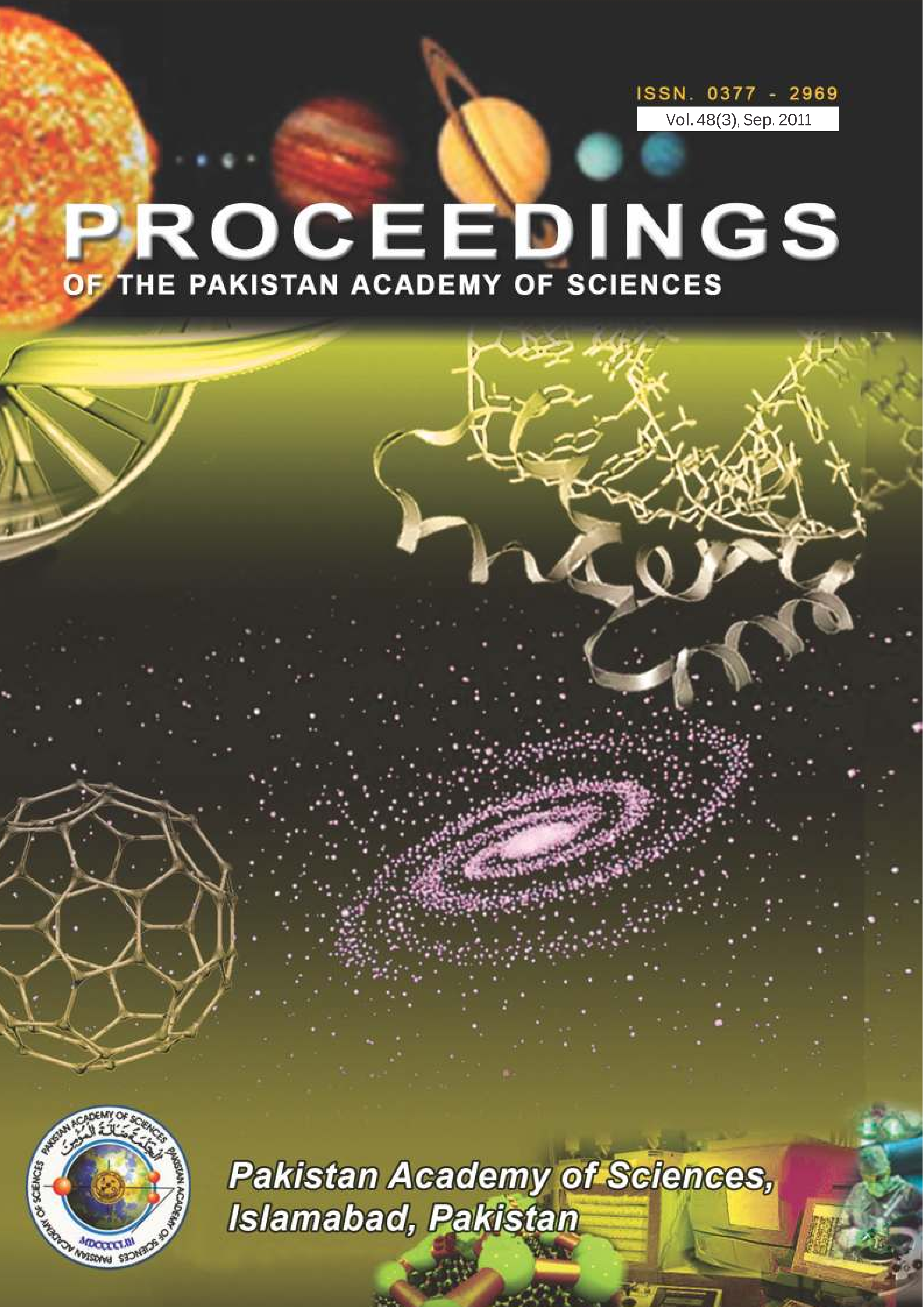

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Original Article

Effect of Bacterivorous and Predatory Nematodes on Macroalgal Detritus Decomposition

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Abstract: Nematodes may contribute to the decomposition of detritus in both aquatic and terrestrial ecosystems. In aquatic sediments, the most prominent role is expected for bacterial-feeding nematodes, since these may in several ways affect the activity and abundance of bacteria, the principal decomposers of organic matter. However, many free-living nematodes have other feeding strategies, and it is unclear whether they may also affect decomposition processes. Predatory nematodes, for instance, may affect the abundance of bacterivores and hence indirectly also of bacteria (i.e. a trophic cascade). This study focuses on the short-term (9 days) effects of the Rhabditis bacteriovorous nematodes *Rhabditis (Pellioditis) marina* and *Panagrolaimus paetzoldi* and of the predatory nematode *Enoploides longispiculosus* in single-species as well as in combination treatments on the decomposition rate of brown algae (*Fucus vesiculosus* L.) detritus through laboratory microcosm experiments. Remarkably, all treatments with the predatory nematode showed higher decomposition rates than treatments without *E. longispiculosus*. Howeve, the mechanism behind this effect remain unclear. In addition, a combination treatment with both bacterivorous species had a higher decomposition rate than treatments with single bacterivore species, inline with other recent studies demonstrating interactions effects between different bacterivore species.

Keywords: Micrococosom, bacteria, nematodes, bacterivores, predators, decomposition, trophic

INTRODUCTION

Nematodes are the most numerous metazoan organisms in the world. They are important components of soil and aquatic sediment communities [1, 2]. They are very abundant in marine and brackish sediments [2]. Many kinds of free-living nematodes feed predominantly on bacteria, and appear mainly in spots with high microbial activity; other species are grazers of microphytobenthos or predators of protozoans and/or small metazoans [3, 4].

