a, b\}\} \) are called \( \tau_{1,2} \)-open and the sets in \( \{\phi, X, \{c\}\} \) are called \( \tau_{1,2} \)-closed. Also the sets in \( \{\phi, X, \{c\}, \{a, c\}, \{b, c\}\} \) are called (1,2)*-αĝ-closed in \( X \) and the sets in \( \{\phi, X, \{a\}, \{b\}\} \) are called (1,2)*-αĝ-open in \( X \). Let \( Y = \{a, b, c\} \), \( \sigma_1 = \{\phi, Y, \{a\}\} \) and \( \sigma_2 = \{\phi, Y, \{b\}\} \). Then the sets in \( \{\phi, Y, \{a\}, \{b\}, \{a, b\}\} \) are called \( \sigma_{1,2} \)-open and the sets in \( \{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\} \) are called \( \sigma_{1,2} \)-closed. Also the sets in \( \{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\} \) are called (1,2)*-αĝ-closed in \( Y \) and the sets in \( \{\phi, Y, \{a\}, \{b\}, \{a, b\}\} \) are called (1,2)*-αĝ-open in \( Y \). Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be the identity function. Then \( f \) is a (1,2)*-αĝ-homeomorphism but \( f \) is not a (1,2)*-homeomorphism.

3.5. Proposition
For any bijective function \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) the following statements are equivalent.

(1) \( f^{-1} : (Y, \sigma_1, \sigma_2) \rightarrow (X, \tau_1, \tau_2) \) is (1,2)*-αĝ-continuous function.

(2) \( f \) is a (1,2)*-αĝ-open function.

(3) \( f \) is a (1,2)*-αĝ-closed function.

Proof
(1) \( \Rightarrow (2) \): Let \( U \) be an \( \tau_{1,2} \)-open set in \( X \). Then \( X - U \) is \( \tau_{1,2} \)-closed in \( X \). Since \( f^{-1} \) is (1,2)*-αĝ-continuous, \( (f^{-1})^{-1}(X - U) \) is (1,2)*-αĝ-closed in \( Y \). That is \( f(X - U) = Y - f(U) \) is (1,2)*-
αĝ-closed in Y. This implies f(U) is (1,2)*-αĝ-open in Y. Hence f is (1,2)*-αĝ-open function.

(2) \( \Rightarrow \) (3): Let \( F \) be a \( \tau_{1,2} \)-closed set in \( X \). Then \( X - F \) is \( \tau_{1,2} \)-open in \( X \). Since f is (1,2)*-αĝ-open, \( f(X - F) \) is (1,2)*-αĝ-open set in Y. That is Y - \( f(F) \) is (1,2)*-αĝ-closed in Y. This implies that \( f(F) \) is (1,2)*-αĝ-closed in Y. Hence f is (1,2)*-αĝ-closed function.

(3) \( \Rightarrow \) (1): Let \( V \) be a \( \tau_{1,2} \)-closed set in \( X \). Since f is (1,2)*-αĝ-closed function, \( f(V) \) is (1,2)*-αĝ-closed in Y. That is (\( f^{-1} \))^{-1}(V) is (1,2)*-αĝ-closed in Y. Hence \( f^{-1} \) is (1,2)*-αĝ-continuous.

3.6. Proposition

Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a bijective and (1,2)*-αĝ-continuous function. Then the following statements are equivalent:

(1) \( f \) is a (1,2)*-αĝ-open function.
(2) \( f \) is a (1,2)*-αĝ-homeomorphism.
(3) \( f \) is a (1,2)*-αĝ-closed function.

Proof

(1) \( \Rightarrow \) (2): Let \( f \) be a (1,2)*-αĝ-open function. By hypothesis, \( f \) is bijective and (1,2)*-αĝ-homeomorphism. Hence \( f \) is a (1,2)*-αĝ-homeomorphism.

(2) \( \Rightarrow \) (3): Let \( f \) be a (1,2)*-αĝ-homeomorphism. Then \( f \) is (1,2)*-αĝ-open. By Proposition 3.5, \( f \) is (1,2)*-αĝ-closed function.

(3) \( \Rightarrow \) (1): It is obtained from Proposition 3.5.

3.7. Theorem

Every (1,2)*-α-homeomorphism is (1,2)*-αĝ-homeomorphism.

Proof

Let \( f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a (1,2)*-α-homeomorphism. Then \( f \) is bijective, (1,2)*-α-irresolute and pre-(1,2)*-α-closed. Let \( F \) be \( \tau_{1,2} \)-closed in \( X \). Then \( F \) is (1,2)*-α-closed in \( X \). Since \( f \) is pre-(1,2)*-α-closed, \( f(F) \) is (1,2)*-α-closed in Y. Every (1,2)*-α-closed set is (1,2)*-αĝ-closed and hence \( f(F) \) is (1,2)*-αĝ-closed in Y. This implies \( f \) is (1,2)*-αĝ-closed function. Let \( V \) be a \( \sigma_{1,2} \)-closed set of \( Y \). Thus \( V \) is (1,2)*-α-closed in Y. Since \( f \) is (1,2)*-α-irresolute \( f(V) \) is (1,2)*-α-closed in X. Thus \( f^{-1}(V) \) is (1,2)*-αĝ-closed in X. Therefore \( f \) is (1,2)*-αĝ-continuous. Hence \( f \) is a (1,2)*-αĝ-homeomorphism.

3.8. Remark

The following Example shows that the converse of Theorem 3.7 need not be true.

3.9. Example

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\emptyset, X\} \) and \( \tau_2 = \{\emptyset, X, \{a\}\} \). Then the sets in \( \{\phi, X, \{a\}\} \) are called \( \tau_{1,2} \)-open and the sets in \( \{\phi, X, \{b, c\}\} \) are called \( \tau_{1,2} \)-closed. Also the sets in \( \{\phi, X, \{b\}, \{a, c\}, \{b, c\}\} \) are called (1,2)*-αĝ-closed in X and the sets in \( \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\} \) are called (1,2)*-αĝ-open in X. Moreover, the sets in \( \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\} \) are called (1,2)*-α-closed in X and the sets in \( \{\phi, X, \{b\}, \{c\}, \{b, c\}\} \) are called (1,2)*-α-open in X. Let \( Y = \{a, b, c\} \), \( \sigma_1 = \{\emptyset, Y\} \) and \( \sigma_2 = \{\emptyset, Y, \{a, b\}\} \). Then the sets in \( \{\phi, Y, \{a, b\}\} \) are called \( \sigma_{1,2} \)-open and the sets in \( \{\phi, Y, \{c\}\} \) are called \( \sigma_{1,2} \)-closed. Also the sets in \( \{\phi, Y, \{a, c\}, \{a, b\}\} \) are called (1,2)*-αĝ-closed in Y and the sets in \( \{\phi, Y, \{a, b\}\} \) are called (1,2)*-αĝ-open in Y. Moreover, the sets in \( \{\phi, Y, \{a\}, \{a, b\}\} \) are called (1,2)*-α-closed in Y and the sets in \( \{\phi, Y, \{c\}\} \) are called (1,2)*-α-open in Y.

3.10. Remark

Next Example shows that the composition of two (1,2)*-αĝ-homeomorphisms is not always a (1,2)*-αĝ-homeomorphism.

3.11. Example

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\emptyset, X\} \) and \( \tau_2 = \{\emptyset, X, \{a\}\} \). Then the sets in \( \{\phi, X, \{a\}\} \) are called \( \tau_{1,2} \)-open and the sets in \( \{\phi, X, \{b, c\}\} \) are called \( \tau_{1,2} \)-closed. Also the sets in \( \{\phi, X, \{b\}, \{a, c\}, \{b, c\}\} \) are called (1,2)*-αĝ-closed in X and the sets in \( \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\} \) are called (1,2)*-αĝ-open in X. Moreover, the sets in \( \{\phi, X, \{a\}, \{a, b\}\} \) are called (1,2)*-α-closed in X and the sets in \( \{\phi, Y, \{a\}\} \) are called (1,2)*-α-open in Y.
\{a, b\}, \{a, c\}, \{b, c\}\} are called \((1,2)^*\alpha\partial\)-closed in \(Y\) and the sets in \(\{\phi, Y, \{a\}, \{b\}, \{c\}\}\) are called \((1,2)^*\alpha\partial\)-open in \(Y\). Let \(Z = \{a, b, c\}\), \(\eta_1 = \{\phi, Z\}\) and \(\eta_2 = \{\phi, Z, \{a, b\}\}\). Then the sets in \(\{\phi, Z, \{a\}\}\) are called \(\eta_1\)-open and the sets in \(\{\phi, Z, \{c\}\}\) are called \(\eta_2\)-closed. Also the sets in \(\{\phi, Z, \{c\}\}, \{a, c\}, \{b, c\}\\} are called \((1,2)^*\alpha\partial\)-closed in \(Z\) and the sets in \(\{\phi, Z, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-open in \(Z\). Let \(f: (X, \tau_1) \rightarrow (Y, \tau_1, \tau_2)\) be a \((1,2)^*\alpha\partial\)-homeomorphism. Then \(f^{-1}(V)\) is \((1,2)^*\alpha\partial\)-open in \(Y\). Every \((1,2)^*\alpha\partial\)-\(\alpha\partial\)-homeomorphism is \((1,2)^*\alpha\partial\)-gs-homeomorphism.

3.12. Theorem

Every \((1,2)^*\alpha\partial\)-homeomorphism is \((1,2)^*\alpha\partial\)-gs-homeomorphism but not conversely.

Proof
Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be a \((1,2)^*\alpha\partial\)-homeomorphism. Then \(f\) is a bijective, \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-continuous function. Let \(U\) be an \(\tau_{1,2}\)-open set in \(X\). Then \(f(U)\) is \((1,2)^*\alpha\partial\)-open in \(Y\). Every \((1,2)^*\alpha\partial\)-open set is \((1,2)^*\alpha\partial\)-gs-open and hence, \(f(U)\) is \((1,2)^*\alpha\partial\)-gs-open in \(Y\). This implies \(f\) is \((1,2)^*\alpha\partial\)-gs-open function. Let \(V\) be \(\sigma_{1,2}\)-closed set in \(Y\). Then \(f(V)\) is \((1,2)^*\alpha\partial\)-open in \(X\). Hence \(f(V)\) is \((1,2)^*\alpha\partial\)-gs-closed in \(X\). This implies \(f\) is \((1,2)^*\alpha\partial\)-gs-continuous. Hence \(f\) is \((1,2)^*\alpha\partial\)-gs-homeomorphism.

3.13. Remark

The following Example shows that the converse of Theorem 3.12 need not be true.

3.14. Example
Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, X, \{a\}\}\) and \(\tau_2 = \{\phi, X, \{b\}\}\). Then the sets in \(\{\phi, X, \{a\}, \{b\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-open and the sets in \(\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{c\}, \{a, c\}, \{b, c\}\}\) are called \((1,2)^*\alpha\partial\)-closed in \(X\) and the sets in \(\{\phi, X, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-open in \(X\). Moreover, the sets in \(\{\phi, X, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed in \(X\) and the sets in \(\{\phi, X, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-gs-open in \(X\). Let \(Y = \{a, b, c\}\), \(\sigma_1 = \{\phi, Y, \{a\}\}\) and \(\sigma_2 = \{\phi, Y, \{b\}\}\). Then the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \(\sigma_{1,2}\)-open and \(\sigma_{1,2}\)-closed. Also the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-gs-open in \(Y\). Moreover, the sets in \(\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed and \((1,2)^*\alpha\partial\)-gs-open in \(Y\). Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be the identity function. Then \(f\) is a \((1,2)^*\alpha\partial\)-gs-homeomorphism but \(f\) is not a \((1,2)^*\alpha\partial\)-gs-homeomorphism.

