



# Wavelet Characterization of Turbulence for Ionospheric Region of the Pakistan Air Space

M. Ayub Khan Yousuf Zai<sup>1\*</sup> and Khusro Mian<sup>2</sup>

<sup>1</sup>Solar-Terrestrial and Atmospheric Research Center, Department of Applied Physics,  
University Karachi, Karachi, Pakistan

<sup>2</sup>Institute of Space and Planetary Astrophysics, University of Karachi,  
Karachi, Pakistan

**Abstract:** It has been observed that solar activity affects the communication system. This effect is due to the ionospheric turbulence. The turbulence due to the perturbation concentration and parcel velocity were analyzed in upper and lower layers of the ionosphere by Spectral analysis, Fast Fourier Transformation and wavelet transformation in one dimension, Haar 5-Levels for approximation and details. Wavelet analysis is known as the method to study non-stationary data. Wavelet analysis is used to quantify both signal of variability, non-stationarity and unification between ionospheric turbulence and solar activity. The results are presented under changing ionospheric conditions.

**Keywords:** Ionosphere, turbulence, perturbation, spectral analysis, wavelet, Fast Fourier Transformation

## 1. INTRODUCTION

Continuous flow of solar wind interacts with geomagnetic field to create the magnetosphere. Ionosphere is part of magnetosphere, and is created by the ionization of neutral particles. Ionospheric perturbations can be enumerated and life time fluctuations can be estimated; doing so, we can monitor this important region of our globe because it is critical for the future progress as it was in the past. Turbulence means unstable and disorderly movements and the ultimate result of the current of air and water whirling against the main current. It has been observed that Earth Science and Space Science are linked together in a peculiar way. The ionosphere lies in the upper atmospheric region of the earth. The ionosphere has diurnal and seasonal changes and its density varies with the solar radiation interaction. The structure of the ionosphere differs with its altitude and the density of ions and electrons. At the base of ionosphere, about 80 km above the earth surface, there are about  $10^4$  ions per cubic cm. In comparison, at the same point in the atmosphere, there are about  $2 \times 10^{16}$  non-ionized or neutral particles per cubic cm. Thus, at 80 km

above the earth, the ratio of neutral particles to ions is quite significant. The turbulence precursors as a result of the influence of solar activity on the ionosphere were analyzed by spectral analysis and wavelet analysis. This is a new approach of exploring ionospheric turbulence at the Pakistan air space [1–5].

## 2. MATERIAL AND METHODS

The study was based on the analyses of ionospheric turbulence at Pakistan air space such as perturbation concentration and parcel velocity. These were analyzed in upper and lower ionospheric layers using Spectral analysis, Fast Fourier Transformation and wavelet approach for one dimension Haar 5-Levels for approximation and details.

## 3. RESULTS AND DISCUSSION

### 3.1. Computation of Turbulence Flux

We computed Turbulence flux in the ionospheric layer at the Pakistan region by using perturbed electron concentration ( $N'$ ) and perturbed parcel

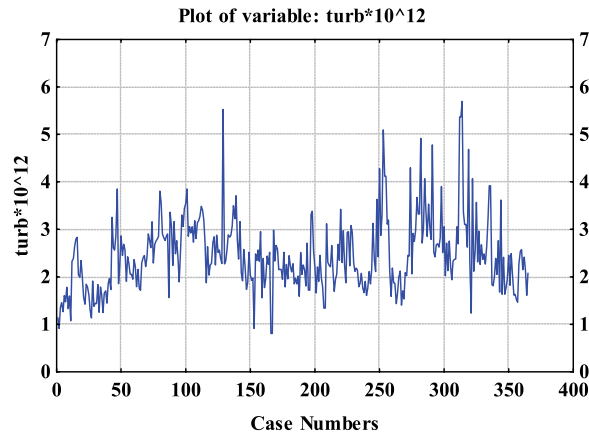


Fig. 1. Time plot for F<sub>2</sub> layer, ionospheric turbulence.