Bacterivorous nematodes do not feed directly on soil organic matter but on the bacteria associated with it. Presence of predaceous nematodes may influence decomposition processes through trophic cascades from predators over bacterivores to bacteria [5]. In addition, some predatory nematodes are facultative predators and may complete their diet with, among other things, bacterial food [6, 7]. They may also facilitate the decomposition of complex organic molecules by bacteria through the secretion of digestive enzymes [8]. Bacterivores may stimulate microbial activity and hence organic matter decomposition through microbioturbation, resulting in a higher diffusion rate of oxygen and nutrients [9, 10, 11]. They also secrete mucus which may act as a substratum for specific bacteria [12, 13] and their grazing on bacteria may keep the bacterial community from reaching stationary growth, implying an additional contribution to the re-mineralization of nutrients [9, 14] and the decomposition rate of the organic matter [9, 15, 16]. Because of their intricate interaction with microbes and with the physical environment [17, 18, 19], nematodes are better indicators of the rate of decomposition of organic matter than the abundance of bacteria

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[20]. Increased abundance of bacterivores indicates an abundance of easily decomposable organic matter [21]. Nematodes respond rapidly to changes in their environment. Increased microbial activity leads to an increased proportion of opportunistic bacterial feeders [22]. The rate of litter decomposition may be influenced by soil moisture, temperature, interaction between the sediment microorganisms and fauna involved in the decomposition process, sediment structure [23], the placement of the organic matter [24] and most importantly, the chemical composition of the substratum such as the N and lignin content of the litter [25]. Most of these factors influence nematode populations and activity as well, including their efficiency and contribution to the decomposition process.

While several studies have looked into the role of nematodes in decomposition processes, only few have addressed the importance of inter-specific interactions among nematodes. Those that have done so [5, 26, 27] have largely focused on horizontal interactions, i.e. between species belonging to the same trophic guild, bacterivores. Functional implications of interactions across trophic levels have only rarely been studied. The present study addresses the effects of bacterivorous nematodes, and of their interactions with predatory nematodes, on organic matter decomposition.

MATERIALS AND METHODS

Collection of Algae, Sediment, Nematodes and Bacteria

The brown algae (*Fucus vesiculosus* L.) and sediments were collected from the eastern edge of the Paulina salt marsh, Westerscheldt Estuary, The Netherlands. The nematodes *Rhabditis (Pellioditis) marina* and *Panagrolaimus paetzoldi* were obtained from monospecific laboratory cultures established with nematodes from the same field site [28]. *R. (P.) marina* is a complex of cryptic species, and our cultures correspond to cryptic species PM I [29, 30]. Both *R. (P.) marina* and *P. paetzoldi* belong to the nematode order of the Rhabditida. The predatory (Pre) nematode *Enoploides longispiculosus* was collected live from freshly sampled sediments at the same field site. A bacterial suspension to inoculate all experimental plates was prepared by mixing habitat water, sediment, decomposed and fresh algae leaves, and agar from the nematode cultures. This mixture was filtered several times over Millipore filters (pore diam. 0.8µm) to remove flagellates and other small eukaryotes [11]. The reason for preparing this inoculum of multiple origins is that we wanted to minimize the risk of introducing specific bacterial strains into any treatment when inoculating it with nematodes, which often have bacteria associated with their cuticles. By ensuring that all bacterial strains from the nematode cultures, as well as a wide variety of strains from the field site, were present in the inoculum, the chances of introducing any strain uniquely into one treatment were quite small.

Brown algae were rinsed thoroughly with fresh water and oven dried at 60˚C to constant weight for 48 hrs. The sediment was washed with tap water over a 1 mm sieve and the fraction that was retrieved on a 38-µm sieve was dried overnight at 180 °C.

Experiment Set-up

We had 17 treatments: a control without nematodes (T1), a treatment inoculated with *R. (P.) marina (RM),* one with *P. paetzoldi (PP),* one with both *R. (P.) marina* and *P. paetzoldi*, a treatment with *Enoploides longispiculosus (EL),* one with *E. longispiculosus* and *P. marina*, and one with *E. longispiculosus* and *P. paetzoldi*. There were 3 replicates for each treatment. These microcosms were composed of brown algae (1.2±.0.1g dry weight) on sediment (20±0.5g dry weight), rehydrated with 9±1 ml sterile artificial seawater [31] with a salinity of 20, and inoculated with 500 µl of the above described bacterial suspension (500µl) in Petri dishes (90 mm in diameter). Extra nutrients were added as a 1/1000 (vol/vol) diluted Killian solution [32] in artificial seawater. Each microcosm except for the control, was inoculated with 30 individuals of *Rhabditida* while 20 for predators. Nematodes were handpicked with the tip of a fine needle. Experimental Petri plates were placed in the dark in an incubator at 22 °C for 9 days after which the decomposition of *Fucus* litter was measured.

Analysis of detritus decomposition and nematode abundance

After 9 days of incubation, the entire content of each microcosm was poured on sieves with 1mm and 38µm pore sizes and thoroughly rinsed with tap water. The algal detritus was retained on the 1 mm sieve; all pieces of algal detritus were picked up from the 1 mm sieve with forceps and stored in pre-weighed Petri plates for drying. Sediment with nematodes retained on the 38µm sieve, was thoroughly mixed with 2 liter of tap water and subsequently decanted again over the 38µm sieve. The decanting and sieving was repeated 10 times to ensure the collection of all nematodes. Finally, the material retained on the 38 µm sieve was stored in 4% buffered formalin. All nematodes in these samples were counted under a binocular microscope. The predatory nematode *E. longispiculosus* can be easily recognized from both Rhabditida under the binocular, but differentiation between both Rhabditida is more difficult. Hence, we recorded abundances of predatory nematodes and of Rhabditida, the latter without further species identification.

The partially decomposed brown algal fragments of each microcosm were dried at 60°C until constant weight and their weight was measured. Weight losses of algal detritus (ΔW) were calculated by subtracting the final dry weight of the fragments from the initial dry weight.

$$\Delta W = 100*(dryweight_{initial} - dryweight_{final}) / dryweight_{initial}$$

Data Analysis

Data were analyzed using the statistical software STATISTICA 7 (Statsoft). One-way analysis of variance (ANOVA) was used to test for differences among the treatments. Weight loss data were square root transformed to meet the assumptions of normality and homoscedasticity. When ANOVA indicated significant differences, the Turkey HSD test was used for pairwise post hoc comparisons.