3.15. Remark

The following Examples show that the concepts of \((1,2)^*\alpha\partial\)-homeomorphisms and \((1,2)^*\alpha\partial\)-gs-homeomorphisms are independent of each other.

3.16. Example
Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, X, \{a\}, \{b\}\}\) and \(\tau_2 = \{\phi, X, \{a\}\}\). Then the sets in \(\{\phi, X, \{a\}, \{b\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-open and the sets in \(\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) are called \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-gs-open in \(X\). Moreover, the sets in \(\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed and \((1,2)^*\alpha\partial\)-gs-open in \(X\). Let \(Y = \{a, b, c\}\), \(\sigma_1 = \{\phi, Y, \{a\}\}\) and \(\sigma_2 = \{\phi, Y, \{b\}\}\). Then the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \(\sigma_{1,2}\)-open and \(\sigma_{1,2}\)-closed. Also the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-gs-open in \(Y\). Moreover, the sets in \(\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed and \((1,2)^*\alpha\partial\)-gs-open in \(Y\). Let \(f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be the identity function. Then \(f\) is a \((1,2)^*\alpha\partial\)-gs-homeomorphism but \(f\) is not a \((1,2)^*\alpha\partial\)-gs-homeomorphism.

3.17. Example
Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, X, \{a\}\}\) and \(\tau_2 = \{\phi, X, \{a\}\}\). Then the sets in \(\{\phi, X, \{a\}\}\) are called \(\tau_{1,2}\)-open and the sets in \(\{\phi, X, \{b\}, \{c\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) are called \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-gs-open in \(X\). Moreover, the sets in \(\{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed and \((1,2)^*\alpha\partial\)-gs-open in \(X\). Let \(Y = \{a, b, c\}\), \(\sigma_1 = \{\phi, Y, \{a\}\}\) and \(\sigma_2 = \{\phi, Y, \{b\}\}\). Then the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \(\sigma_{1,2}\)-open and \(\sigma_{1,2}\)-closed. Also the sets in \(\{\phi, Y, \{a\}, \{b\}\}\) are called \((1,2)^*\alpha\partial\)-open and \((1,2)^*\alpha\partial\)-gs-open in \(Y\). Moreover, the sets in \(\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}\) are called \((1,2)^*\alpha\partial\)-gs-closed and \(f(a) = b, f(b) = a\) and \(f(c) = c\). Then \(f\) is a \((1,2)^*\alpha\partial\)-gs-homeomorphism but \(f\) is not a \((1,2)^*\alpha\partial\)-gs-homeomorphism.
and (1,2)*-α̃g-open in Y. Moreover, the sets in \{φ, Y, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\} are called (1,2)*-gs-closed and (1,2)*-gs-open in Y. Define a function \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) by \(f(a) = b, f(b) = c, f(c) = a\). Then \(f\) is a (1,2)*-g-homeomorphism but \(f\) is not a (1,2)*-α̃g-homeomorphism.

3.18. Remark

(1,2)*-α̃g-homeomorphisms and (1,2)*-sg-homeomorphisms are independent of each other as shown below.

3.19. Example

The function \(f\) defined in Example 3.16 is (1,2)*-α̃g-homeomorphism but not (1,2)*-sg-homeomorphism.

3.20. Example

Let \(X = \{a, b, c\}, τ_1 = \{φ, X, \{a\}\}\) and \(τ_2 = \{φ, X, \{a, c\}\}\). Then the sets in \{φ, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\} are called \(τ_{1,2}\)-open and (1,2)*-α̃g-open in \(X\). The sets in \{φ, X, \{c\}, \{a, c\}, \{b, c\}\} are called \(τ_{1,2}\)-closed and (1,2)*-α̃g-closed in \(X\). Let \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) be the identity function. Then \(f\) is (1,2)*-α̃g-homeomorphism but not (1,2)*-sg-homeomorphism.

4. STRONGLY (1,2)*-Α̃G-HOMEOMORPHISMS

4.1. Definition

A bijection \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) is said to be strongly (1,2)*-α̃g-homeomorphism if \(f\) is (1,2)*-α̃g-irresolute and its inverse \(f^{-1}\) is also (1,2)*-α̃g-irresolute.

4.2. Theorem

Every strongly (1,2)*-α̃g-homeomorphism is (1,2)*-α̃g-homeomorphism.

Proof

Let \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) be strongly (1,2)*-α̃g-homeomorphism. Let \(U\) be \(τ_{1,2}\)-open in \(X\). Then \(U\) is (1,2)*-α̃g-open in \(X\). Since \(f^{-1}\) is (1,2)*-α̃g-irresolute, \((f^{-1})^{-1}(U)\) is (1,2)*-α̃g-open in \(Y\). That is \(f(U)\) is (1,2)*-α̃g-open in \(Y\). This implies \(f\) is (1,2)*-α̃g-homeomorphism.

4.3. Remark

The following Example shows that the converse of Theorem 4.2 need not be true.

4.4. Example

Let \(X = \{a, b, c\}, τ_1 = \{φ, X, \{a\}\}\) and \(τ_2 = \{φ, X, \{a, c\}\}\). Then the sets in \{φ, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\} are called \(τ_{1,2}\)-open and (1,2)*-α̃g-open in \(X\). The sets in \{φ, X, \{c\}, \{a, c\}, \{b, c\}\} are called \(τ_{1,2}\)-closed and (1,2)*-α̃g-closed in \(X\). Let \(Y = \{a, b, c\}, σ_1 = \{φ, Y, \{a\}\}\) and \(σ_2 = \{φ, Y, \{a\}\}\). Then the sets in \{φ, Y, \{a\}, \{b\}, \{c\}\} are called \(σ_{1,2}\)-open and \(σ_{1,2}\)-closed. Also the sets in \{φ, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\} are called (1,2)*-α̃g-closed in \(X\) and the sets in \{φ, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\} are called (1,2)*-α̃g-open in \(X\). Let \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) be the identity function. Then \(f\) is (1,2)*-α̃g-homeomorphism but \(f\) is not strongly (1,2)*-α̃g-homeomorphism.

4.5. Theorem

The composition of two strongly (1,2)*-α̃g-homeomorphisms is a strongly (1,2)*-α̃g-homeomorphism.

Proof

Let \(f : (X, τ_1, τ_2) \rightarrow (Y, σ_1, σ_2)\) and \(g : (Y, σ_1, σ_2) \rightarrow (Z, η_1, η_2)\) be two strongly (1,2)*-α̃g-homeomorphisms. Let \(F\) be a (1,2)*-α̃g-closed set in \(Z\). Since \(g\) is (1,2)*-α̃g-irresolute, \(g^{-1}(F)\) is (1,2)*-α̃g-closed in \(Y\). Since \(f\) is a (1,2)*-α̃g-irresolute, \((f^{-1})^{-1}(F)\) is (1,2)*-α̃g-closed in \(X\). That is \((g \circ f)^{-1}(F)\) is (1,2)*-α̃g-closed in \(X\). This implies \((g \circ f)\) is a (1,2)*-α̃g-homeomorphism.
\( \eta_i, \eta_j \) is (1,2)-\( \alpha \)-irresolute. Let \( V \) be a (1,2)-\( \alpha \)-\( \alpha \)-closed in \( X \). Since \( f^{-1} \) is a (1,2)-\( \alpha \)-\( \alpha \)-irresolute, \((f^{-1})\) \( i(V) \) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Y \). That is \( f(V) \) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Y \). Since \( g^{-1} \) is a (1,2)-\( \alpha \)-\( \alpha \)-irresolute, \((g^{-1})\) \( i(f(V)) \) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Z \). That is \( g(f(V)) \) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Z \). So, \((g \circ f)(V)\) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Z \). This implies that \((g \circ f)^{-1}\) \( i(V) \) is (1,2)-\( \alpha \)-\( \alpha \)-closed in \( Z \). This shows that \((g \circ f)^{-1} \) is (1,2)-\( \alpha \)-\( \alpha \)-irresolute. Hence \( g \circ f \) is a strongly (1,2)-\( \alpha \)-\( \alpha \)-homeomorphism.

We denote the family of all strongly (1,2)-\( \alpha \)-\( \alpha \)-homeomorphisms from a bitopological space \((X, \tau_1, \tau_2)\) onto itself by (1,2)-\( \alpha \)-\( \alpha \)-homeomorphisms.

4.6. Theorem

The set (1,2)-\( \alpha \)-\( \alpha \)-homeomorphisms is a group under composition of functions.

**Proof**

By Theorem 4.5, \( g \circ f \in (1,2)-\alpha \)-homeomorphisms for all \( f \), \( g \in (1,2)-\alpha \)-homeomorphisms. We know that the composition of functions is associative. The identity function belonging to (1,2)-\( \alpha \)-homeomorphisms serves as the identity element. If \( f \in (1,2)-\alpha \)-homeomorphisms, then \( f^{-1} \in (1,2)-\alpha \)-homeomorphisms such that \( f \circ f^{-1} = f^{-1} \circ f = I \) and so inverse exists for each element of (1,2)-\( \alpha \)-homeomorphisms. Hence (1,2)-\( \alpha \)-homeomorphisms is a group under the composition of functions.

4.7. Theorem

Let \( f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2) \) be a strongly (1,2)-\( \alpha \)-\( \alpha \)-homeomorphism. Then \( f \) induces an (1,2)-\( \alpha \)-isomorphism from the group (1,2)-\( \alpha \)-homeomorphisms onto the group (1,2)-\( \alpha \)-homeomorphisms.

**Proof**

Using the function \( f \), we define a function \( \theta_f : (1,2)-\alpha \)-homeomorphisms \( (X, \tau_1, \tau_2) \rightarrow (1,2)-\alpha \)-homeomorphisms \( (Y, \sigma_1, \sigma_2) \) by \( \theta_f(k) = f \circ k \circ f^{-1} \) for every \( k \in (1,2)-\alpha \)-homeomorphisms \( (X, \tau_1, \tau_2) \). Then \( \theta_f \) is a bijection. Further, for all \( k, k_1, k_2 \in (1,2)-\alpha \)-homeomorphisms \( (X, \tau_1, \tau_2) \), \( \theta_f(k_1 \circ k_2) = f \circ (k_1 \circ k_2) \circ f^{-1} = (f \circ k_1 \circ f^{-1}) \circ (f \circ k_2 \circ f^{-1}) = \theta_f(k_1) \circ \theta_f(k_2) \). Therefore \( \theta_f \) is an (1,2)-\( \alpha \)-isomorphism induced by \( f \).

4.8. Remark

The concept of strongly (1,2)-\( \alpha \)-\( \alpha \)-homeomorphisms and (1,2)-\( \alpha \)-homeomorphisms are independent notions as shown in the following examples.