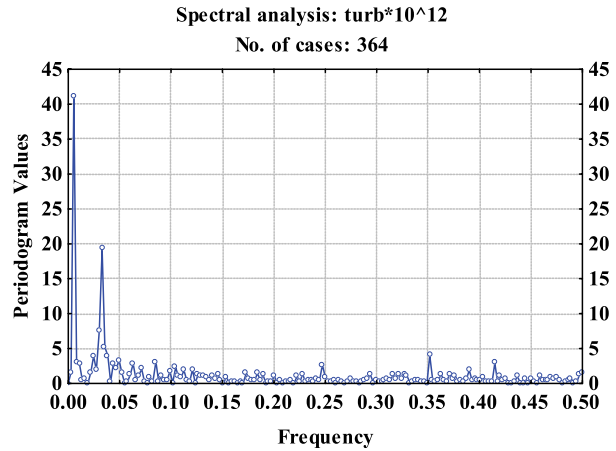


Fig. 2. Illustration of period-o-gram value and frequency for ionospheric turbulence F<sub>2</sub> Layer data.

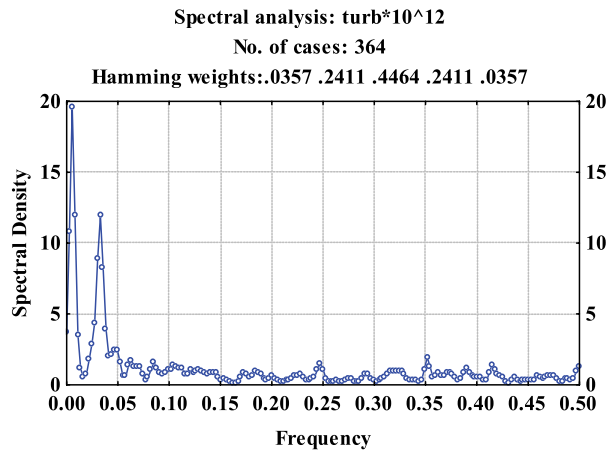


Fig. 3. Illustration of spectral density and frequency for ionospheric turbulence F<sub>2</sub>-layer data.

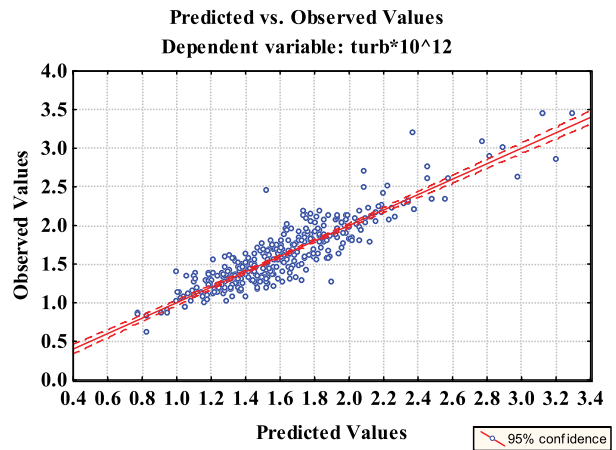


Fig. 4. Predicted value vs. observed value obtained from the model fitted values or turbulence and electron concentration at ionospheric F<sub>2</sub> layer.

velocity ( $V'$ ). Turbulent flow varies randomly with time at a location (Fig. 1); thus, turbulent flow is unsteady. The product  $\overline{U'N'}$  represents the transport of Kinematics Turbulent Flux:

$$\overline{U'N'} = \frac{U'_E N'_E + U'_{F2} N'_{F2}}{2}$$

where

$U'_{F2}$  is Instantaneous and Local Perturbation Scalar Velocity content

$U'_E$  is Instantaneous and Local Perturbation Scalar Velocity content

$N'_{F2}$  is Instantaneous and Local Perturbation

Plasma Concentration content

$N'_E$  is Instantaneous and Local Perturbation Plasma Concentration content [5].

### 1.2. Spectral Analysis

The ionospheric turbulence clearly appears to follow a cyclical pattern (Fig. 1), the line spectrum on periodogram (Fig. 2) constructed to identify the randomness in the ionospheric turbulence due to influences of solar activity and other factors.

