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# Supra $\beta$-connectedness on Topological Spaces

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**Abstract.** In this paper, supra $\beta$-connectedness are researched by means of a supra $\beta$- separated sets.

**Keywords and Phrases:** Supra $\beta$ -separated, supra $\beta$ -connectedness and supra topological space.
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## 1. INTRODUCTION

Some types of sets play an important role in the study of various properties in topological spaces. Many authors introduced and studied various generalized properties and conditions containing some forms of sets in topological spaces. In 1983, Mashhour et al [2] developed the supra topological spaces and studied $S$-continuous maps and $S^*-$continuous maps. We will use the term supra-continuous maps instead of $S$-continuous maps. In 2008, Devi et al [1] introduced and studied a class of sets and maps between topological spaces called supra $\alpha-$open sets and supra $\alpha-$continuous maps, respectively. In 2010, Sayed and Noiri [4] introduced the concepts of supra b-open sets, supra b-continuity, supra b-open maps and supra b-closed maps and studied some of their properties. In [3] the concepts of supra $\beta$-open sets, supra $\beta$-continuity, supra $\beta$-open maps and supra $\beta$-closed maps were introduced and some of their properties were investigated. The purpose of this paper is to introduce the concept of supra $\beta$-connectedness based on supra $\beta$-separated sets. We prove that supra $\beta$-connectedness is preserved by supra $\beta$-continuous bijections.

Throughout this paper, $(X, \tau)$ , $(Y, \sigma)$ and $(Z, \upsilon)$ (or simply, $X$ , $Y$ and $Z$) denote topological spaces on which no separation axioms are assumed unless explicitly stated. All sets are assumed to be subsets of topological spaces. The closure and the interior of a set $A$ are denoted by $Cl(A)$ and $Int(A)$, respectively. A sub collection $\mu \in 2^X$ is called a supra topology [2] on $X$ if $X \in \mu$ and $\mu$ is closed under arbitrary union. $(X, \mu)$ is called supra topological space. The elements of $\mu$ are said to be supra open in $(X, \mu)$ and the complement of a supra open set is called supra closed. The supra closure of a set $A$, denoted by $Cl^\mu(A)$, is the intersection of supra closed sets including $A$. The supra interior of a set $A$, denoted by $Int^\mu(A)$, is the union of supra open sets included in $A$. The supra topology $\mu$ on $X$ is associated with the topology $\tau$ if $\tau \subset \mu$. A set $A$ is supra $\beta$-open [3] if $A \subseteq Cl^\mu(Int^\mu(Cl^\mu(A)))$. The complement of a supra $\beta$-open set is called supra $\beta$-closed. Thus A is supra $\beta$-closed if and only if $Int\mu(Cl\mu(Int\mu(A))) \subseteq A$. The supra $\beta$-closure of a set $A$ [3], denoted by $Cl^\mu_\beta(A)$, is the intersection of the supra $\beta$-closed sets including $A$. The supra $\beta$-interior of a set $A$ [3],denoted by $Int^\mu_\beta(A)$, is the union of the supra $\beta$-open sets included in $A$. Let $(X, \tau)$ and $(Y, \sigma)$ be two topological spaces and $\mu$ be an associated supra topology with $\tau$ . A map $f : X \rightarrow Y$ is called a supra $\beta$-continuous map [3] if the inverse image of each open set in $Y$ is a supra $\beta$-open set in $X$.

The following theorem was given by Ravi et al [3]:

**Theorem 1.1.** *Let* $(X, \tau)$ *and* $(Y, \sigma)$ *be two topological spaces and* $\mu$ *be an associated supra*

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*topology with $\tau$ . Let f be a map from X into Y. Then the following are equivalent:*

*(1) f is a supra $\beta$-continuous map;*

*(2) The inverse image of each closed set in Y is a supra $\beta$-closed set in X;*

*(3) $Cl_\beta^\mu(f^{-1}(A)) \subseteq f^{-1}(Cl(A))$ for every set A in Y;*

*(4) $f(Cl_\beta^\mu(A)) \subseteq Cl(f(A))$ for every set A in X;*

*(5) $f^{-1}(Int(B)) \subseteq Int_\beta^\mu(f^{-1}(B))$ for every B in Y .*

## 2. SUPRA $\beta$-SEPARATED SETS

In this section, we shall research supra $\beta$-separated sets in topological spaces.