4.9. Example

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, X, \{a\}\} \) and \( \tau_2 = \{\phi, X, \{a, b\}\} \). Then the sets in \( \{\phi, X, \{a\}\} \) and \( \{\phi, X, \{a, b\}\} \) are called (1,2)-\( \alpha \)-open and (1,2)-\( \alpha \)-closed. Also the sets in \( \{\phi, X, \{a, c\}\} \) and \( \{\phi, X, \{b, c\}\} \) are called (1,2)-\( \alpha \)-open and (1,2)-\( \alpha \)-closed.

Moreover, the sets in \( \{\phi, X, \sigma_1, \sigma_2\} \) are called (1,2)-\( \alpha \)-open and (1,2)-\( \alpha \)-closed.

4.10. Example

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, X, \{a\}\} \) and \( \tau_2 = \{\phi, X, \{a, b\}\} \). Then the sets in \( \{\phi, X, \{a\}\} \) and \( \{\phi, X, \{a, b\}\} \) are called (1,2)-\( \alpha \)-open and (1,2)-\( \alpha \)-closed. Also the sets in \( \{\phi, X, \{a, c\}\} \) and \( \{\phi, X, \{b, c\}\} \) are called (1,2)-\( \alpha \)-open and (1,2)-\( \alpha \)-closed.
4.12. Remark

The concept of strongly \((1, 2)^*\)-\(\alpha g\)-homeomorphisms and \((1, 2)^*\)-g-homeomorphisms are independent of each other as the following examples show.

4.13. Example

Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, X, \{a\}\}\) and \(\tau_2 = \{\phi, X, \{a, b\}\}\). Then the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-open and the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{b\}, \{a, c\}, \{b, c\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-closed in \(X\) and the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{b\}, \{a, c\}, \{b, c\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-closed in \(X\) and the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{b\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-closed and \((1, 2)^*\)-\(\alpha g\)-closed in \(Y\) and the sets in \(\{\phi, Y, \{a\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-open and \((1, 2)^*\)-\(\alpha g\)-open in \(Y\). Let \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) be the identity function. Then \(f\) is a strongly \((1, 2)^*\)-\(\alpha g\)-homeomorphism but not \((1, 2)^*\)-g-homeomorphism.

4.14. Example

Let \(X = \{a, b, c\}\), \(\tau_1 = \{\phi, X, \{a\}\}\) and \(\tau_2 = \{\phi, X, \{b\}\}\). Then the sets in \(\{\phi, X, \{a\}, \{b\}\}\) are called \(\tau_{1,2}\)-open and the sets in \(\{\phi, X, \{a\}, \{b\}\}\) are called \(\tau_{1,2}\)-closed. Also the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-closed and \((1, 2)^*\)-\(\alpha g\)-closed in \(X\) and the sets in \(\{\phi, X, \{a\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-open and \((1, 2)^*\)-\(\alpha g\)-open in \(X\). Let \(Y = \{a, b, c\}\), \(\sigma_1 = \{\phi, Y, \{a\}, \{a, b\}\}\) and \(\sigma_2 = \{\phi, Y, \{a\}, \{a, b\}\}\). Then the sets in \(\{\phi, Y, \{a\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-closed and \((1, 2)^*\)-\(\alpha g\)-closed in \(Y\) and the sets in \(\{\phi, Y, \{a\}, \{a, b\}\}\) are called \((1, 2)^*\)-\(\alpha g\)-open and \((1, 2)^*\)-\(\alpha g\)-open in \(Y\). Let \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) be the identity function. Then \(f\) is a strongly \((1, 2)^*\)-\(\alpha g\)-homeomorphism but not \((1, 2)^*\)-g-homeomorphism.

5. REFERENCES


Seasonal Variability in Virtual Height of Ionospheric $F_2$ Layer at the Pakistan Atmospheric Region

Akbar Ali Jilani, M. Ayub Khan Yousuf Zai*, Faisal Ahmed Khan Afridi, Khusro Mian and Syed Nazeer Alam

1Institute of Space and Planetary Astrophysics, University of Karachi, Karachi, Pakistan
2Department of Applied Physics, Solar-Terrestrial & Atmospheric Research Wing and Institute of Space & Planetary Astrophysics, University of Karachi, Karachi, Pakistan
3National University of Sciences & Technology-Pakistan Navy Engineering College, Department of Electronics & Power Engineering, Karachi, Pakistan

Abstract: The aim of this study was to assess the seasonal variability in virtual height of ionospheric $F_2$ layer for Pakistan’s atmospheric region (PAR). In this communication virtual height variations have been analyzed by the descriptive statistical techniques. These methodologies comprise an autoregressive strategy, linear regression and polynomial regression. The relevance of these models has been illuminated using predicted values of different parameters under the seasonal variation of ionospheric $F_2$ layer in virtual height that affect the radio wave propagation through the ionosphere. These techniques are implemented to theorize the physical process of varying the virtual heights that leads this study towards formulating the variations due to interaction of radio wave propagation with this ionospheric layer.

Keywords: autoregressive process, variability in virtual height, linear regression, polynomial regression

1. INTRODUCTION

The sun is a main source of ionization in the extraterrestrial atmosphere, so that the variability depends upon the solar activity and geomagnetic conditions. The ionosphere is very important in radio wave communications, which absorbs large quantities of radiant energy from the sun. It is earth’s upper atmosphere and the $F_2$ layer is the outer layer of the ionosphere. The virtual height of an ionospheric $F_2$ layer could be explained as the wave is refracted down gradually rather sharply. Incident and refracted rays below the ionized layer follow the same path as if the reflection has taken place from the greater height. The spot is called virtual height of this layer in other words when the incidence and returned rays are extrapolated to a vertex they meet at a height $h'$ is called virtual height. It has been noted that if the virtual height of the layer is known then it is quite simple to calculate the angle of incidence required for the wave to return to ground at a selected point. The $F_2$ layer has a permanent existence, even though the height varies on a daily basis. The frequency range of this layer is 3 to 40 MHz. It is the most important reflecting medium for high frequency radio waves and the height of $F_2$ layer varies from 250-400 km. Since sun’s presence is essential, ionization can take place in any location only during the day time. Even during day time, sun’s radiation will not be from morning till evening because sun’s zenith angle changes. We may expect maximum ionization at noon, and then falling of both sides of the day’s daylight hours. Every event in the sun in respect of thermal content, magnetic field, radiation variation due to sun’s rotation around itself should be reflected in the ionospheric properties. The other

Received, April 2012; Accepted, April 2013
*Corresponding author: M. Ayub Khan Yousuf Zai; Email: ayubzai@yahoo.com
influential factors are latitude, sunspot cycle and magnetic storms [1-3].

The ionosphere seasonal variation is related to a solar zenith angle change, while its solar cycle variation is linked to a change in the solar extreme Ultraviolet (EUV) and x-ray radiation. The important feature is that the maximum electron density (\(N_{\text{m}F_2}\)) of \(F_2\) layer is greater in winter than in summer, despite the fact that the solar zenith angle is smaller in summer. Specifically the summer-to-winter neutral circulation results in an increase in the \(O/N_2\) ratio in the winter hemisphere and a decrease in the summer hemisphere. The day-time wavelengths are much shorter then at the night-time. The point is that the day-time electron density of the \(F_2\) layer is very high and it can reflect the higher frequencies. Conversely, the night-time electron density of \(F_2\) layer goes down. Thus, the time delay is used to determine the altitude of reflection and the frequency is an indicator of the electron density at that location. The height which is calculated by assuming that the waves travel with the velocity of light is called the virtual height [4-7].

The different geomagnetic storms can be significantly different and even for a given storm the system’s response can be very different in the latitudinal and longitudinal regions. However it is educational to show the ionospheric response to the large magnetic storm that was triggered by a solar flare which appeared at 1229UT on October 19, 1989 and it was 22 maximum cycles. In response to this storm, there were long-lasting electron density depletions at high latitudes as measured DG-Sonde Suparco station Karachi and worldwide Ionosondes. During the incidence of ionosphere disturbances the electron density of the \(F_2\) layer takes a dip and the virtual heights go up due to the heating so that the maximum usable frequencies are decreased. The regular structure of \(F_2\) layer is disrupted and strata appear in it. Through very strong ionosphere disturbances, the ionization of the \(F_2\) layer may drop to a point where the layer will not reflect short waves any longer. The \(F_2\) layer is destroyed during ionosphere disturbances at high geomagnetic Latitudes. We have a recorded data of the virtual height of ionosphere \(F_2\) layer over a period of one year (January to December 1989) which is being used in this study [8-13].

2. MATERIALS AND METHODS

The study was based on the analysis of seasonal variability in virtual height and electron concentration (Ne) for Pakistan's atmospheric region. We have used some statistical tools such as Minitab and Statistica for analyzing time series. For this purpose we have utilized ionospheric data recorded on 256 Digisonde for evolution, analysis and forecasting. Spectral analysis, autocorrelation and partial autocorrelation functions have been effectively implemented on the basis of these statistical software packages.

3. RESULTS AND DISCUSSION

The results presented in this study establish that Auto regressive AR (1) order one character of modeling, the seasonal variability in virtual height of ionospheric \(F_2\) layer as a physical process is an appropriate position. The reported data suggest quantitatively that the seasonal variability in virtual height of ionospheric \(F_2\) layer is occurring Fig. 1 is an original time plot of virtual height of \(F_2\) layer data, Fig. 2 illustrates negative correlation between the electron concentration and the seasonal variability in virtual height of \(h'F_2\) layer Fig. 3. Is a scatter plot of virtual height of \(h'F_2\) layer data. From this graph it appears that the seasonal variability in virtual height of \(h'F_2\) layer in the period (t-1) is useful in predicting the value of the total value of virtual height deliberation in period t. It seems that \(X_t\) can be explained as significance of \(X_{t-1}\).To identify the idea we can illustrate the case of autoregressive model which is normally used in time series model.

\[
X_t = \alpha_0 + \phi X_{t-1} + \varepsilon_t
\]

where, \(X_t\) is expressed as a linear combination of its two immediately preceding values, \(\varepsilon_t\) is an error value and \(\alpha_0\) is constant (parameters) value

\[
X_t = \phi X_{t-1} + \alpha_0
\]

Regression analysis describes the assessment of the unknown value of one variable from the known value of the other variable. We have the data which consists of two variables x and y that we want to find the linear function of electron concentration (Ne) of \(F_2\) layer and the virtual height of \(h'F_2\)
Variability in Virtual Height of Ionospheric $F_2$ Layer

Fig. 1. The temporal variation of virtual height of ionospheric $F_2$ layer on 19 October 1989.

![Plot of variable: $h'F_2$ 1989](image1)

(Trend equation $y = 328.815 - 5.1954 + x$)

Fig. 2. Correlation between the $h'F_2$ and Ne.

![Plot of correlation between $h'F_2$ and Ne](image2)

(Trend equation $y = 328.815 - 5.1954 + x$)
Fig. 3. Virtual height of $h'F_2$ layer data plotted past values $X_{t-1}$ and present $X_t$.

Fig. 4. Periodogram to classify randomness in the seasonal variability of $h'F_2$. 
Variability in Virtual Height of Ionospheric F2 Layer

Layer. The linear regression analysis is the key for building forecasting models to produce forecast of seasonal variability of virtual height of ionosphere h’F2 layer.

\[ \bar{X} = 296.615 - 0.2837 X_{t-1} \]

t statistics: (11.421) (12.074)

R² = 3.2 %

R² is the amount of shared variance between the two variables and useful for the fit of the model.