Fig. 3 shows spectral density at ionospheric turbulence F<sub>2</sub>-layer data. Fig. 4 shows predicted values vs. observed values of fitting the model for turbulence and electron concentration at ionospheric F<sub>2</sub> layer. In fact, it is described to show seasonality

among the time series event. The purpose of spectral analysis is to present ionosphere that examines data series in term of their frequency content. Frequency domain analysis has become much more central to decode the cause and effect of upper atmospheric changes. Fast Fourier Transformation (FFT) techniques further aided the application of frequency domain analysis in upper atmosphere. It has been shown that the result of five largest peak frequencies are periodogram peaks. The large peaks identified the presence of fluctuations in the turbulence flux. The number of observations 365 and number of observations after padding is 364 and Transformations: Mean = 2.4458 subtracted. Largest peak frequencies corresponding values are:

|        |           |
|--------|-----------|
| Value: | Frequency |
| 41.21: | 0.006     |
| 19.28: | 0.033     |
| 07.64: | 0.030     |
| 05.19: | 0.035     |
| 04.01: | 0.351     |

### 1.3. Fourier Analysis

We have presented ionospheric turbulence in the form of astrophysical signals as depicted in Fig. 3 and listed their parameters in table.1. These are frequencies, Periods, coefficient Cosine, coefficient Sine, Periodogram, Density, Hamming Weight. A goal of time series analysis in the frequency domain is to separate, periodic oscillations from the random and a-periodic fluctuations. Fourier analysis is one of the famous methods for identifying periodic components. Fourier series,

$$f(t) = \overline{f(t)} + \sum_p [A_p \text{Cos}(\omega_p t) + B_p \text{Sin}(\omega_p t)]$$

where  $\overline{f(t)}$  is the mean value of the function,  $A_p, B_p$  are the coefficient of trigonometric function of  $\text{Sin}(\omega_p t)$  and  $\text{Cos}(\omega_p t)$  and the angular frequency  $\omega_p$  are integer  $p = 1, 2, 3, \dots$ , multiples of the total length of the time series. This concept is central to spectral analysis technique, the collection of Fourier coefficient having amplitude  $A_p, B_p$ , form a period-o-gram [6, 7].

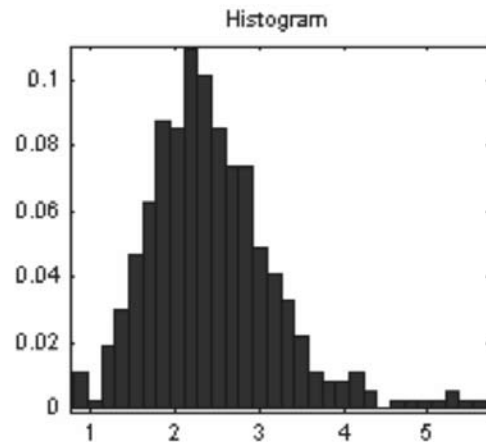
**Table 1.** Illustration of ionospheric turbulence as frequency (F), period (P), coefficient cosine and coefficient of sine and period-o-gram (Pgm).

| F     | P   | Cos    | Sin    | Pgm   |
|-------|-----|--------|--------|-------|
| 0.000 |     | 0.003  | 0.000  | 0.002 |
| 0.002 | 364 | -0.045 | 0.082  | 1.607 |
| 0.005 | 182 | -0.437 | -0.186 | 41.20 |
| 0.008 | 121 | -0.088 | -0.092 | 2.987 |
| 0.010 | 091 | -0.071 | 0.099  | 2.734 |