**Definition 2.1.** *Let $(X, \tau)$ be a topological space and A,B be two non-empty subsets of X. Then A and B are said to be supra $\beta$-separated if $A \cap Cl_\beta^\mu(B) = \phi$ and $Cl_\beta^\mu(A) \cap B = \phi$.*

The following result is immediate from the above definition:

**Theorem 2.1.** *Let C and D are two non-empty subsets of the supra $\beta$- separated sets A and B, respectively. Then C and D are also supra $\beta$-separated in X.*

**Theorem 2.2.** *Let A,B be two non-empty subsets of X such that $A \cap B = \phi$ and A,B are either they both are supra $\beta$-open or they both are supra $\beta$-closed. Then A and B are supra $\beta$-separated.*

**Proof.** If both $A$ and $B$ are supra $\beta$-closed sets and $A \cap B = \phi$, then $A$ and $B$ are supra $\beta$-separated. Let $A$ and $B$ be supra $\beta$-open and $A \cap B = \phi$. Then $A \subseteq X - B$. So $Cl_\beta^\mu(A) \subseteq Cl_\beta^\mu(X - B) = X - Int_\beta^\mu(B) = X - B$. Hence $Cl_\beta^\mu(A) \cap B = \phi$. Similarly, $A \cap Cl_\beta^\mu(B) = \phi$. Thus $A$ and $B$ are supra $\beta$- separated.

**Theorem 2.3.** *Suppose that A and B are two non-empty subsets of X such that either they both are supra $\beta$-open or they both are supra $\beta$-closed. If $C = A \cap (X - B)$ and $D = B \cap (X - A)$,then C and D are supra $\beta$-separated, provided they are non-empty.*

**Proof.** First suppose $A$ and $B$ are both supra $\beta$-open. Now, $D = B \cap (X - A)$ implies $D \subseteq X - A$. Then $Cl_\beta^\mu(D) \subseteq Cl_\beta^\mu(X - A) = X - Int_\beta^\mu(A) = X - A$. Hence $A \cap Cl_\beta^\mu(D) = \phi$. Therefore $C \cap Cl_\beta^\mu(D) = \phi$. Similarly, $Cl_\beta^\mu(C) \cap D = \phi$. Thus $C$ and $D$ are supra $\beta$- separated.

Next, suppose that $A$ and $B$ are both supra $\beta$-closed sets. Then $C = A \cap (X - B)$, implies $C \subseteq A$. Hence $Cl_\beta^\mu(C) \subseteq Cl_\beta^\mu(A) = A$. Therefore $Cl_\beta^\mu(C) \cap D = \phi$. Similarly, $Cl_\beta^\mu(C) \cap D = \phi$. Thus $C$ and $D$ are supra $\beta$- separated.

**Theorem 2.4.** *Two non-empty subsets A and B of X are supra $\beta$- separated if and only if there exists two supra $\beta$-open sets U and V such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$, $B \cap U = \phi$.*

**Proof.** Suppose that $A$ and $B$ are two supra $\beta$-separated. Now, $A \cap Cl_\beta^\mu(B) = \phi$ and $Cl_\beta^\mu(A) \cap B = \phi$. Then $A \subseteq X - Cl_\beta^\mu(B) = U$ (say); and $B \subseteq X - Cl_\beta^\mu(A) = V$ (say). Since both $Cl_\beta^\mu(A)$ and $Cl_\beta^\mu(B)$ are supra $\beta$-closed, then both $U$ and $V$ are supra $\beta$-open. Therefore $A \subseteq Cl_\beta^\mu(A) = X - V$ and $B \subseteq Cl_\beta^\mu(B) = X - U$. Hence $A \cap V = \phi$ and $B \cap U = \phi$.