RESULTS

A one-way ANOVA showed significant differences (P=0.004) in decomposition among the treatments (Fig. 1). Surprisingly, the fastest decomposition was found in treatments with predators, both those including predators with bacterial-feeding nematodes and the one with only *E. longispiculosus*. Treatments with only bacterial-feeding nematodes had decomposition rates not significantly different from the control without nematodes, albeit that the treatment with both species of Rhabditida on average had a considerably higher decomposition rate than the control.

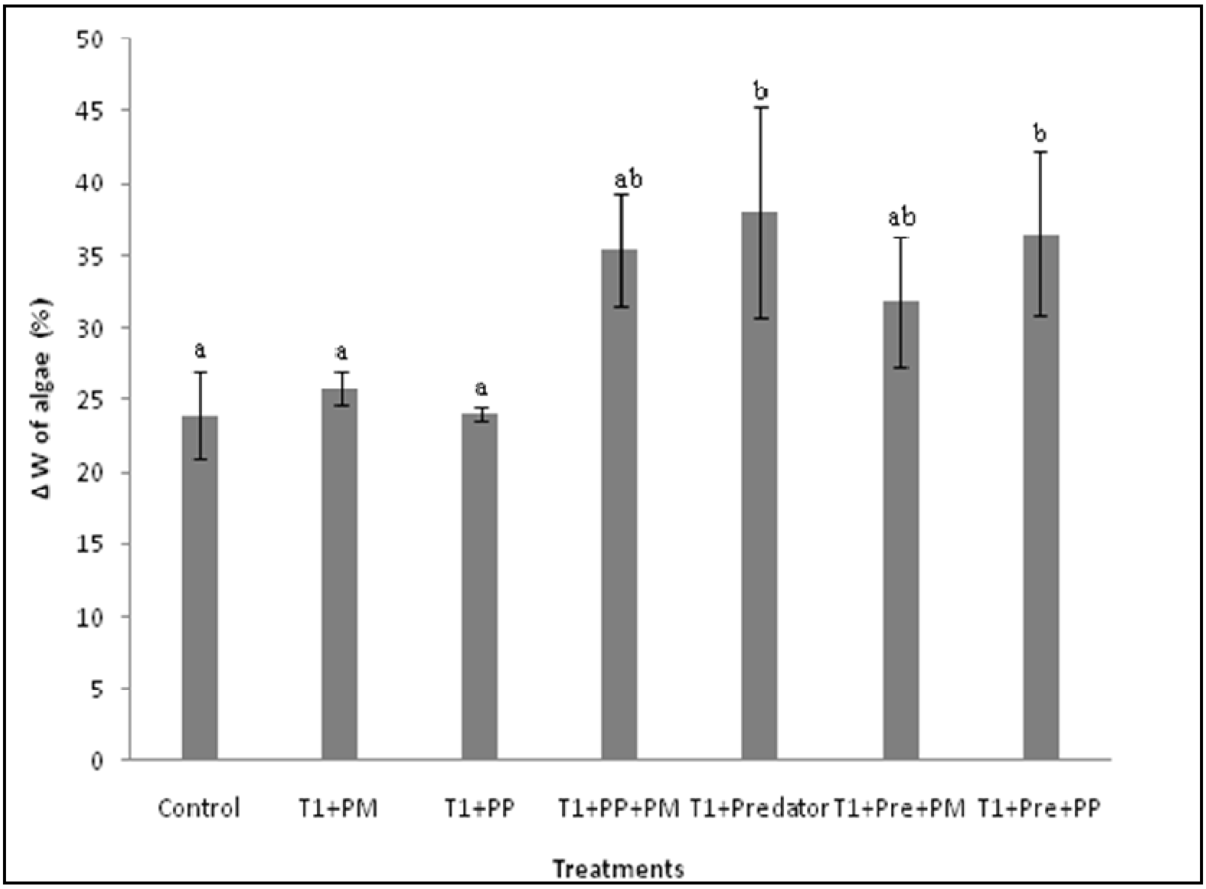

Fig. 1. Weight loss of alge after 9 days of incubation. Data are means ± 1 stdev of 3 replicates per treatment. Treatments sharing the same letter (a, b) did not differ significantly.

DISCUSSION

It is tempting to attribute the higher decomposition rates in predator treatments to a trophic cascade effect, predators depressing bacterivore abundance and thus releasing bacteria from grazing by rhabditid nematodes. However, two arguments invalidate this conclusion: first, the treatment with only predatory nematodes also had a higher decomposition rate than the control treatment. Secondly, numbers of bacterivorous nematodes in our experiment remained low because of the short incubation time, and we consider it unlikely that at such low abundances, they could have depressed bacterial abundance. This is also supported by the absence of differences in decomposition rates between controls without nematodes and treatments with single rhabditid species. Hence, like the only similar study hitherto published, our results do not confirm a clear trophic cascading effect in this experimental detritus food web [33].

The predatory feeding ecology of *E. longispiculosus* has been well-documented [34, 35, 36, 37], whereas Moens et al. [7] were unable to demonstrate any bacterivory in this species. This is further supported by the observation that at the end of our experiment, *E. longispiculosus* in the predator-only treatment had more transparent guts than in the predator + bacterivores treatment, which suggests lack of feeding activity. Hence, we expected that the decomposition rate of the predator-only treatment would be similar to that of the control, but this was clearly contradicted by our results. Since we used a bacterial inoculum with very diverse strains, it is unlikely that we systematically introduced one or more new bacterial strains in all the treatments with predators. We also did not find evidence of co-transfer of flagellates into the predator microcosms, although we cannot fully exclude this possibility. Many flagellates are bacterial feeders and may thus affect microbial activity [38]. Nematodes may, however, also indirectly affect microbial activity and community composition, for instance through the production of mucus tracks which may favour the settlement and growth of specific types of bacteria [13], or through the secretion of digestive enzymes which may initiate substratum decomposition and thus facilitate growth of bacteria [17].