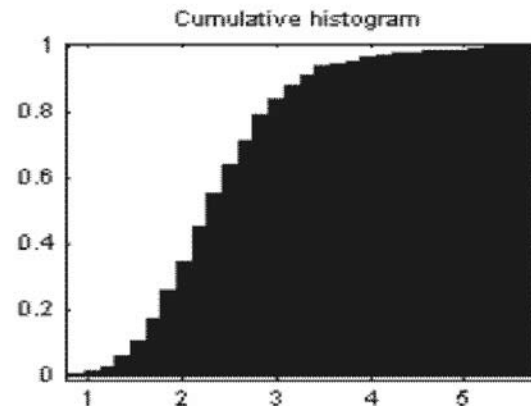
### 1.4. Wavelet Analysis

We consider ionospheric turbulence data at Pakistan air space as presented in Fig 5 and descriptive statistics is shown in Table 2 and 3 histogram and

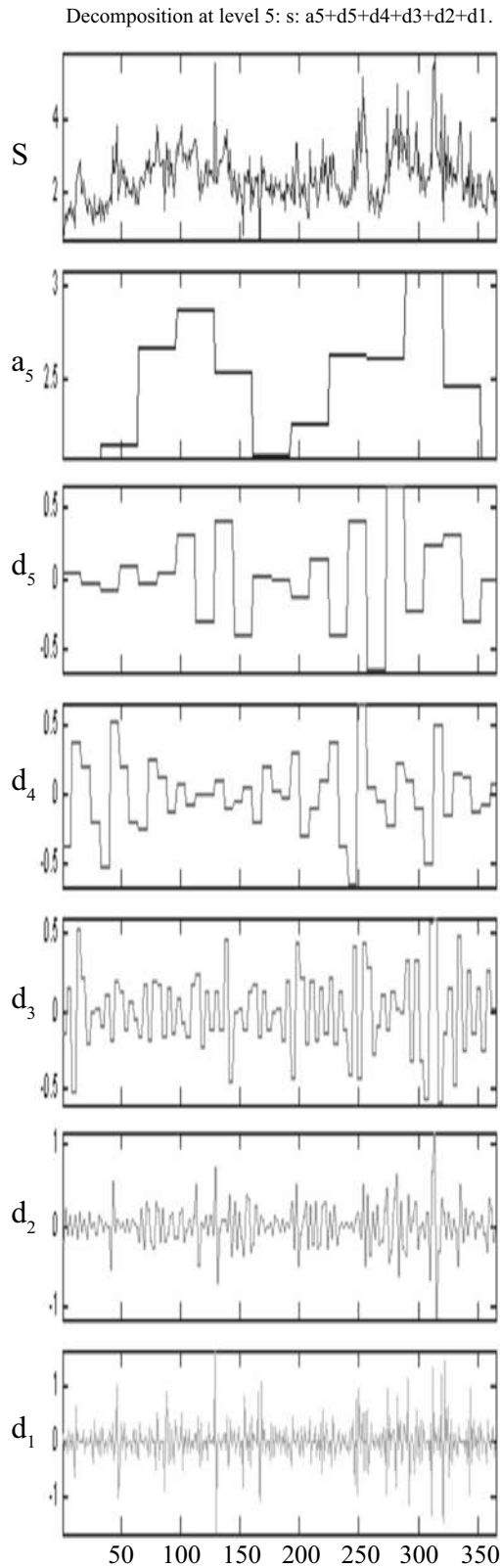
(i)