Conversely, let $U$ and $V$ be supra $\beta$-open such that $A \subseteq U$, $B \subseteq V$, $A \cap V = \phi$ and $B \cap U = \phi$. Then $X - U$ and $X - V$ are supra $\beta$-closed. Also, $A \cap V = \phi$ implies $A \subseteq X - V$. Thus $Cl_\beta^\mu(A) \subseteq Cl_\beta^\mu(X - V) = X - V$. Hence $Cl_\beta^\mu(A) \cap V = \phi$. Similarly, $U \cap Cl_\beta^\mu(B) = \phi$. Thus $A$ and $B$ are supra $\beta$-separated.

## 3. SUPRA $\beta$-CONNECTEDNESS

In this section, we research supra $\beta$-connectedness by means of supra $\beta$-separated.

**Definition 3.1.** *A subset A of X is supra $\beta$-connected if it can't be represented as a union of two non-empty supra $\beta$-separated sets. When A = X is supra $\beta$-connected, then X is called supra $\beta$-connected space.*

**Theorem 3.1.** *A non-empty subset C of X is supra $\beta$-connected if and only if for every pair of supra $\beta$-separated sets A and B in X with $C \subseteq A \cup B$, exactly one of the following possibilities holds:*

*(a) $C \subseteq A$ and $C \cap B = \phi$,*

*(b) $C \subseteq B$ and $C \cap A = \phi$.*

**Proof.** Let $C$ be supra $\beta$-connected. Since $C \subseteq A \cup B$, then both $C \cap A = \phi$ and $C \cap B = \phi$can not hold simultaneously. If $C \cap A \neq \phi$ and $C \cap B \neq \phi$, then by Theorem 2.1 they are also supra $\beta$-separated and $C = (C \cap A) \cup (C \cap B)$ which goes against the supra $\beta$-connectedness of $C$. Now, if $C \cap A = \phi$, then $C \subseteq B$, while $C \subseteq A$ holds if $C \cap B = \phi$.

Conversely, suppose that the given condition holds. Assume by contrary that $C$ is not supra $\beta$-connected. Then there exist two non-empty supra $\beta$-separated sets $A$ and $B$ in $X$ such that $C = A \cup B$. By hypothesis, either $C \cap A = \phi$ or $C \cap B = \phi$. So, either $A = \phi$ or $B = \phi$, none of which is true. Thus $C$ is supra $\beta$-connected.

**Theorem 3.2.** *The following are equivalent:*

*(1) A space $X$ is not supra $\beta$-connected.*

*(2) There exist two non-empty supra $\beta$-closed sets $A$ and $B$ such that $A \cup B = X$ and $A \cap B = \phi$.*

*(3) There exist two non-empty supra $\beta$-open sets $A$ and $B$ such that $A \cup B = X$ and $A \cap B = \phi$.*

**Proof.** (1) ⇒ (2): Suppose that $X$ is not supra $\beta$-connected. Then there exist two non-empty subsets $A$ and $B$ such that $Cl^{\mu}{}_{\beta}(A) \cap B = A \cap Cl^{\mu}{}_{\beta}(B) = \phi$ and $A \cup B = X$. It follows that $Cl_{\beta}^{\mu}(A) = Cl_{\beta}^{\mu}(A) \cap (A \cup B) = (Cl_{\beta}^{\mu}(A) \cap A) \cup (Cl_{\beta}^{\mu}(A) \cap B) = A \cup \phi = A$. Hence $A$ is supra $\beta$-closed set. Similarly, $B$ is supra $\beta$-closed. Thus (2) is held.(2) ⇒ (3) and (2) ⇒ (1): Obvious.