The periodogram illustrated in Fig. 4 is used to classify randomness in the data series. Also it helps in identifying seasonality in the given time series, and in recognizing the predominance of positive or negative autocorrelation (for positive autocorrelation low-frequency amplitudes should dominate, and for negative autocorrelation, high frequencies should dominate. Thus the following inferences are incurred:

(a) Autoregressive processes may be established – pattern of autocorrelation, of partials, and within the line scale, will show a explanation of a potential model.

(b) The graph of the data set is a visual support to recognize the behavior of the pattern. The autocorrelations and the line scale are the review of the pattern presented in the data. They can expose a great agreement about the data and their characteristics.

(c) The model can be used in the present case to state the dependence between X_i and X_{t-1} in the pair (X_i, X_{t-1}), and to thus relate X_i with X_{t-1}, X_{t-2} with X_{t-1} and so on. The plot of X_i and X_{t-1} for t = 2, 3. n = 365 is depicted in Fig. 5. It can be examined that the points are scattered around a straight line. The above model expresses the dependence of the variable on itself at different times for model under consideration \( \alpha \), at different t are independent, that \( \alpha \) is independent of \( \alpha_{t-1} \), so that just like \( \varepsilon \) the distribution of \( \alpha \) is unspecified to be normal. [14, 15]

\[ \alpha \sim ND(0, \sigma_a^2) \]  

Normal (N) distribution (D) with mean zero (0) and variance error term (\( \sigma_a^2 \)). It has been distinguished that predictable model is entirely specified only when \( \sigma^2_a \) is given in calculation to \( \phi_1 \), \( \alpha \) is understood to be normal. The value of X_i may increase or decrease without bound, because \( \alpha \) have fixed finite variance and can not continually increase in magnitude to keep X_i within bound as depicted from Fig. 6 that explains the residual analysis specified for this model and also confirms that this model is sufficient.

Fig. 5. Residuals analysis of seasonal variability.
Fig. 6. Plot of Autocorrelation function for seasonal variability.

Fig. 7. Partial Autocorrelation function for the residuals of seasonal variability.
While a condition tells that if \( \phi_1 > 1 \) or \( \phi_1 < -1 \), then the series will be non-stationary or unsteady time series. For a stationary stable time series, \( X_t \) remains surrounded in the motive, it has finite variance, we would need \( \phi_1 < 1 \). Fig. 6 and Fig. 7 illustrate the estimated auto-correlation function and the partial autocorrelation function from lag 1 to 15 respectively. Fig. 6 depicts the estimated correlation between the y–axis vs the lag number on the x-axis and can be used to determine the pattern (AR) in the set of data. Similarly, Fig. 7 shows partial autocorrelation plot for the residuals of the seasonal variability of ionospheric F\(_2\) layer. Partial autocorrelation is used to measure the degree of association between \( X_t \) and \( X_{t-1} \), when the effects of other time lags 2, 3, … up to \( X_{t-1} \) are somehow partial lead out. Their singular purpose in time series analysis is identifying an appropriate AR model for forecasting. For the residuals of the virtual height of ionospheric F\(_2\) layer, that showing an adequacy of the constructed model of seasonal variability in virtual height of F\(_2\) for atmospheric region of Pakistan. The dashed lines in the above mentioned figures demonstrate an approximate 95% confidence interval for an individual estimated partial autocorrelation. In these cases the partial autocorrelation plots strongly recommend the presence of serial correlation in the ionospheric F\(_2\) layer data and measure the degree of association if the process is an autoregressive order one then the partial autocorrelations can be examined to determine the order of the process. In table 1 observed value are plotted against predicted values as depicted in Fig. 8. These illustrations are verifying the results of estimates obtained from the estimating techniques.

Following parametric tests were performed for the model verification through regression and autoregressive AR (1):

1. Standard error test as computed from regression models and autoregressive AR (1) order one model.
2. MSE (Mean Square test). It is a ratio sum of square error and number of sample.
3. SSE test (Sum of the square test). It is called the total sum of the square deviations.
4. t-test (t- distribution test). It is to estimate the confidence interval for the mean of a variable, to make various assumptions about the data.
5. F-test (Fraser test). It is a ratio of the larger variance and the smaller variance.
6. P-value confidence interval (from +95% to -95% confidence level) for models [16, 17].

### Table 1. Comparison of observed and predicted values.

<table>
<thead>
<tr>
<th>Days</th>
<th>Observed Values</th>
<th>Predicted values</th>
</tr>
</thead>
<tbody>
<tr>
<td>366</td>
<td>293.89</td>
<td>251.69</td>
</tr>
<tr>
<td>367</td>
<td>293.29</td>
<td>233.67</td>
</tr>
<tr>
<td>368</td>
<td>292.68</td>
<td>219.74</td>
</tr>
<tr>
<td>369</td>
<td>292.68</td>
<td>207.94</td>
</tr>
<tr>
<td>370</td>
<td>291.47</td>
<td>197.50</td>
</tr>
<tr>
<td>371</td>
<td>290.88</td>
<td>188.03</td>
</tr>
<tr>
<td>372</td>
<td>290.28</td>
<td>179.31</td>
</tr>
<tr>
<td>373</td>
<td>289.68</td>
<td>171.17</td>
</tr>
<tr>
<td>374</td>
<td>289.85</td>
<td>163.51</td>
</tr>
<tr>
<td>375</td>
<td>288.49</td>
<td>156.26</td>
</tr>
</tbody>
</table>

### Table 2. Tests of statistical significance.

<table>
<thead>
<tr>
<th>S.#</th>
<th>Model / Test</th>
<th>Simple Regression</th>
<th>Multiple Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Estimate ( \beta_1 )</td>
<td>-0.126</td>
<td>-0.0283</td>
</tr>
<tr>
<td>2</td>
<td>St. Error</td>
<td>25.970</td>
<td>2.340</td>
</tr>
<tr>
<td>3</td>
<td>t-Statistics</td>
<td>11.421</td>
<td>2.22</td>
</tr>
<tr>
<td>4</td>
<td>Constant ( \beta_0 )</td>
<td>325.85</td>
<td>311.213</td>
</tr>
<tr>
<td>5</td>
<td>Estimate ( \beta_2 )</td>
<td>-</td>
<td>-0.096</td>
</tr>
<tr>
<td>6</td>
<td>MSE</td>
<td>3996.9</td>
<td>1026</td>
</tr>
<tr>
<td>7</td>
<td>SSE</td>
<td>3996.9</td>
<td>371443</td>
</tr>
<tr>
<td>8</td>
<td>( F_{0.05} )-value</td>
<td>12.074</td>
<td>4.929</td>
</tr>
<tr>
<td>9</td>
<td>P – value</td>
<td>0.000573</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>Corr. Coefficient</td>
<td>Negative</td>
<td>Negative</td>
</tr>
<tr>
<td>11</td>
<td>( R^2 )</td>
<td>3.2 %</td>
<td>1.7 %</td>
</tr>
</tbody>
</table>

While a condition tells that if \( \phi_1 > 1 \) or \( \phi_1 < -1 \), then the series will be non-stationary or unsteady time series. For a stationary stable time series, \( X_t \) remains surrounded in the motive, it has finite variance, we would need \( \phi_1 < 1 \). Fig. 6 and Fig. 7 illustrate the estimated auto-correlation function and the partial autocorrelation function from lag 1 to 15 respectively. Fig. 6 depicts the estimated correlation between the y–axis vs the lag number on the x-axis and can be used to determine the pattern (AR) in the set of data. Similarly, Fig. 7 shows partial autocorrelation plot for the residuals of the seasonal variability of ionospheric F\(_2\) layer. Partial autocorrelation is used to measure the degree of association between \( X_t \) and \( X_{t-1} \), when the effects of other time lags 2, 3, … up to \( X_{t-1} \) are somehow partial lead out. Their singular purpose in time series analysis is identifying an appropriate AR model for forecasting. For the residuals of the virtual height of ionospheric F\(_2\) layer, that showing an adequacy of the constructed model of seasonal variability in virtual height of F\(_2\) for atmospheric region of Pakistan. The dashed lines in the above mentioned figures demonstrate an approximate 95% confidence interval for an individual estimated partial autocorrelation. In these cases the partial autocorrelation plots strongly recommend the presence of serial correlation in the ionospheric F\(_2\) layer data and measure the degree of association if the process is an autoregressive order one then the partial autocorrelations can be examined to determine the order of the process. In table 1 observed value are plotted against predicted values as depicted in Fig. 8. These illustrations are verifying the results of estimates obtained from the estimating techniques.

Following parametric tests were performed for the model verification through regression and autoregressive AR (1):

1. Standard error test as computed from regression models and autoregressive AR (1) order one model.
2. MSE (Mean Square test). It is a ratio sum of square error and number of sample.
3. SSE test (Sum of the square test). It is called the total sum of the square deviations.
4. t-test (t- distribution test). It is to estimate the confidence interval for the mean of a variable, to make various assumptions about the data.
5. F-test (Fraser test). It is a ratio of the larger variance and the smaller variance.
6. P-value confidence interval (from +95% to -95% confidence level) for models [16, 17].

### CONCLUSIONS

In this study, we have estimated the seasonal variation of virtual height h’ of F\(_2\) layer and the deviation in virtual height of h’F\(_2\) layer due to solar maximum and geomagnetic field. With the predication equation obtained from the time plot, the simple linear regression model may provide a very good fit to data. The autoregressive AR (1) model is clearer and easier to handle than the moving average from the virtual height data set
which we have analyzed. In the case of finding the appropriate model for seasonal variation in virtual height $h'F_2$ of ionosphere layer, we have looked into the major parametric values of model. It can be seen that the order one is suitable for making prediction and finding forecasts for the atmospheric region of Pakistan.

5. ACKNOWLEDGEMENTS

The authors pay their gratitude to Director and Staff of the SUPARCO Ionosphere Station Karachi for providing the data.

6. REFERENCES

Extreme Rainfall Incidents over Sindh Province, against Different Return Periods

Muhammad Ali¹* and M.J. Iqbal²

¹Mathematical Sciences Research Centre, Federal Urdu University of Arts, Science & Technology, Karachi, Pakistan
²Institute of Space and Planetary Astrophysics, University of Karachi, Karachi, Pakistan

Abstract: High frequency and intensity of rainfall events are the major causes of flooding in Pakistan. Statistical distributions, i.e., Gumbell Maximum (GM) distribution & Generalized Extreme Value distribution (GEV), were used to analyze annual extreme rainfalls in different cities of Sindh (Karachi, Badin, Chor, Rohri). Different return periods of extreme annual rainfall were calculated by using these distributions. This article also indicated that GM distribution better explains the yearly incidence of rainfall in Karachi and Badin while the GEV distribution better explains that in Chor and Rohri. We also estimated 200 mm return level for Sindh province in the next 100-year period.