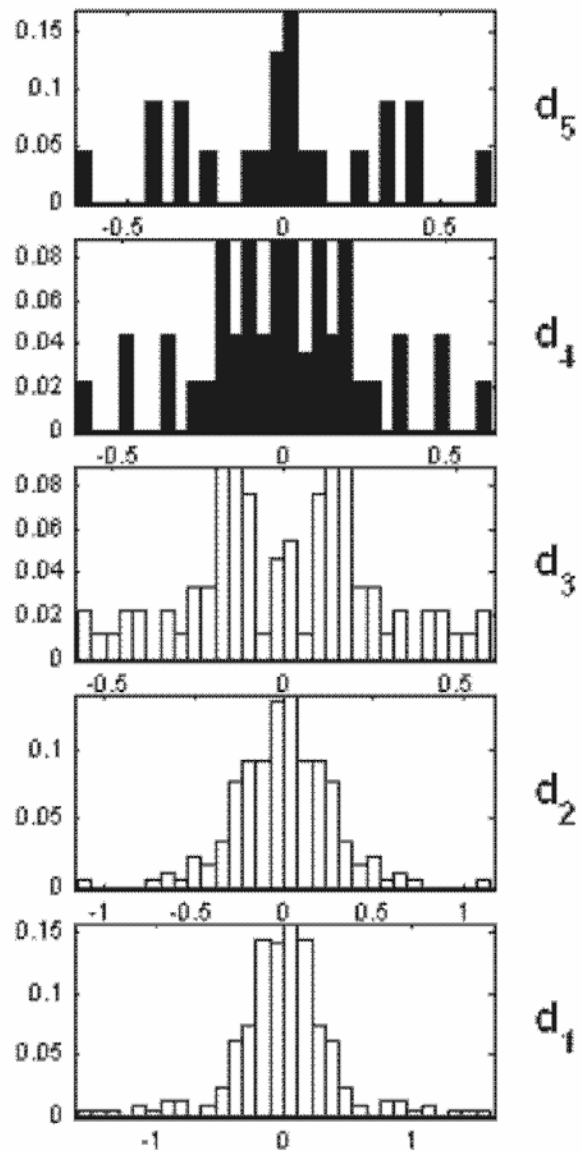
(ii)



**Fig. 5.** Descriptive statistics: (i) Histogram; (ii) Cumulative histogram of ionospheric turbulence at the Pakistan air space.

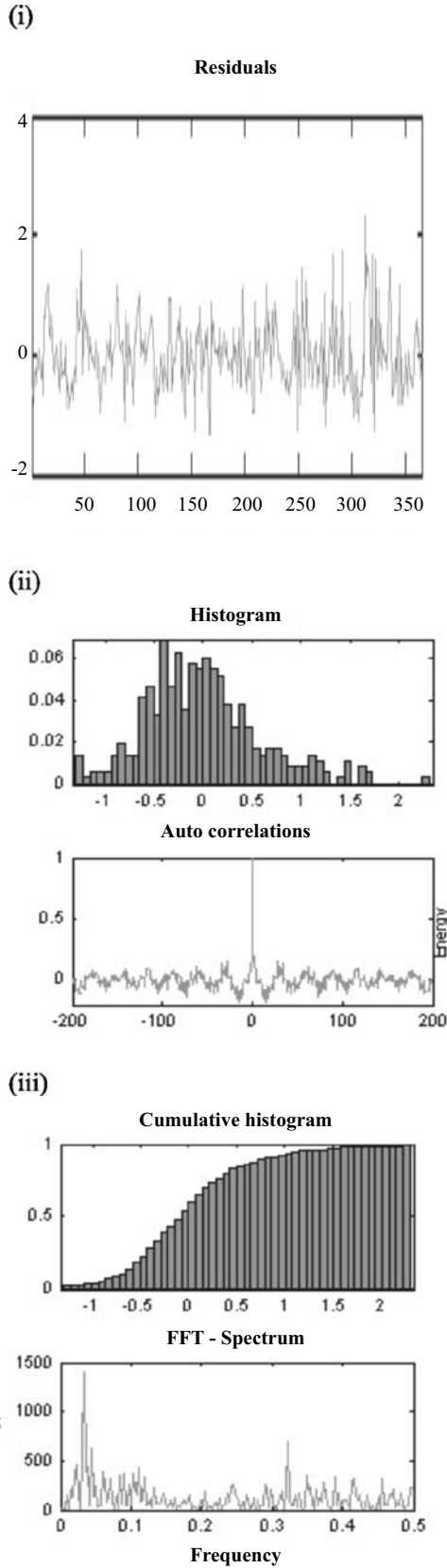


**Fig. 6.** Decomposition at level 5,  $s = a_5 + d_5 + d_4 + d_3 + d_2 + d_1$  of ionospheric turbulence at Pakistan air space.