**Corollary 3.1.** *The following are equivalent:*

*(1) A space $X$ is supra $\beta$-connected.*

*(2) If $A$ and $B$ are supra $\beta$-open sets, $A \cup B = X$ and $A \cap B = \phi$, then $\phi \in \{A,B\}$.*

*(3) If $A$ and $B$ are supra $\beta$-closed sets, $A \cup B = X$ and $A \cap B = \phi$, then $\phi \in \{A,B\}$.*

**Theorem 3.3.** *For a subset $G$ of $X$, the following conditions are equivalent:*

*(1) $G$ is supra $\beta$-connected.*

*(2) There does not exist two supra $\beta$-closed sets $A$ and $B$ such that $A \cap G \neq \phi$, $B \cap G \neq \phi$, $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*

*(3) There does not exist two supra $\beta$-closed sets $A$ and $B$ such that $G \subseteq A$, $G \subseteq B$, $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$.*

**Proof.** (1) $\Rightarrow$ (2): Suppose that $G$ is supra $\beta$-connected and there exist two supra $\beta$-closed sets $A$ and $B$ such that $A \cap G \neq \phi$, $B \cap G \neq \phi$, $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $(A \cap G) \cup (B \cap G) = (A \cup B) \cap G = G$. Also, $Cl_{\beta}^{\mu}(A \cap G) \cap (B \cap G) \subseteq Cl_{\beta}^{\mu}(A) \cap (B \cap G) = A \cap B \cap G = \phi$. Similarly, $(A \cap G) \cap Cl_{\beta}^{\mu}(B \cap G) = \phi$. This shows that $G$ is not supra $\beta$-connected, which is a contradiction.

(2) $\Rightarrow$ (3): Suppose by opposite that there exist two supra $\beta$-closed sets $A$ and $B$ such that $G \subseteq A$, $G \subseteq B$, $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Then $A \cap G \neq \phi$ and $B \cap G \neq \phi$. This is a contradiction.

(3) $\Rightarrow$ (1): Suppose that (3) is satisfied and $G$ is not supra $\beta$-connected. Then there exist two non-empty supra $\beta$-separated sets $C$ and $D$ such that $G = C \cup D$. Thus $Cl_{\beta}^{\mu}(C) \cap D = C \cap Cl_{\beta}^{\mu}(D) = \phi$. Assume that

$A = Cl_{\beta}^{\mu}(C)$ and $B = Cl_{\beta}^{\mu}(D)$. Hence $G \subseteq A \cup B$ and $Cl_{\beta}^{\mu}(C) \cap Cl_{\beta}^{\mu}(D) \cap (C \cup D) = (Cl_{\beta}^{\mu}(C) \cap Cl_{\beta}^{\mu}(D) \cap C) \cup (Cl_{\beta}^{\mu}(C) \cap Cl_{\beta}^{\mu}(D) \cap D) = (Cl_{\beta}^{\mu}(D) \cap C) \cup (Cl_{\beta}^{\mu}(C) \cap D) = \phi \cup \phi = \phi$. Now we prove that $G \subseteq A$ and $G \subseteq B$. In fact, if $G \subseteq A$, then $Cl_{\beta}^{\mu}(D) \cap G = B \cap G = B \cap (G \cap A) = \phi$, a contradiction. Thus $G \subseteq A$. Analogously we have $G \subseteq B$. This contradicts (3). Therefore $G$ is supra $\beta$-connected.

**Corollary 3.2.** *A space $X$ is supra $\beta$-connected if and only if there does not exist two non-empty supra $\beta$-closed sets $A$ and $B$ such that $A \cup B = X$ and $A \cap B = \phi$.*

**Theorem 3.4.** *For a subset $G$ of $X$, the following conditions are equivalent:*

*(1) $G$ is supra $\beta$-connected.*

*(2) For any two supra $\beta$-separated sets $A$ and $B$ with $G \subseteq A \cup B$, we have $G \cap A = \phi$ or $G \cap B = \phi$.*

*(3) For any two supra $\beta$-separated sets $A$ and $B$ with $G \subseteq A \cup B$, we have $G \subseteq A$ or $G \subseteq B$.*

**Proof.** (1) $\Rightarrow$ (2): Suppose that $A$ and $B$ are supra $\beta$-separated and $G \subseteq A \cup B$. Then by Theorem 2.1 we have $G \cap A$ and $G \cap B$ are also supra $\beta$-separated. Since $G$ is supra $\beta$-connected and $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B)$, then $G \cap A = \phi$ or $G \cap B = \phi$.