The observation of a generally higher decomposition rate in the treatment with both species of Rhabditida combined as compared to single-species treatments, suggests a possible complementarity between both species of bacterivores. Even between closely related nematode species, different feeding preferences for different types of bacteria exist, which may result in differential effects on the microbial community [37, 38]. Perhaps as a result of this, but also as a result of intricate interactions between species of bacterivorous nematodes, combinations of two or more bacterivore nematode species elicited different effects on the decomposition of cordgrass (*Spartina*) detritus than did monospecific treatments [27]. We suggest that the same mechanisms may be responsible for the result of our experiment, but further work, including a detailed analysis of nematode population growth and microbial community structure over different time intervals, is needed to confirm this hypothesis.

The absence of a significant effect of single rhabditid species on decomposition rate may result from the relatively short incubation time of this experiment. Even though under the temperature conditions of our experiment, generation times of *R. (P.) marina* and *P. paetzoldi* are considerably shorter than 1 week, we did not observe a substantial population growth of either species in our experiment. Since we expect that effects of bacterivores on detritus decomposition are mainly through their grazing on bacteria [9, 39] and through microbioturbation [11], such effects are likely heavily dependent on the abundance of bacterivorous nematodes [39], and the limited duration of our experiment did not allow a sufficient population build-up.

Decomposition process in aquatic environment may be influenced by many other factors like moisture, temperature etc, and interaction between the other microorganisms and fauna involved in, the placement of the organic matter [24] and, most importantly, the chemical composition of the substrate such as the N and lignin content of the litter. Most of these factors influence nematode populations and activity as well, including their efficiency and contribution to the decomposition process. So microcosm experiment can give an idea about nematodes role and their interaction but to know the real fact *in vivo* is the best to conduct experiment.

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Original Article

Evaluating Drought Impact on Vegetation Cover of *Rarkan Rod-Kohi* Area, Balochistan using Remote Sensing Technique

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Abstract: The impact of the 1998-2004 prolonged drought was studied on the vegetation cover of Rarkan Rod-kohi area, Balochistan through using multi-sensor remote sensing data in Geographic Information System (GIS) environment. The satellite image data of drought (2001) and post drought (2006) periods were analyzed in order to assess change in landuse and vegetation cover through hybrid (visual and digital) interpretation technique. The results of the study show a distinct change in vegetation cover of Rarkan area in Balochistan in response to the last drought occurred during 1998-2004. The change is indicative from 11 percent decrease in coverage of shrubland and conversion of 12 percent rangeland into bare soil area due to shortage of water resource in the drought period. Although, water availability is one of the limiting factors for low vegetation growth in the dry condition, other factors like woodcutting and over grazing might had contributed to the change in vegetation cover. Regular monitoring and reliable prediction of climate are required for better risk mitigation and development of water management strategies in order to cope with the drought impacts in Balochistan. The study would provide base for developing better strategies for monitoring drought impacts for future landuse planning and adaptation.

Keywords: Remote sensing, vegetation cover, drought impact, Balochistan

INTRODUCTION

The Rod-kohi (hill-torrent) region in Balochistan is characterized by low rainfall, extreme drought and flood conditions. Balochistan was the worst drought-affected province in Pakistan during 1998-2004. About 88 percent of it was directly affected by drought worsening significantly its economic situation. Droughts are caused due to lack of precipitation during a length of period. It is an episode of unusually low precipitation that causes damage to agriculture, ecosystems, and freshwater supplies [1]. Drought is a regional phenomenon whose characteristics vary from one climate regime to another [2]. Its severity would depend on the duration, moisture deficiency, and size of the area affected. In drought years vegetation growth gets limited and chances of occurrence of sparse vegetation also become less. Although, availability of water is the foremost factor governing the land use, but factors like soil conditions and socio economic conditions of an area may also influence the land use to some extent. Although droughts can persist for several years, even a short, intense drought can cause significant damage and harm the local economy.

Remote sensing platforms can provide large amounts of data quickly and inexpensively relative to other means of collection. The spatial and spectral resolution provided by the polar orbiting earth resources satellites such as the Landsat and SPOT have been proven to be of immense use in monitoring agricultural condition [3]. Further GIS can bring together vast amounts of information from a wide variety of sources and make the information quickly visible and applicable in emergency situations [4]. Mostly the data is available in digital format i.e. remote sensing images, digital elevation model (DEM), field data of Global Positioning System (GPS), data integration is a common method used for interpretation and analysis. Environmental, urban, precision farming, and agriculture are applications that have benefited

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from this integration [5] and [6]. Landuse is one area where remote sensing has become indispensable not only for baseline mapping but also for repeated measurement to monitor changes [7]. Attempts had been made by different scientists to study risks of drought at micro and macro levels using remote sensing technique [8], [9] and [10]. In the present study, multi-sensor remote sensing data is analyzed to assess landcover changes occurred due to impact of prolonged drought in the Balochsitan area. The study would provide base for future risk assessment of droughts and effective landuse planning and adaptation for risk mitigation in the area.

Drought in Balochistan

Below normal rainfall in December/January of years 1997-98 and 1999-00 resulted in emergence of severe drought conditions in Balochistan area. The province received almost 50 percent of the normal rains during summer but winter rains turned out to be only 37 percent of the normal. Since the end of March 2000, the entire Balochistan province was experiencing drought. About 2.18 million livestock died due to shortage of supply of fodder, drying up of water reservoirs and vanishing of grasslands in the area. Since livestock is the major source of income of the province mass killing of livestock due to drought badly affected the economy over a long period of time. A total of 1.973 million acres of cultivable land were also affected by the drought [11]. Dry conditions continued to prevail during Apr-May 2003 over most of the southern half of the country. However During June 2003, moderate to heavy showers were recorded over most of Sindh, Punjab and eastern Balochistan. Above normal rains over most of Punjab, NWFP and upper parts of Sindh during April-Jun 2004 compensated the deficit of winter 2004 rains in these areas but eastern Sindh and most of Balochistan remained dry that aggravated the situation in the area [11]. Largely below than average rainfall during July 2004 over monsoon belt had created an alarming situation and drought like conditions had emerged over eastern Sindh and most of the Balochistan. In these arid and semi-arid regions, rainfall is usually insufficient to support dryland and irrigated crops. Much of the soil is lost because of surface run-off or rapid evaporation due to high temperature [12]. This causes drought, which is coupled with prolonged dry seasons, influences growth, and physiological activities of plants [13]. There is need to monitor such changes in landuse and vegetation in order to adopt effective management options for drought risk mitigation in future.