**Fig. 7.** Reconstructed details analyzed at level 5, residual part of ionospheric turbulence model at Pakistan air space.

cumulative histogram of ionospheric turbulence at Pakistan air space in Fig. 6–8. Wavelet approach is a recently developed signal processing tool enabling us on several timescale of the local properties of complex signals that can present non stationary zone. Wavelet analysis is used to find the mode of variations and also to study how it varies with time by decomposing time series into frequency space. Finally, powerful multi resolution with respect to Haar is carried out. This decomposes actual signal into different signals to be analyzed into principal and residual part. By mathematical expression



$$S = A_j + \sum_j D_j$$

$S, A_j$  and  $D_j$  are the signal, principal part  $j$  level and residual part  $j$  level. Mathematically, the relation between principal and residual part  $j$  level by mathematical is:

$$A_{j-1} = A_j + D_j$$

The wavelet used in the study by Haar of the level 5, the dyadic scale is  $a = 2^j$  for level 5 the resolution is given by  $1/a$ , or  $2^{-j}$ . In order to approach the cyclic study maximum and minimum values, we have carried out wavelet analysis of ionospheric turbulence. In Fig. 6 the variation is presented in the form of different resolutions at level 5 of Haar wavelet type in the detailed and approximated part the cyclic variation is also presented at different level. In the detailed and approximated part at the lowest resolution several peaks are appeared. The expression of Haar wavelet in continuous time series function is represented as follows:

$$\begin{cases} 1; 0 \leq t < \frac{1}{2}t_0 \\ -1; \frac{1}{2}t_0 \leq t < t_0 \\ 0; otherwise, \end{cases}$$

The corresponding wavelet in discrete time series

$$\begin{cases} 1; n = 1 \\ h_n = h_0 \\ -1; n = 2 \\ 0; otherwise \end{cases}$$

The quantity  $h_0 = 1/\sqrt{2}$  is a constant. Fig. 7 shows wavelet 1-D analysis of ionospheric turbulence at Pakistan air space. Present data is non-linear there is local minimum in this period which depicts a positive skewness of frequency histogram:

$$s = a_5 + d_5 + d_4 + d_3 + d_2 + d_1$$

The Fig. 8 manifests the residual, autocorrelation, FFT-spectrum, histogram, cumulative histogram of ionospheric turbulence  $F_2$  layer. For an advance and more recent analysis of ionospheric turbulence at Pakistan air space, between 1 to 365 components data point, we have constructed model of ionospheric turbulence. We have used wavelet transformation which is a common method of analyzing ionospheric plasma turbulence [8-10].

Fig. 8. The graph of residual, autocorrelation, FFT-spectrum, histogram, cumulative histogram obtained from wavelet analysis  $F_2$  layer. for ionospheric turbulence.

**Table 2.** Descriptive statistics, quantities of ionospheric turbulence at Pakistan air space.

|           |     |
|-----------|-----|
| Mean      | 2.5 |
| Median    | 2.3 |
| Mode      | 2.2 |
| Max       | 5.7 |
| Min       | 0.8 |
| Range     | 4.9 |
| St. Dev.  | 0.7 |
| Median AD | 0.5 |
| Mean AD   | 0.6 |

**Table 3.** Descriptive statistics, residual quantities of ionospheric turbulence at Pakistan air space.

|           |        |
|-----------|--------|
| Mean      | -0.001 |
| Median    | -0.051 |
| Mode      | -0.400 |
| Max       | 2.306  |
| Min       | -1.300 |
| Range     | 3.605  |
| St. Dev.  | 0.600  |
| Median AD | 0.338  |
| Mean AD   | 0.450  |

### 3. CONCLUSIONS

In this article we have presented an idea to find out the hidden periodicities in different processes quantifying the ionospheric turbulence that was analyzed in upper layers of ionosphere by spectral analysis, FFT and wavelet transformation one dimension Haar 5-Levels for approximation and decomposition details at the Pakistan air space. The more precise results were obtained using tests of randomness and normality in datasets. These results

might be useful in astrophysical plasma.

### 4. ACKNOWLEDGEMENTS

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