(2) $\Rightarrow$ (3): If $G \cap A = \phi$, then $G = G \cap (A \cup B) = (G \cap A) \cup (G \cap B) = G \cap B$. So, $G \subseteq B$. Similarly, $G \cap B = \phi$ implies $G \subseteq A$.

(3) $\Rightarrow$ (1): Suppose that $A$ and $B$ are supra $\beta$-separated and $G = A \cup B$. Then by (3) either $G \subseteq A$ or $G \subseteq B$.

If $G \subseteq A$, then $B = B \cap G \subseteq B \cap A \subseteq B \cap Cl^{\mu}_{\beta}(A) = \phi$. Similarly, if $G \subseteq B$, then $A = \phi$. So $G$ can't be represented as a union of two non-empty supra $\beta$-separated sets. Therefore $G$ is supra $\beta$-connected.

**Theorem 3.5.** *Let $G$ be a supra $\beta$-connected subsets of $X$. If $G \subseteq H \subseteq Cl^{\mu}_{\beta}(G)$, then $H$ is also supra $\beta$-connected.*

**Proof.** Suppose that $H$ is not supra $\beta$-connected. By Theorem 3.3 there exist two supra $\beta$-closed sets $A$ and $B$ such that $H \subseteq A$, $H \subseteq B$, $H \subseteq A \cup B$ and $A \cap B \cap H = \phi$. Since $G \subseteq H$, then $G \subseteq A \cup B$ and $A \cap B \cap G = \phi$. Now we prove that $G \subseteq A$ and $G \subseteq B$. In fact, if $G \subseteq A$, then $Cl^{\mu}_{\beta}(G) \subseteq Cl^{\mu}_{\beta}(A) = A$. Therefore by hypothesis $H \subseteq A$ which is a contradiction. Hence, $G \subseteq A$. Similarly, $G \subseteq B$. This contradicts that $G$ is supra $\beta$-connected.

**Theorem 3.6.** *Let $G$ and $H$ be supra $\beta$-connected. If $G$ and $H$ are not supra $\beta$-separated, then $G \cup H$ is supra $\beta$-connected.*

**Proof.** Suppose that $G \cup H$ is not supra $\beta$-connected. By Theorem 3.3 there exist two supra $\beta$-closed $A$ and $B$ such that $G \cup H \subseteq A$, $G \cup H \subseteq B$, $G \cup H \subseteq A \cup B$ and $(G \cup H) \cap (A \cap B) = \phi$. So, either $G$ $A$ or $H$ $A$. Assume $G \subseteq A$. Then $G \subseteq B$ because $G$ is supra $\beta$-connected. Hence $H \subseteq B$ and $H \subseteq A$. Thus $A \cap G \subseteq A \cap B \cap (G \cap H) = \phi$. Therefore $Cl^{\mu}_{\beta}(H) \cap G \subseteq Cl^{\mu}_{\beta}(A) \cap G = A \cap G = \phi$. Similarly, $H \cap Cl^{\mu}_{\beta}(G) = \phi$. This shows that $G$ and $H$ are supra $\beta$-separated, a contradiction.

**Theorem 3.7.** *Let $\{G_i : i \in I\}$ be a family of supra $\beta$-connected subsets of $X$. If there is $j \in I$ such that $G_i$ and $G_j$ are not supra $\beta$-separated for each $i \neq j$, then $\bigcup_{i \in I} G_i$ is supra $\beta$-connected.*