Keywords: Return period, return level, yearly rainfall, Gumbel Maximum Distribution

1. INTRODUCTION

The enhanced greenhouse effect is causing global climate change at a much faster rate compared to its natural pace. There is an increasing change in ambient temperature and rainfall which results in an increase in the frequency and intensity of droughts and floods. Pakistan is now experiencing heavy rainfalls and floods due to global warming. This asks for analyzing the pattern of extreme rainfalls all over Pakistan. This study analyzed the extreme rainfalls and its variability over Sindh province of Pakistan, by selecting Karachi (24.54° N, 67.08° E), Badin (24.38° N, 68.58° E), Chor (26.58° N, 69.47° E) and Rohri (27.40° N, 68.90° E) (Fig. 1). Two approaches have been suggested to analyze extreme rainfall [1, 2]. The first step used a quartile method to evaluate the extreme rainfall [2, 3], while in second step statistical distributions have been used to demonstrate yearly extremes with their return periods [4, 5, 6]. Several literatures also depict that log-Pearson type III

Fig. 1. Geographical location of Rohri, Chor, Karachi & Badin.
Fig. 2(A). Scatter plots of yearly maximum rainfall.

Fig. 2(B). Multiple bar diagram of annual extreme rainfall (mm).
Extreme Rainfalls over Sindh Province

161

analysis, Generalized Extreme Value distribution and Gumbel distribution have been commonly used to examine extreme rainfall events [7]. This study used the second approach to calculate the long return periods and return levels by using the best fit to the extreme value distributions.

2. EXPLORING DAILY RAIN DATA

Daily data from 1981 to the end of 2010, acquired from Pakistan Meteorological Department, has been used for the analysis of annual extreme rainfall over Sindh province. Scatter plots and bar diagram of yearly extreme rainfall values are plotted in Fig. 2(A) and 2(B) respectively. Fig. 2(A) shows that there is no any observed trend. From Fig. 2(a) we get twelve values more than normal (53 mm) value and yearly maximum rain (142.5 mm) occurred on 19th July 2009, at Karachi station, while it also receives rainfall more than 100 mm i.e. (113.0 mm, 108.4 mm, 124.2 mm and 142.5 mm) at different time intervals, i.e., on 7 Aug, 1984, 29 July 2003, 10 Aug 2007 and 19 July 2009, respectively. Fig. 2(b) depicts 12 values greater than normal value (i.e., 76 mm) and Badin receives annual maximum rainfall of 176.5 mm on 3rd Aug 1994, while the other two higher rainfall are above 150 mm per day (i.e., 159.8 mm and 176.5 mm) which occurred on 3 Sep. 1990 and 3 Aug 1994. Fig. 2(c) illustrates the highest frequency of annual rainfall, i.e., it has 13 rainfall incidences of greater than normal value (i.e., 78 mm) and the heaviest rainfall of 251.2 mm was received on 20 Sep 1998; while 2nd, 3rd and 4th highest rainfalls were of 214.6 mm, 141.2 mm and 130.4 mm which occurred on 7 Aug, 1990, 23Aug, 2006 and 26 July, 2003, respectively. Fig. 2 (d) exhibits 11 rainfall incidents of greater than the mean value and the highest rainfall was 173.7 mm on 3 July, 1992, while 2nd, 3rd and 4th highest were 111.2 mm, 95.8 mm and 71 mm which occurred on 3 July 1994, 7 Aug 1988 and 9 Aug 1986. It is obvious that the highest amount of rainfall (251.2 mm per day) occurred at Chor, compare to the other stations, during the study period [Fig. 2(B)].

2.1. Extreme Value Distribution

Let \( R_1, R_2, \ldots, R_n \) represent daily rainfall within a year. An appropriate model for extremes can be obtained by taking \( R = \max \{ R_1, R_2, \ldots, R_n \} \). Where \( n = 365 \) and \( R \) shows annual maxima.

Asymptotic considerations recommend the GEV family [6], who’s PDF, is

\[
 f(R) = \frac{1}{\beta} [1 + S]^{1 - \frac{1}{\kappa}} \exp \{ -[1 + S] \}
\]

\( 1+S > 0 \) (1)

where

\[
 S = \frac{k (R - \mu)}{\beta}
\]

(2)

where \( \mu, \beta \) and \( k \) are Location parameter, scale parameter, and shape parameter, respectively. From analytically integrating Equation (1) we get CDF, i.e.,

\[
 F(R) = e^{-[1+S]} \frac{1}{S}
\]

(3)

The inverted CDF gives us the below quintile function.

\[
 R = F^{-1}(p) = \mu + \frac{\beta}{k} \left\{ [-\ln(P)]^{-k} - 1 \right\}
\]

(4)

Here \( F(R) = P \) shows cumulative probability.

In fitting the GEV distribution, either the method of L-moment is used or maximum likelihood (MLH) method is used [9]. L-moment fitting is favorable for mainly small data samples [8]. The modification of MLH is easy to include additional influences. The special case of GEV distribution which is independent of parameter \( k \), is called Gumbel Distribution which is unbounded. Gumbel found the application of extreme value theory to solve engineering problems and meteorological modeling phenomenon. The PDF of Gumbel distribution is:

\[
 f(R) = e^{-e^{-S^*}} - S^*
\]

(5)

where

\[
 S^* = \frac{(R - \mu)}{\beta}
\]

(6)

with location parameter ‘\( \mu \)’, scale parameter ‘\( \beta \)’ >0 and independent of shape parameters ‘\( k \)’. Due to its huge utilization to represent the statistics of extremes it is sometimes called “the” extreme value distribution. The Gumbel PDF is rightly skewed and exhibits its maximum at \( x = \mu \). CDF of Gumbel distribution is:

\[
 F(R) = e^{-e^{-S^*}}
\]

(7)

If we invert this CDF, we get quintile function as;

\[
 R = F^{-1}(p) = \mu - \beta \ln\{ -\ln(P) \}
\]

(8)
The above parameters \( \mu \) and \( \beta \) can be calculated by using the mean and standard deviation as;

\[
\hat{\beta} = \frac{s \sqrt{6}}{\pi} \tag{9}
\]

and

\[
\mu = \bar{X} - \gamma \hat{\beta}, \tag{10}
\]

where \( \gamma \) is Euler’s constant and whose value is 0.5771.

### 2.2. Distribution Fitting

This section compares the GEV and Gumbel Maximum distribution for extreme annual rainfall and investigates that which distribution is the best fitted. Here three methods are used.

1) Histogram with fitted probability density function.
2) Chi square test and
3) Probability probability (p-p) plots.

The histogram plots shown in Fig. 3 depict that more area has been covered by GEV distribution as compared to Gumbel Maximum distribution. Thus GEV provides the best fit for yearly extreme rainfalls in Karachi and Badin while Fig. 3(c) and Fig 3(d) show that Gumbel Max provides the best fit for other two cities of Sindh.

Then Chi square test was applied to get the best fitted distribution for extreme annual rainfalls in Karachi, Badin, Chor, and Rohri; the result is summarized in Table 1. As for as Karachi is concerned, the estimated \( \chi^2 = 0.17932 \) for Gumbel maximum distribution. Under the negative null hypothesis, the statistic is designed from a \( \chi^2 \) distribution having degree of freedom \( \nu = 6 - 3 - 1 = 2 \). Thus, from Table (1) it is clear that the estimated chi squared values for the above stations are smaller than 95th percentile value 5.99 with 5% level of significance. Hence, the yearly extreme values of rainfall of considered stations have GEV distribution. As, calculated values of chi square for Gumbel Maximum are smaller than that of GEV distribution for Karachi and Badin, thus Gumbel maximum is the best fitted distribution for Karachi and Badin, whereas the calculated values of chi square for GEV distribution are smaller that of Gumbel maximum distribution for Chor and Rohri. Therefore GEV distribution is the best fitted for Chor and Rohri.

Next, the best fitted test for GEV distribution for which \( \chi^2 \) is 5.99 at 5% level, with \( \nu = 6 - 3 - 1 = 2 \). Thus, from Table (1) it is clear that the estimated chi squared values for the above stations are smaller than 95th percentile value 5.99 with 5% level of significance. Hence, the yearly extreme values of rainfall of considered stations have GEV distribution. As, calculated values of chi square for Gumbel Maximum are smaller than that of GEV distribution for Karachi and Badin, thus Gumbel maximum is the best fitted distribution for Karachi and Badin, whereas the calculated values of chi square for GEV distribution are smaller that of Gumbel maximum distribution for Chor and Rohri. Therefore GEV distribution is the best fitted for Chor and Rohri.

In order to display the goodness of fit test results p-p Plots are drawn for the yearly maximum rainfall of above cities in Fig. 4. Here we check only the deviation of observed data points from theoretical values. Fig. 4 indicates more deviation in GEV distribution while it is lesser in Gumbel Maximum distribution for Badin and Karachi, and vice versa for Chor and Rohri. So Gumbel Max. is best fitted for Badin and Karachi while GEV is best for Chor and Rohri. Hence, the p-p plots supported the above chi-squared test.

### 4. RESULTS AND DISCUSSION

This section calculates the annual return period of extreme rainfall of four mentioned cities of Sindh province. The average return period is calculated by the formula:

\[
\text{Return (} R \text{)} = \frac{1}{\omega [\text{1-}F(R)]} \tag{11}
\]
Fig. 3 Comparison of pdf for GEV and Gumbel max. distribution.
Fig. 4 P-P plot of GEV & Gumbel maximum distribution of four major cities of Sindh.
This Eq. represents that the Return (R) is a function of the Cumulative Distribution Function calculated at R and the average sampling frequency \( \omega \), for yearly maximum value \( \omega = 1/\text{year} \).

### 4.1. Return (R) for Karachi

Our calculation reveals that the heaviest rainfall during the period 1981 to 2010, i.e., 142.5 mm, occurred on 19th July, 2009. Obviously, it is desirable to know that when such a high rainfall or even a more heavy rainfall can reoccur in Karachi. So by using Gumbel Distribution (being the best distribution), we calculated the parameter values as \( \mu = 36.814 \) and \( \beta = 27.943 \).

The cumulative probability is computing using Eq. (6):

\[
S^* = \frac{(R - \mu)}{\beta}
\]

\( S^* = 3.782 \),

and using Eq. (7)

\[
F(R) = P(R \leq 142.5) = e^{-e^{-S^*}}
\]

\( F(R) = 0.98 \)

Now Eq. (11) gives

\[
\text{Return (R)} = \frac{1}{\omega[1 - F(R)]}
\]

Return (R) = 44 years

This implies that, it will take 44 years to reoccur the rainfall equal to 142.5 mm or more per day. Similarly return period for Chor and Rohri was calculated by using GEV.

### 4.2. Return Levels against Given Return Periods

This section provides the calculation of return level for \( T = 2 \) years, with \( p = 0.5 \) and \( F(X) = P = 1 - p = 0.5 \). From Eq. (8), we have 47.06 mm which reveals that in the next two years Karachi may receive 47.06 mm rain approximately. Different return levels 78.73 mm, 99.7 mm, 119.81 mm, 134.37 mm, 145.85 mm and 165.36 mm for Karachi are calculated respectively for return periods, 5, 10, 20, 30, 50 and 100 years.

Hence, return levels are determined against different return period for Karachi and Badin by using Gumbel distribution (Eq. 6, 7 and 8 ) and for Chor and Rohri by using GEV ( Eq. 2, 3 and 4). The calculations are summarized in Table 2. Our results demonstrate that above cities of Sindh have 100 mm daily rainfall against the return period of 20-year and 140 mm rainfall against 50-year, suggesting the necessity of appropriate flood forecasting and for acquiring the requisite improvements in river structure in the country.