The Study Area

The Rarkan Rod-kohi area is located within longitudes 69° 44' 36" - 70° 01' 35"E and latitudes 29° 59' 39" - 30° 21' 16"N in the

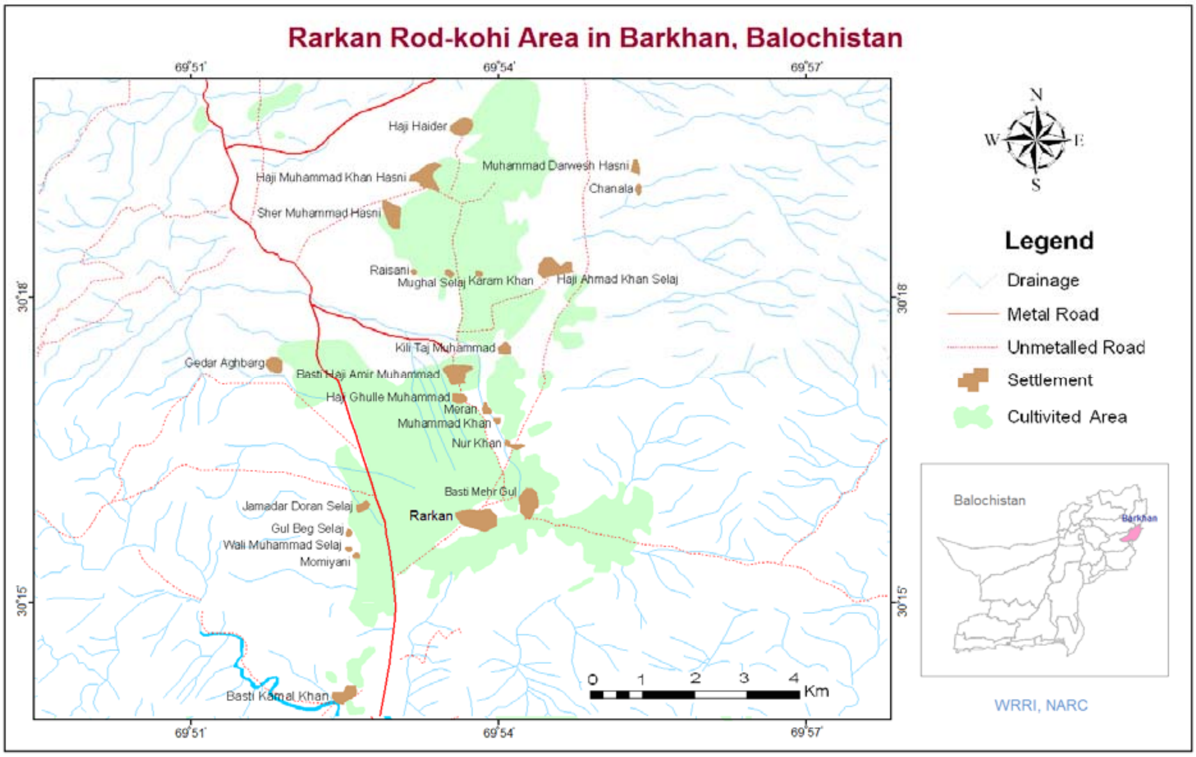

Fig. 1. Location map of Rarkan Rod-kohi area, Balochistan

eastern part of Barkhan district of Balochistan (Fig. 1). It stretches over an area of about 776 sq. km within elevation range of 1052 meters and 1853 meters above mean sea level (Fig. 2). The area is characterized by numerous dry nullahs and intermittent streams exhibiting mostly dendritic type of drainage pattern. The soils are excessively drained, moderately deep to shallow over a stone or gravel bed. Gravelly clay loams or sandy clay loams are most common. The irrigation sources are not reliable. The duration and quantum of available flows entirely depend upon local precipitation. The area is mostly dry with implications on environmental fragility, minimum recharge of aquifers and slow vegetative recovery. During a year of low rainfall, groundwater table goes enormously low which affects the quantity as well as quality of the agricultural produce. The main sources of income are livestock and agriculture production based on perennial flows and hill-torrent flood irrigation. The population density is very low due to the mountainous terrain and scarcity of water. More than 90 percent of the population is rural.

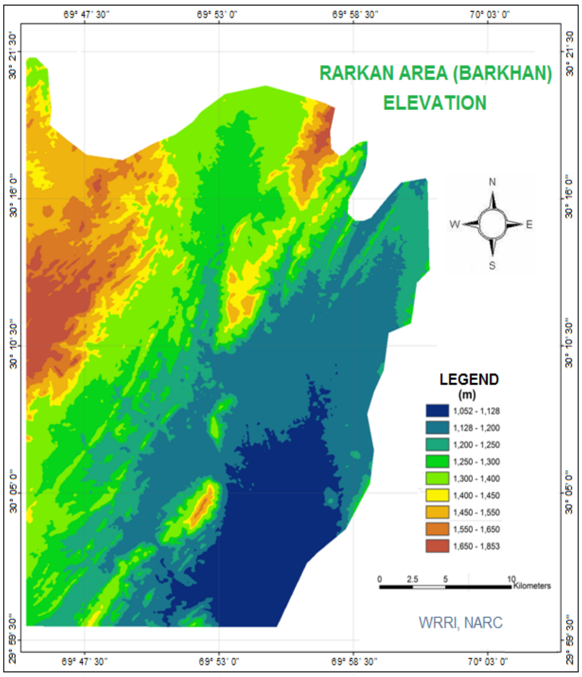

Fig. 2. Elevation map of the study area.