**Proof.** Suppose that $\bigcup_{i \in I} G_i$ is not supra $\beta$-connected. Then there exist two non-empty supra $\beta$-separated subsets $A$ and $B$ of $X$ such that $\bigcup_{i \in I} G_i = A \subseteq B$. For each $i \in I$, $G_i$ is supra $\beta$-connected and $G_i \subseteq A \cup B$. Then by Theorem 3.1 either $G_i \subseteq A$ and $G_i \cap B = \phi$, or else $G_i \subseteq B$ and $G_i \cap A = \phi$. If possible, let for some $r, s \in I$ with $r \neq s$, $G_r \subseteq A$ and $G_s \subseteq B$. Then $G_r$, $G_s$ being non-empty of supra $\beta$-separated sets which is not the case. Thus either $G_i \subseteq A$ with $G_i \cap B = \phi$ for each $i \in I$ or else $G_i \subseteq B$ with $G_i \cap A = \phi$ for each $i \in I$. In the first case $B = \phi$ (since $B \subseteq \bigcup_{i \in I} G_i$) and in the second case $A = \phi$. Non of which is true. Thus $\bigcup_{i \in I} G_i$ is supra $\beta$-connected.

**Corollary 3.3.** *Let $\{G_i : i \in I\}$ be a family of supra $\beta$-connected sets. If $\bigcap_{i \in I} G_i \neq \phi$, then $\bigcup_{i \in I} G_i$ is supra $\beta$-connected.*

**Theorem 3.8.** *A non-empty subset $G$ of $X$ is supra $\beta$-connected if and only if for any two elements $x$ and $y$ in $G$ there exists a supra $\beta$-connected set $H$ such that $x, y \in H \subseteq G$.*

**Proof.** The necessity is obvious. Now we prove the sufficiency. Suppose by contrary that G is not supra $\beta$-connected. Then there exist two non-empty supra $\beta$-separated $P$,$Q$ in $X$ such that $G = P \cup Q$. Choose $x \in P$ and $y \in O$. So, $x, y \in G$ and hence by hypothesis there exists a supra $\beta$-connected set $H$ such that $x, y \in H \subseteq G$. Thus $H \cap P$ and $H \cap Q$ are non-empty supra $\beta$-separated sets with $H = (P \cap H) \cup (Q \cap H)$, a contradict to the supra $\beta-$connectedness of $H$.

**Theorem 3.9.** *If $f : X \rightarrow Y$ is a supra $\beta$-continuous surjective map and $C$, $D$ are supra $\beta$-separated sets in $Y$, then $f^{-1}(C)$, $f^{-1}(D)$ are supra $\beta$-separated in $X$.*

**Proof.** Since $f$ is surjective, then $f^{-1}(C)$ and $f^{-1}(D)$ are non-empty sets in $X$. Suppose by contrary that $f^{-1}(C)$ and $f^{-1}(D)$ are not supra $\beta$-separated sets in $X$. Then $f^{-1}(C) \cap Cl^{\mu}_{\beta}(f^{-1}(D)) \neq \phi$. Since $f$ is a supra $\beta$-continuous map, then by Theorem 1.1 we have $f^{-1}(C) \cap f^{-1}(Cl(D)) \neq \phi$. Thus $C \cap Cl(D) \neq \phi$. Therefore $C \cap Cl^{\nu}_{\beta}(D) \neq \phi$. Similarly, $Cl^{\nu}_{\beta}(C) \cap D \neq \phi$. Thus $C$ and $D$ are not supra $\beta$-separated in $Y$, a contradiction. Hence $f^{-1}(C)$ and $f^{-1}(D)$ are supra $\beta$-separated in $X$.

**Theorem 3.10.** *If $f : X \rightarrow Y$ is supra $\beta$-continuous bijective and $A$ is supra $\beta$-connected in $X$, then $f(A)$ is supra $\beta$-connected in $Y$.*

**Proof.** Suppose by contrary that $f(A)$ is not supra $\beta$-connected in $Y$. Then $f(A) = C \cup D$, where $C$ and $D$ are two non-empty supra $\beta$-separated in $Y$. By Theorem 3.9, we have $f^{-1}(C)$ and $f^{-1}(D)$ are not supra $\beta$-separated in $X$. Since $f$ is bijective, then $A = f^{-1}(f(A)) = f^{-1}(C) \cup f^{-1}(D)$.
Hence $A$ is not supra $\beta$-connected in $X$, a contradiction. Thus $f(A)$ is supra $\beta$-connected in $Y$.

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