### 5. CONCLUSIONS

This study employed probability distributions for the calculation of return periods of yearly rainfall in four major cities of Sindh province of Pakistan, viz. Karachi, Badin, Chor and Rohri. Daily data of rainfall of these cities have been employed from first Jan 1981 to 31 Dec 2010. Analyses of time series of these cities shows no obvious trend.

Our calculations explain that Gumbel maximum distribution provides the best fit for coastal areas of the country, like Karachi and Badin, while the generalized extreme value (GEV) depicts the actual pattern of rainfall in Chor and Rohri.
Rohri. Our results also revealed that the considered cities of Sindh will have 130 mm return levels against 20-year return period, while rainfall of 170 mm or more per day is expected in the next 50-year return period. So it suggests for upgrading the flood forecasting system, by using modern technology like GIS, improving the river flows, and constructing new dams in Pakistan.

6. ACKNOWLEDGEMENTS

We are grateful to the Pakistan Meteorological Department, Karachi for providing the daily rainfall data.

7. REFERENCES


Properties of Univalent Solution for Complex Fractional Differential Equation

Rabha W. Ibrahim\textsuperscript{1*} and Maslina Darus\textsuperscript{2}

\textsuperscript{1}Institute of Mathematical Sciences, Universiti Malaya, 50603, Kuala Lumpur, Malaysia
\textsuperscript{2}School of Mathematical Sciences, Faculty of Science and Technology, University Kebangsaan Malaysia, Bangi 43600, Selangor Darul Ehsan, Malaysia

Abstract: In this note, we discuss some properties of univalent solutions for fractional differential equation in the unit disk in the sense of Srivastava-Owa operators. By employing the differential subordination concept, we will study the upper bound of these solutions. Furthermore, by applying the Rogosinski theorem and Goluzin theorem, we illustrate some inequalities involving the coefficients and integral representation of these solutions. Moreover, the uniqueness is studied by using Rouche's theorem.

Keywords: Fractional calculus; fractional differential equation; unit disk; univalent function; analytic function; subordination; superordination

AMS Mathematics Subject Classification: 30C45.

1. INTRODUCTION

In [1], Srivastava and Owa, gave definitions for fractional operators (derivative and integral) in the complex $z$-plane $\mathbb{C}$ as follows:

Definition 1.1 The fractional derivative of order $\alpha$ is defined, for a function $f(z)$ by

$$D_\alpha^\alpha f(z) := \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta; \quad 0 \leq \alpha < 1,$$

where the function $f(z)$ is analytic in simply-connected region of the complex $z$-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $\log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

Definition 1.2 The fractional integral of order $\alpha$ is defined, for a function $f(z)$, by

$$I_\alpha^\alpha f(z) := \frac{1}{\Gamma(\alpha)} \int_0^z f(\zeta)(z-\zeta)^{\alpha-1} d\zeta; \quad \alpha > 0,$$

where the function $f(z)$ is analytic in simply-connected region of the complex $z$-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $\log(z-\zeta)$ to be real when $(z-\zeta) > 0$.

Let $A$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in U.$$  \hspace{1cm} (1)

Also, let $S$ and $C$ denote the subclasses of $A$ consisting of functions which are, respectively, univalent and convex in $U$. It is well known that; if the function $f(z)$ given by (3) is in the class $S$, then $|a_n| \leq n, \quad n \in \mathbb{N} \setminus \{1\}$. Moreover, if the
function \( f(z) \) given by (1) is in the class \( C \), then \( |a_n| \leq 1, \ n \in \mathbb{N} \).

**Definition 1.3** (Subordination Principal). For two functions \( f \) and \( g \) analytic in \( U \), we say that the function \( f(z) \) is subordinate to \( g(z) \) in \( U \) and we write as \( f(z) \prec g(z) (z \in U) \), if there exists a Schwarz function \( w(z) \) analytic in \( U \) with \( w(0) = 0 \), and \( |w(z)| < 1 \), such that \( f(z) = g(w(z)), z \in U \). In particular, if the function \( g(z) \) is univalent in \( U \), the above subordination is equivalent to \( f(0) = g(0) \) and \( f(U) \subset g(U) \).

**Definition 1.4** (Differential subordination ). Let \( \phi : \mathbb{C}^2 \to \mathbb{C} \) and let \( h \) be univalent in \( U \). If \( p \) is analytic in \( U \) and satisfies the differential subordination \( \phi(p(z)), zp'(z) \prec h(z) \) then \( p \) is called a solution of the differential subordination. The univalent function \( q \) is called a dominant of the solutions of the differential subordination, if \( p \prec q \). If \( p \) and \( \phi(p(z)), zp'(z) \) are univalent in \( U \) and satisfy the differential superordination \( h(z) \prec \phi(p(z)), zp'(z) \), then \( p \) is called a solution of the differential superordination. An analytic function \( q \) is called subordinant of the solution of the differential superordination if \( q \prec p \).

In [2] the authors imposed a linear fractional differential operator based on \( D^\alpha_z f(z) \) as follows:

\[
D^\alpha_z f(z) = f(z) = z + \sum_{n=1}^{\infty} a_n z^n,
\]

\[
D^\alpha_\beta_z f(z) = (\beta - \lambda)\Phi^\alpha f(z) + \lambda z(\Phi^\alpha f(z))' + (1 - \beta)z = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)} [\lambda(n-1) + \beta] a_n z^n = D^\alpha_\beta_z (f(z)),
\]

\[
D^\alpha_\beta_z f(z) = D^\alpha_\beta_z (D^\alpha_\beta_z f(z))
\]

\[
= z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)} [\lambda(n-1) + \beta] a_n z^n,
\]

\[
D^\alpha_\beta_z f(z) = D^\alpha_\beta_z (D^\alpha_\beta_z f(z))
\]

\[
= z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\alpha)}{\Gamma(n+1-\alpha)} [\lambda(n-1) + \beta] a_n z^n,
\]

for \( 0 \leq \alpha < 1, \beta \geq 1, \lambda \geq 0 \) and \( k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\} \) with \( D^\alpha_\beta_z f(0) = 0 \).

Corresponding to the differential operator \( D^\alpha_\beta_z f(z) \), the fractional integral operator is given as follows

\[
I^\alpha_\beta f(z) = z + \sum_{n=1}^{\infty} \frac{a_n z^n}{\Gamma(n+1-\alpha)} [\lambda(n-1) + \beta] z^n
\]

and hence we obtain

\[
I^\alpha_\beta f(z) = f(z).
\]

**Remark 1.1.** The operator (2) generalizes various types of operators such as Sălăgean’s differential operator [3], Al-Oboudi’s differential operator [4], Srivastava-Owa fractional differential operator, the linear multiplier fractional differential operator which introduced in [5]. Finally, when \( \beta = 1 \), we obtain the linear fractional operator as in [6].

Here, we consider the following class of fractional differential equation:

\[
z(D^\alpha_\beta f(z))' = p(z)
\]

such that \( p(z) \prec q(z) \) and \( q(0) = p(0) = 1 \).

We denote this class by \( S^\alpha_\beta(q) \).

We need the following preliminaries in the sequel.

The Libera-Pascu integral operator \( L_a : A \to A \) defined by

\[
F(z) := L_a f(z) = \frac{1}{z^a} \int_0^z t^{a-1} f(t) dt, a \in \mathbb{C}, \Re(a) \geq 0.
\]

For \( a = 1 \) we obtain the Libera integral operator, for \( a = 0 \) we obtain the Alexander integral operator and in the case \( a = 1,2,3,... \) we obtain the Bernardi integral operator.

**Lemma 1.1** [7] Let \( h \) be convex in \( U \) and \( \theta, \phi \) be analytic in domain \( D \). Let \( p \) be analytic in \( U \), with \( h(0) = \theta(p(0)) \) and \( p(U) \subset D \). If the differential equation

\[
\theta(q(z)) + zq'(z)\phi(q(z)) = h(z)
\]
has a univalent solution in $U$ that satisfies $q(0) = p(0)$ and $\theta[q(z)] < h(z)$ then the differential subordination

$$\theta[p(z)] + zp'(z)\phi[p(z)] < h(z)$$

implies that $p(z) < q(z)$. The function $q$ is the best dominant.

**Lemma 1.2** (Rogosinski Theorem) [8] Let $f(z) = \sum_{n=1}^{\infty} a_n z^n$ and $g(z) = \sum_{n=1}^{\infty} b_n z^n$ be analytic in $U$ and suppose $g < f$. Then

$$\sum_{n=1}^{k} |b_n|^2 \leq \sum_{n=1}^{k} |a_n|^2.$$ 

**Lemma 1.3** (Goluzin Theorem) [8] If $g < f$ and $\lambda_1 \geq \lambda_2 \geq \ldots \geq 0$, then

$$\sum_{n=1}^{\infty} \lambda_n |b_n|^2 \leq \sum_{n=1}^{\infty} \lambda_n |a_n|^2.$$ 

**2. MAIN RESULTS**

Our main results are provided in this section.

**Theorem 2.1** Let $f \in S_{k,\alpha}^{k,\alpha}(q)$,

$$P(z) := \frac{z(D_{\beta,\lambda}^{k,\alpha}F(z))'}{D_{\beta,\lambda}^{k,\alpha}F(z)},$$

where $F$ is the Libera-Pascu integral operator, be analytic in $U$ such that $P(U) \subset D$ (a domain) and

$$h(z) = \left(\frac{a}{1+a} + \frac{q^2(z) + zq'(z)}{1+a}\right)\phi(z), \quad \phi(0) = 1$$

has a univalent solution in the unit disk $U$ such that $P(0) = q(0)$. Then the subordination

$$\left(\frac{a}{1+a} + \frac{P^2(z) + zP'(z)}{1+a}\right)\phi(z) < h(z)$$

implies $P(z) < q(z)$ for some analytic function $\phi$ and $q$ is the best dominant.

**Proof.** Since $f \in S_{k,\alpha}^{k,\alpha}(q)$, then

$$\frac{z(D_{\beta,\lambda}^{k,\alpha}f(z))'}{D_{\beta,\lambda}^{k,\alpha}f(z)} < q(z).$$

From the definition of the Libera-Pascu integral operator we have

$$(1+a)f(z) = aF(z) + zF'(z),$$

and by using the linear operator $D_{\beta,\lambda}^{k,\alpha}$, we have

$$(1+a)D_{\beta,\lambda}^{k,\alpha}f(z) = aD_{\beta,\lambda}^{k,\alpha}F(z) + D_{\beta,\lambda}^{k,\alpha}(zF'(z)), \quad \Re(a) \geq 0.$$ 

By making use the first derivative of the last assertion, we obtain

$$z(D_{\beta,\lambda}^{k,\alpha}f(z))' = \frac{a}{1+a}z(D_{\beta,\lambda}^{k,\alpha}F(z))'$$

$$+ \frac{1}{(1+a)}z[D_{\beta,\lambda}^{k,\alpha}(zF'(z))]'$$

and using the fact that

$$D_{\beta,\lambda}^{k,\alpha}(zF'(z)) = z(D_{\beta,\lambda}^{k,\alpha}F(z))',$$

yields

$$z(D_{\beta,\lambda}^{k,\alpha}f(z))' = \frac{a}{1+a}z(D_{\beta,\lambda}^{k,\alpha}F(z))' + \frac{1}{(1+a)}z[D_{\beta,\lambda}^{k,\alpha}(zF'(z))]'$$

$$\cdot \frac{1}{(1+a)}z[D_{\beta,\lambda}^{k,\alpha}(zF'(z))]'$$

$$= \frac{a}{1+a}z(D_{\beta,\lambda}^{k,\alpha}F(z))' + \frac{1}{(1+a)}z[D_{\beta,\lambda}^{k,\alpha}(zF'(z))]'$$

A computation implies that

$$\frac{z(D_{\beta,\lambda}^{k,\alpha}f(z))'}{D_{\beta,\lambda}^{k,\alpha}f(z)} = \theta[P(z)] + zP'(z)\phi(P(z))$$

where

$$\theta[P(z)] := \left(\frac{a}{1+a} + \frac{P^2(z)}{1+a}\right)\phi(z),$$

$$\phi(P(z)) := \frac{\phi(z)}{1+a}$$

and
\[ \varphi(z) := \frac{D^{k,\varphi}_{\beta,\varphi} F(z)}{D^{k,\varphi}_{\beta,\varphi} f(z)} \]

are analytic in \( U \). It is clear that \( \theta[q(z)] \prec h(z) \).
Hence in view of Lemma 1.1, we have \( P(z) \prec q(z) \) and \( q \) is the best dominant.