Climate

The climate of Barkhan is moderate and not very hot in summer. The winters are cold, especially when the winds blow from north-west. Precipitation has two peaks in a year i.e. the monsoon in summer and western storms in winter. The average annual rainfall in the district is about 403 mm measured over period of 1967-2006. The mean monthly rainfall is maximum in July while it is minimum in the month of November (Fig. 3a). The maximum annual precipitation recorded was 784 mm received during 2006 and maximum monthly rainfall was about 294 mm received in the month of August, 1984.

The temporal variability of annual rainfall and temperature was analyzed at Barkhan Meteorological Station lying nearest to the study area. The high variability in rainfall at Barkhan is indicative of uncertainty in rainfall occurrence. Although an overall rising trend in annual rainfall was observed at Barkhan during 1967-2006 period, but the rainfall pattern from 1990 onward had indicated a downward sub trend (Fig. 3b). A cycle of depression in rainfall data indicative of last drought period was prominent during 1998-2004 period. This shows that drought had started prevailing in this area during 1998 year. The average annual temperature during 1967-2006 was 21.9°C while average monthly minimum and maximum temperatures were 10.2°C and 31.2°C, respectively. January is usually the coldest and June the hottest months of the year.

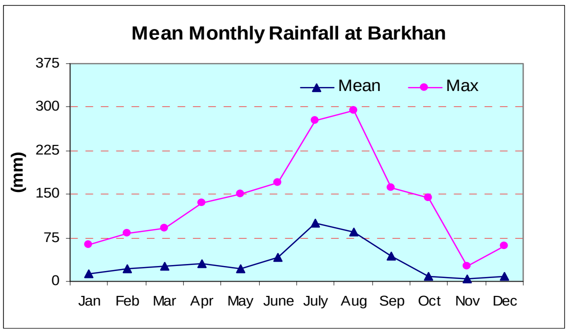

Fig. 3a. Mean monthly rainfall (max and mean) at Barkhan during 1967-2006.

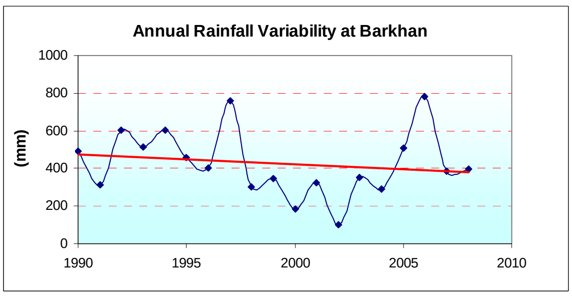

Fig. 3b. Variability of annual rainfall at Barkhan.

Agriculture and Landuse

In nearly level to gently sloping parts of alluvial deposits, favorable temperature throughout the year makes the area suitable for crop production. The irrigated area is of a minor extent only, mostly occurring along streams or where the sub soil water is available at shallow depths. Wheat, maize, vegetables, chilies, melon and fruits orchards are the important crops under irrigation. The area under dry farmed agriculture fluctuates from year to year depending upon adequacy and timing of the precipitation. Yields also vary greatly and only in years of exceptionally good rainfall, really satisfactory crops are produced. Wheat is the main dry-farmed crop. The yields of various crops in different periods are shown in Fig. 4. The wheat yield had indicated a decline during the prevailing drought period. On contrary, the cotton had shown an increase in yield during that period. The crop data of jawar had shown more or less uniform trend in yields in different periods.

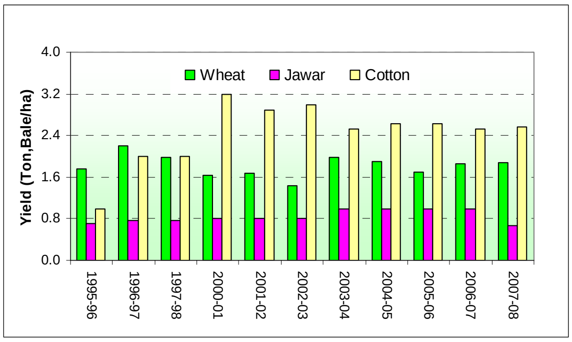

Fig. 4. Variations of major crops yield in Barkhan during 1995-2008.

In Barkhan, the rangeland and exposed rocks were found collectively over 0.3 M.ha [14]. The forest cover exists in scattered form over 0.01 M.ha only. The area generally supports a mixed vegetation dominated by low shrubs comprising: *Cynodon dactylon (khabal), Desmotachya bipinata (dab) Peganum harmala (harmal, spand), Haloxylon griffithii (lana), Zizyphus nummularia (malla), Rosa beggeriana (surai), Caragana ambigua (makhae), Nannhorhops ritehiana (mazri), Acacia modesta (phulai), Acacia arabica (Kandero) and Saueda fruticosa (zamai, lani)* [15]. The natural vegetation provides an ample source for rearing livestock which is a good source of income for the local community. The exposed rocks were found in the mountainous terrain over 0.12 M.ha. The cropped area and bare soil covers about 0.02 Mha each in this district.

Water Resources

Precipitation is the only source of moisture in the area. Due to dissected nature of terrain, unfavorable aquifer conditions and a well developed drainage system, a large part of the precipitation water drains out of the area. The ground water reservoir capacity as well as recharge is therefore limited. The main sources of water supply are channels diverting water from streams, wells/ tubewells, Karezes, and springs. River floods are not very common. During heavy rains main streams build up huge discharges. The groundwater exists in pockets of limited extent within and close to the alluvium deposits.

MATERIALS AND METHODS

Data Used

The satellite image data of SPOT XS (February 5, 2001) and SPOT-5 (January 6, 2006) acquired from SUPARCO, Islamabad were used as primary data in the present study. The SPOT XS image has three spectral bands with spatial resolution of 20 meters while SPOT-5 has four spectral bands with spatial resolution of 2.5 meters. The secondary data includes topographic maps of 1:50,000 scale acquired from Survey of Pakistan (SoP); soils and landform data/maps from Soil survey of Pakistan; crop data (area and production) of major crops from Agricultural Statistical Division of Pakistan and climatic data of Barkhan acquired from Pakistan Meteorological Department (PMD), Islamabad. The GPS survey was carried out to collect ground control points (GCPs), landcover/landuse information, socioeconomic data to support landcover analysis in the study.

Methodology

The drought and post drought images of Rarkan Rod-kohi area, Barkhan were processed to study the response of vegetation to the drought condition. The post drought period of 2006 was selected as base to analyze the vegetation change in the drought period. The reason for selecting post drought year as base includes availability of landcover information collected through field survey conducted during January 2006 in the study area. The GIS was used to develop geo-databases and spatial data layers of infrastructure, topography, hydrology, soils and landuse etc. in ArcGIS 9.3 software. The remote sensing image data and thematic maps were geo-referenced using Geographic coordinate system i.e. Lat./Long. WGS84. The multi-sensor SPOT image data were resampled to a common resolution of 20m for consistency during image interpretation and analysis. The ancillary data of topography and thematic maps and statistical data was used to supplement the RS image processing and analysis.

The satellite image data was analyzed for spatial variability of landuse through hybrid (visual and digital) interpretation technique. The visual interpretation was performed for qualitative analysis while digital interpretation for quantitative analysis of the image data. For image classification, unsupervised classification was used initially to develop classes identified by the computer on the basis of similar spectral characteristics of the group of pixels. The signatures were evaluated using contingency matrix. Maximum likelihood rule was followed for supervised classification of the image. Preprocessing analysis is required before implementing change detection algorithms [16]. In this respect, the precise registration of multi-temporal images was applied to make sure that they properly align to each other. Afterwards, the created thematic maps were compared by a pixel-by-pixel comparison to detect changes. Generally two basic methods are adopted for change detection studies. The first one is detection of changes in the raw images (pixel-to-pixel comparison) and the other is comparing two classified images (post-classification comparison) [17]. This paper involves detection of change by comparing two independently classified landuse/landcover maps from multi-date images of the study area. The principal advantage of post-classification comparison lies in the fact that the bi-temporal images are separately classified, thereby the problem of radiometric calibration between dates is minimized [18]. The temporal images were separately classified using maximum likelihood technique. The thematic maps were then compared pixel by- pixel basis using matrix and than recoding functions of the ERDAS software.

RESULTS

The rainfall data available for the meteorological station in Barkhan was evaluated for the identification of meteorological drought conditions. The percentage deviation of annual rainfall from average rainfall was found predominant on the negative side during 1998-2004 indicating a prolong drought condition (Fig. 5). The decline in annual rainfall was maximum i.e. over 75% in the year 2002 followed by 54% during 2000. During 2001, the shortfall in rainfall was 19% while it was about 13% each during years 1999 and 2003. The drought, which was started with 25% decline in average rainfall in 1998, faded off with an over average rainfall shower during 2005.

In high resolution RS image data of SPOT-5 (2.5 m resolution) of 2006 period, crop cover in the fields were observed in different shades of fresh green color when using false color composite of 4, 1, 2 (RGB). The image resolution provides adequate information of the geometry and size of the fields. The density and vigor of the crop cover like of wheat was seen in medium green color in the image. The bluish green color in some fields indicated presence of high soil moisture or irrigation water. The well irrigated fields are visible in homogenous patches of crop cover formed nearby the source well(s) in the study area.

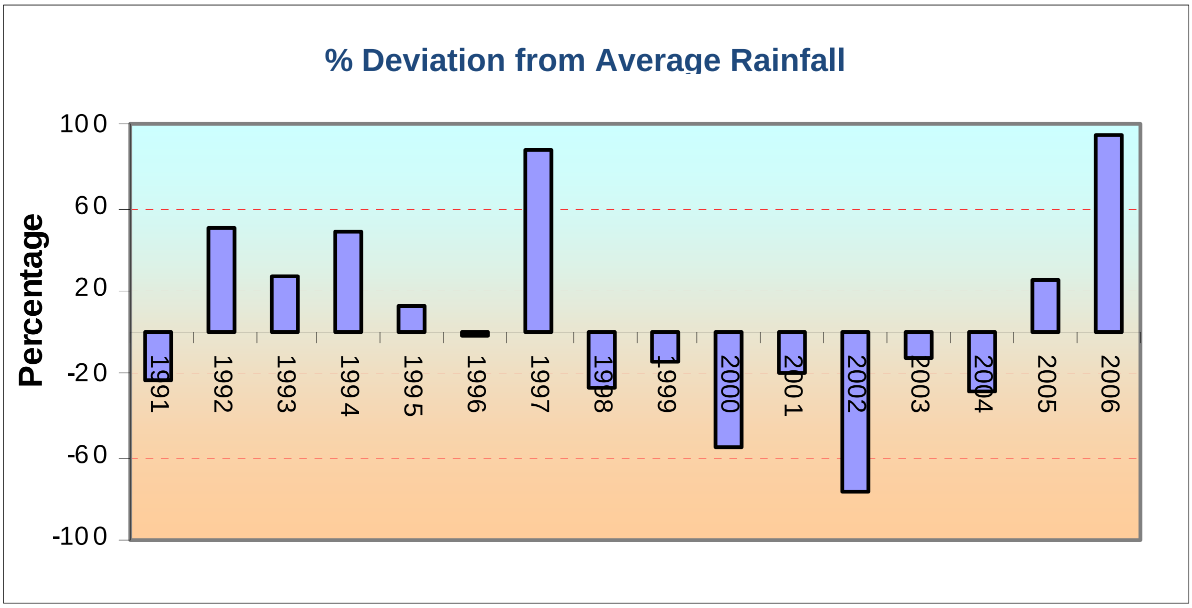

Fig. 5. Deviation of annual rainfall from average rainfall during 1990-2006 at Barkhan, Balochistan.