**Corollary 2.1** Let the assumptions of Theorem 2.1 hold. Then the Libera-Pascu integral operator \( F(z) \in S^{k,\alpha}_{\beta,\alpha}(q) \).

Next, using Lemma 1.2 and Lemma 1.3, we introduce the coefficient inequalities as follows.

**Theorem 2.2** Let \( f \in S^{k,\alpha}_{\beta,\alpha}(q) \). Then

\[ \sum_{n=1}^{k} |b_n|^2 + 1 \leq \sum_{n=1}^{k} |q_n|^2 + 1, \quad (4) \]

where \( q(z) = \sum_{n=0}^{\infty} q_n z^n, q(0) = 1 \) and

\[
\begin{align*}
\hat{b}_1 := & \Psi_{2,k}(\alpha,\beta,\lambda) a_z, \\
\hat{b}_2 := & 2 \Psi_{3,k}(\alpha,\beta,\lambda) a_z - [\Psi_{2,k}(\alpha,\beta,\lambda) a_z]^2, \\
& \ldots 
\end{align*}
\]

**Proof.** Since \( f \in S^{k,\alpha}_{\beta,\alpha}(q) \), then we have

\[
\frac{z(D^{k,\alpha}_{\beta,\alpha} f(z))'}{D^{k,\alpha}_{\beta,\alpha} f(z)} \prec q(z).
\]

A computation implies that

\[
z\left( \frac{D^{k,\alpha}_{\beta,\alpha} f(z)}{D^{k,\alpha}_{\beta,\alpha} f(z)} \right)' = 1 + \Psi_{2,k}(\alpha,\beta,\lambda) a_z z + [\Psi_{2,k}(\alpha,\beta,\lambda) a_z]^2 z^2 + \ldots + q(z) + q(z)^2 + \ldots.
\]

which is equivalent to

\[
1 + \Psi_{2,k}(\alpha,\beta,\lambda) a_z z + [2 \Psi_{3,k}(\alpha,\beta,\lambda) a_z - [\Psi_{2,k}(\alpha,\beta,\lambda) a_z]^2] z^2 + \ldots + q(z) + q(z)^2 + \ldots.
\]

or

\[
\Psi_{2,k}(\alpha,\beta,\lambda) a_z z + [2 \Psi_{3,k}(\alpha,\beta,\lambda) a_z - [\Psi_{2,k}(\alpha,\beta,\lambda) a_z]^2] z^2 + \ldots + q(z) + q(z)^2 + \ldots.
\]

thus in view of Lemma 1.2

\[
\sum_{n=1}^{k} |b_n|^2 \leq \sum_{n=1}^{k} |q_n|^2,
\]

and consequently we obtain (4).

As applications of Lemma 1.3, we have the following theorems:

**Theorem 2.3** Let \( f \in S^{k,\alpha}_{\beta,\alpha}(q) \). Then for \( m\lambda + \beta > 0, \ m = 1,2,3, \ldots \)

\[
\sum_{n=1}^{\infty} \lambda_n |b_n|^2 + 1 \leq \sum_{n=1}^{\infty} |q_n|^2 \lambda_n + 1, \quad (5)
\]

where

\[
\begin{align*}
\lambda_1 &= \left( \frac{2(\lambda + \beta)}{2 - \alpha} \right)^k, \\
\lambda_2 &= \left( \frac{6(2\lambda + \beta)}{(2 - \alpha)(3 - \alpha)} \right)^k, \\
\lambda_3 &= \left( \frac{24(3\lambda + \beta)}{(2 - \alpha)(3 - \alpha)(2 - \alpha)} \right)^k, \\
& \ldots
\end{align*}
\]

such that \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots \geq 0. \)

**Theorem 2.4** Let \( f \in A \) such that \( |a_1| \leq |a_2| \leq \ldots \). Assume that the solution of the fractional differential equation

\[
D^{k,\alpha}_{\beta,\alpha} f(z) = g_{\alpha}(z) \tag{6}
\]

satisfying the subordination \( f(z) \prec G(z), G(0) = 0. \) Then the solution is bounded.

**Proof.** The solution of the equation (6) has the form

\[
f(z) = I^{k,\alpha}_{\beta,\alpha} g_{\alpha}(z).
\]

Since \( f(z) \prec G(z) \), then Lemma 1.3 implies

\[
|f(z)| = \left| \sum_{n=1}^{\infty} a_n z^n \right| \leq \sum_{n=1}^{\infty} |a_n|, \quad (a_1 = 1)
\]

\[
= \sum_{n=1}^{\infty} |a_n|^2 \leq \sum_{n=1}^{\infty} |g_n|^2.
\]

But \( \frac{1}{|a_1|} \geq \frac{1}{|a_2|} \geq \ldots \) hence the solution \( f(z) \) is bounded.
Theorem 2.5 If

\[ \sum_{n=1, n \neq k}^{\infty} |a_n| |a_k|, \quad a_k \neq 0, k \geq 2. \]

Then Eq. (3) has a unique solution in the unit disk.

**Proof.** By setting

\[ \phi(z) = -\frac{1}{a_k} \sum_{n=1, n \neq k}^{\infty} a_n z^n, \quad k \geq 2 \]

for \(|z| \leq 1\), implies

\[
|\phi(z)| = \left| -\frac{1}{a_k} \sum_{n=1, n \neq k}^{\infty} a_n z^n \right| \\
= \frac{1}{a_k} \left\| \sum_{n=1, n \neq k}^{\infty} a_n z^n \right\| \\
\leq \frac{1}{a_k} \left\| \sum_{n=1, n \neq k}^{\infty} |a_n| \right\| \\
< 1.
\]

Since \( |\phi(z)| < 1 \) for \(|z| = 1\), and by Rouche's theorem, we observe that (3) has exactly one zero in \( U \).

**Theorem 2.6** Let the assumption of Theorem 2.4 hold. If for positive integer \( m \)

\[ \sum_{n=1}^{\infty} |g_n|^2 < |a_m| \]  

(7)

then the fractional differential equation (6) has a unique solution in \( U \).

**Proof.** From the Proof of Theorem 2.4, we have

\[ |f(z)| \leq \sum_{n=1}^{\infty} \left| \frac{g_n}{a_n} \right|^2. \]

By the assumption of the theorem, there exists a coefficient \( a_m \) such that \( 1/|a_n| \leq 1/|a_m| \) for all \( n \). Therefore by (7), we obtain

\[
|f(z)| \leq \frac{1}{a_m} \left| \sum_{n=1}^{\infty} g_n^2 \right| \\
< 1;
\]

thus by Rouche's theorem, Eq.(6) has a unique solution.

Theorem 2.7 Let the assumption of Theorem 2.6 hold. If

\[ \sum_{n=1}^{\infty} |g_n|^2 < 1, \]  

(8)

then the fractional differential equation (6) has a unique solution in \( U \).

**Proof.** Since

\[ |f(z)| \leq \sum_{n=1}^{\infty} \left| \frac{g_n}{a_n} \right|^2. \]

Then we impose

\[
|f(z)| \leq \sum_{n=1}^{\infty} \left| \frac{1}{a_n} \right| |g_n|^2 \\
\leq \sum_{n=1}^{\infty} \left| \frac{1}{a_1} \right| |g_n|^2 \\
= \sum_{n=1}^{\infty} |g_n|^2 \\
< 1;
\]

hence by Rouche's theorem, Eq.(6) has a unique solution.

3. **CONCLUSIONS**

By employing a linear fractional differential operator in the unit disk in the sense of the Srivastava-Owa differential operator, fractional differential equations (3) and (6) are introduced. In Theorem 2.1, we considered that the Eq. (3) has a univalent solution. The important properties of this solution are described by making use of the Libera-Pascu integral operator, subordination concept, differential subordination and coefficients bound. Theorems 2.5-2.7 are proposing the unique solution of Eq.(3) and (6) in the unit disk by applying the Rouche's theorem. Theorem 2.6 showed the connection between the existence of unique solution and the coefficients bound. Similarly for Theorem 2.7.

4. **ACKNOWLEDGEMENT**

The second author was financially supported by UKM-DLP-2011-050.
5. REFERENCES


Obituary

Professor Dr. Mustafa Shameel
(1941 – 2013)

We are grieved to report the sad demise of our friend and colleague, Prof. Dr. Mustafa Shameel, an eminent Meritorious Professor of Botany, University of Karachi, on 13th May 2013 at his residence, Ouj-e-Suraiya, A-338, Block 7, Gulistan-e-Jauhar, Karachi-75290, Pakistan. He had been suffering from multiple ailments for the last few years, but remained active and delightfully engaged in shaping papers of his students until he last breath. This reflects his keen infatuation for phycological research.

Prof. Dr. Mustafa Shameel was an authority on botany, biology and phycochemistry of seaweeds. He was the developer of a new classification of algae, and founder and patron of International Journal of Phycology & Phycochemistry.

Prof. Dr. Mustafa Shameel was born on 3rd July, 1941 at Rudauli (UP, British India) as the younger son of Mr. Syed Amirul Hasan Quadri. He obtained his MSc in Botany from the University of Karachi in 1962 by securing 1st division and 2nd position and PhD in Marine Botany in 1972 from the University of Kiel, Germany under German Academic Exchange Scholarship. In 1977, he completed 14-month Post-Doctoral research from the same University under Alexander von Humboldt Fellowship. Also, he did MDH in 1963 with 1st division, MBSc with 1st division in 1964 and MSc (H) with Gold Medal in Homoeopathy from International Medical College Lahore, affiliated with Nature Cure Institute, Bradford England.

Dr Shameel served at the Department of Botany, University of Karachi as Lecturer (1962-1972), Assistant Professor (1972-1973), Associate Professor (1978-1979), Professor (1985-1999), Meritorious Professor (1999-2001) and HEC Eminent Professor (2003-2005). He also worked as Director, Institute of Marine Science (1994-1998) and Centre of Excellence in Marine Biology (1999-2001), University of Karachi. He was the President of Pakistan Botanical Society during 2001-2003.

Prof. Dr. Shameel was elected as Fellow of Pakistan Academy of Sciences in 2005 and as Fellow of The Academy of Sciences for the Developing World, Trieste, Italy (TWAS) in 2008.

Prof Shameel had more than 50-year experience of teaching and research. He was a globally recognized authority on marine benthic algae. He published more than 330 scientific research papers in journals of international repute and 7 books with emphasis on marine algae of the Baltic, North, Mediterranean & Arabian seas. He participated in several national and international conferences and presented scientific papers. He supervised research work of 17 PhD students, 15 from Botany and two from Marine Biology. Dr. Shameel presented a new concept of barobiology of seaweeds in 1972 and reported for the first time the effects of high hydrostatic pressure on the physiology and biochemistry of marine algae. Apart from taxonomic accounts of 28 new taxa of marine algae, he has emphasized on seaweed natural products, where 24 new compounds such as sterols, terpenes, glycosides and other secondary metabolites have been isolated and described. Six new species of algae have been named after him like Ulothrix shameelii Faridi in Faridi MAF, Anjum G & Haq 1 1982:183 Pak J Bot 14 (2): 181-188, Scinaia shameelii Afaq-Husain S 1996: 446 Candollea 51 (2): 445-459, Stypopodium shameelii Nizamuddin M & Aisha K 1996:128 Pak J Bot 28 (2): 127-141, Codium shameelii Nizamuddin M 1997:179 Pak J Bot 29 (2): 179-184.

Dr. Shameel has been a member of the Editorial Board of Pakistan Journal of Botany, Pakistan Journal of Marine Sciences, Marine Research, Biological Research Journal and Journal of Natural History & Wildlife and FUUAST Journal of Biology. He served as Editor-in-Chief of Pakistan Journal of Marine Biology (1999-2001), and as Chief Editor of International Journal of Phycology & Phycochemistry (2005-2010). He received “Meritorious Research Award” of the University of Karachi (1997) and “Agha Hasan Abedi Gold Medal” in Botany from Pakistan Academy of Sciences (1998). He was assigned 10th National Ranking in Biology in 1999 as well as 2000, by Pakistan Council for Science & Technology. In recognition of his outstanding contributions to scientific research, Prof. Shameel was awarded “Izaz-i-Fazeelat” on 14th August 2000 and “Tamga-i-Imtiaz” on 14th August 2001 by the President of Pakistan.

Professor Shameel had a charming and amiable personality and had many friends among the scientific community. May Allah the Almighty bestow his countless blessings upon him and strength to his numerous students, colleagues, friends and members of his family within the country and abroad to bear this irreparable loss. Ameen!

Prof. Dr. Viqar Uddin Ahmad, HI, SI
Proceedings of the Pakistan Academy of Sciences

Instructions for Authors

Aims and Scope: Proceedings of the Pakistan Academy of Sciences is official journal of the Academy, published quarterly, in English. This open access journal publishes research papers in Engineering Sciences & Technology, Life Sciences, Medical Sciences, and Physical Sciences. State-of-the-art reviews (~20 pages, supported by recent references) summarizing R&D in a particular area of science, especially in the context of Pakistan, and suggesting further R&D are also considered. Manuscripts undergo double-blind review. Authors are not required to be Fellows or Members of the Pakistan Academy of Sciences or citizens of Pakistan.

Manuscript Format

Manuscript may contain Abstract, Keywords, INTRODUCTION, MATERIALS AND METHODS, RESULTS, DISCUSSION (or RESULTS AND DISCUSSION), CONCLUSIONS, ACKNOWLEDGEMENTS and REFERENCES and any other information that the author(s) may consider necessary. The Manuscript sections must be numbered, i.e., 1. INTRODUCTION, 2. MATERIALS AND METHODS, and so on.

Manuscripts, in Times New Roman, 1.5-spaced (but single-space the Tables), with line numbering and one-inch margins on all sides on A-4 size paper, should not exceed 20 pages including Tables and Figures. Number manuscript pages throughout. The text (in Font Size 11, except for the sections mentioned in Font Size 10) must be typed in a single column across the paper width. All Tables and Figures must be placed after the text, i.e., after REFERENCES section.

Title Page: For Online submission of the manuscript, Title Page should not be included. However, for submitting as an email attachment, the title page must be as under:
(a) Title of the article (Capitalize initial letter of each main word; font size 16; bold), max 160 characters (no abbreviations or acronyms), depicting article’s contents; (b) Author’ first name, middle initial and last name (font size 12, bold), and professional affiliation (i.e., each author’s Department, Institution, Mailing address and Email; but no position titles) (font size 12); (c) Indicate the corresponding author with *; (d) Short running title, max 50 characters (font size 10).

The next Page should start with Title of the Article, followed by entire manuscript.

Headings and Subheadings (font size 11): All flush left

- **LEVEL-1: ALL CAPITAL LETTERS; bold**
- **Level-2: Capitalize each main word; bold**
- **Level-3: Capitalize each main word; Bold, Italic**
- **Level-4: Run-in head; Italics, in the normal paragraph position. Capitalize the initial word only and end in a colon (i.e., :)**

Abstract (font size 10; max 250 words): Must be self-explanatory, stating rationale, objective(s), methodology, main results and conclusions of the study. Abbreviations, if used, must be defined on first mention in the Abstract as well as in the main text. Abstract of review articles may have variable format.

Keywords (font size 10): Three to eight keywords, depicting the article.

For Online submission:
- Separate submission of Abstract along with Title is required
- Author(s)’ names and affiliations should not be included in the manuscript or the Abstract

INTRODUCTION: Provide a clear and concise statement of the problem, citing relevant recent literature, and objectives of the investigation.

MATERIALS AND METHODS: Provide an adequate account of the procedures or experimental details, including statistical tests (if any), in a concise manner but sufficient enough to replicate the study.

RESULTS: Be clear and concise with the help of appropriate Tables, Figures and other illustrations. Data should not be repeated in Tables and Figures, but must be supported with statistics.

DISCUSSION: Provide interpretation of the RESULTS in the light of previous relevant studies, citing published references.

ACKNOWLEDGEMENTS (font size 10): In a brief statement, acknowledge financial support and other assistance.
REFERENCES (font size 10): Cite references in the text by number only in square brackets, e.g. “Brown et al [2] reported ...” or “... as previously described [3, 6–8]”, and list them in REFERENCES section, in the order of citation in the text, Tables and Figures (not alphabetically). Only published (and accepted for publication) journal articles, books, and book chapters qualify for REFERENCES.

List of REFERENCES must be prepared as under:

a. Journal Articles (Name of journals must be stated in full)

b. Books

c. Book Chapters

Tables, with concise but self-explanatory headings must be numbered according to the order of citation (like Table 1, Table 2). Round off data to the nearest three significant digits. Provide essential explanatory footnotes, with superscript letters or symbols keyed to the data. Do not use vertical or horizontal lines, except for separating column heads from the data and at end of the Table.

Figures may be printed in two sizes: column width of 8.0 cm or page width of 16.5 cm; number them as Fig. 1, Fig. 2, … in the order of citation in the text. Captions to Figures must be concise but self-explanatory. Laser printed line drawings are acceptable. Do not use lettering smaller than 9 points or unnecessarily large. Photographs must be of high quality. A scale bar should be provided on all photomicrographs.

Declaration: Provide a declaration that: (i) the results are original; (ii) the same material is neither published nor under consideration elsewhere; (iii) approval of all authors has been obtained; and, (iv) in case the article is accepted for publication, its copyright will be assigned to Pakistan Academy of Sciences. Authors must obtain permission to reproduce copyrighted material from other sources.

Reviewers: Authors may suggest four relevant reviewers, two local and two from scientifically advanced countries.

Online Manuscript Submission:
Proceedings of the Pakistan Academy of Sciences is using a web-enabled online manuscript submission and review system, which offers authors the option to track the progress of manuscripts in real time. For online submission, authors are required to register at website of the Academy (www.paspk.org) by clicking the “Journal and Publications” tab.

Manuscripts must be submitted in Microsoft Word (.doc or .docx format; pdf files not acceptable). Figures can be submitted in Word format, TIFF, GIF, JPEG, EPS, PPT.

NOTE: In case of online submission, authors are not required to submit the manuscript as email attachment. In case of any difficulty while submitting your manuscript, please get in touch with:

Editor-in-Chief
Pakistan Academy of Sciences
3-Constitution Avenue, G-5/2, Islamabad, Pakistan
Email: pas.editor@gmail.com
Tel: +92-51-920 7140
Website: www.paspk.org
## CONTENTS

**Volume 50, No. 2, June 2013**

<table>
<thead>
<tr>
<th>Research Articles</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engineering Sciences</strong></td>
<td></td>
</tr>
<tr>
<td>Parabolic Trough Solar Concentrators: A Technology which can Contribute towards Pakistan’s Energy Future</td>
<td>99</td>
</tr>
<tr>
<td>— Rizwan Masood</td>
<td></td>
</tr>
<tr>
<td>Naturally Fractured Reservoirs – Yet an Unsolved Mystery</td>
<td>109</td>
</tr>
<tr>
<td>— Muhammad Khurram Zahoor and Muhammad Haris</td>
<td></td>
</tr>
<tr>
<td>Classification of Scientific Publications using Swarm Intelligence</td>
<td>115</td>
</tr>
<tr>
<td>— Tariq Ali, Sohail Asghar, Naseer Ahmed Sajid and Munir Ahmad</td>
<td></td>
</tr>
<tr>
<td><strong>Life Sciences</strong></td>
<td></td>
</tr>
<tr>
<td>Evaluation of Relative Efficacy of Hypertonic Saline and Lactated Ringer’s Solutions in Experimentally-induced Hypo-volumic Shock in Dogs</td>
<td>127</td>
</tr>
<tr>
<td>— Muhammad A. Zafar, Nemat Ullah, Imran Sarfaraz and Fahad Raza</td>
<td></td>
</tr>
<tr>
<td>Comparative Phycoremediation of Sewage Water by Various Species of Algae</td>
<td>131</td>
</tr>
<tr>
<td>— Farooq Ahmad, Amin U. Khan and Abdullah Yasar</td>
<td></td>
</tr>
<tr>
<td><strong>Physical Sciences</strong></td>
<td></td>
</tr>
<tr>
<td>Remarks on (1,2)*αg-Homeomorphisms</td>
<td>141</td>
</tr>
<tr>
<td>— T. Salai Parkunan, S. Pious Missier, Ravi and A. Pandi</td>
<td></td>
</tr>
<tr>
<td>Seasonal Variability in Virtual Height of Ionospheric F, Layer at the Pakistan Atmospheric Region</td>
<td>151</td>
</tr>
<tr>
<td>Extreme Rainfall Incidents over Sindh Province, against Different Return Periods</td>
<td>159</td>
</tr>
<tr>
<td>— Muhammad Ali and M.J. Iqbal</td>
<td></td>
</tr>
<tr>
<td>Properties of Univalent Solution for Complex Fractional Differential Equation</td>
<td>167</td>
</tr>
<tr>
<td>— Rabha W. Ibrahim and Maslina Darus</td>
<td></td>
</tr>
<tr>
<td><strong>Obituary: Prof. Dr. Mustafa Shameel</strong></td>
<td>173</td>
</tr>
</tbody>
</table>

**Instructions for Authors** | 175 |