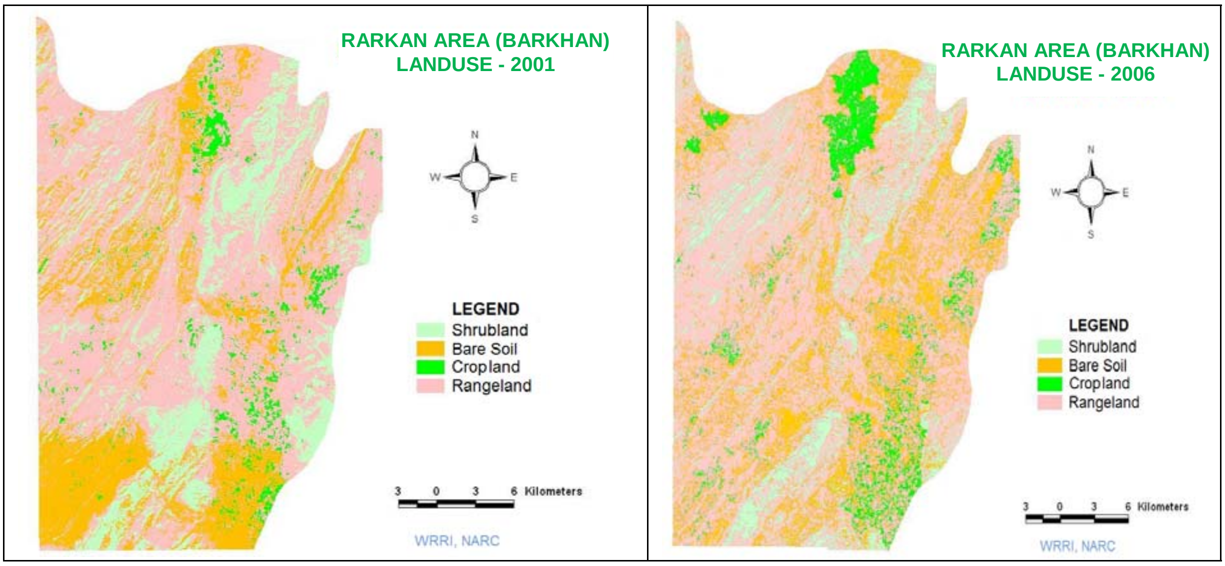

Fig. 6. Landuse maps of Rarkan area indicating landuse change during 2001-2006.

The image of 2001 representing drought condition indicated shrubland over 17% area (Table 1, Fig. 6). This area is dominantly used for grazing and some fuel wood production. The rangeland was dominant over 59% area mainly over level to sloping parts of low hills, alluvial fans and parts of piedmont plains and basins. The vegetation distribution is controlled by the physiographic position and associated environmental conditions especially climate. Climatic conditions favor plain growth round the year. The cropland was found over 3.4% area (Plate 1). The bare soil or culturable waste land was prominent over 21% area during 2001. This land exists generally over piedmont plains and near alluvium fans. Here the soils are generally deep to shallow, calcareous, non-gravelly loamy, silty and clay loams. The culturable waste land can be developed for cultivation through adopting appropriate flood management techniques i.e. diverting excess runoff for irrigation use.

Plate 1. Cropped Area in Rarkan, Barkhan in January 2006.

The rangeland had indicated increase up to 62.4% in the post drought year 2006 (Table 1). Similarly cropland had shown increase up to 6.2% during this period. On the other hand shrubland had shown a decrease in area from 17% to 13% which may be attributed to factors like under growth during drought condition, over grazing and rapid wood cutting etc. The bare soil had indicated a reduction of about 2.4% in 2006, which may change to vegetation class i.e. shrubs/grass/cropland as normal rains started after diminishing of the drought condition. The situation was validated from the landcover information collected during field survey in the target area in early 2006. The interchange of different landuse classes is shown in Fig. 7.

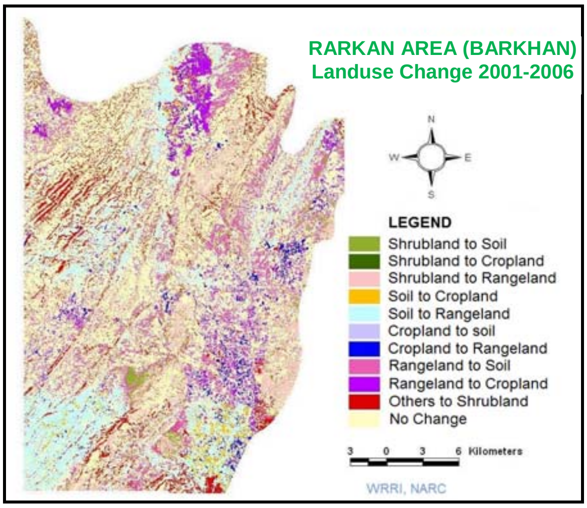

Fig. 7. Landuse change in Rarkan Rod-kohi Area, Barkhan during 2001-2006.

Table 1. Change in landuse between 2001 and 2006.

S. No.	Class	2001		2006		Change	
		Area (ha)	Area %	Area (ha)	Area %	ha	%
1	Shrubland	13219.3	17.0	9920.2	12.8	-3299.1	-4.3
2	Bare soil	16293.4	21.0	14420.0	18.6	-1873.4	-2.4
3	Cropland	2606.9	3.4	4811.6	6.2	2204.6	2.8
4	Rangeland	45486.6	58.6	48454.4	62.4	2967.8	3.8
	Total	77606.2	100.0	77606.2	100.0		

Landuse changes are occurred in 50 percent of the study area (Table 2). About 12 percent of the rangeland changes to bare land during the drought condition in 2001. Similarly about 2 percent of the shrubland was converted into bare soil area during this period. About 9 percent of the shrubland was changed into rangeland due to factors like wood cutting, over grazing or water shortage. In the post drought year 2006, about 13 percent of the bare soil land was recovered by rangeland. Similarly, about 1.7 percent of the soil area was converted into cropland indicating availability of sufficient water for irrigation use. Furthermore, about 7 percent of land under different landuse classes were changed to shrubland probably being saved from outside influential factors like over grazing and wood cutting, etc or availability of sufficient water for plant growth.

Table 2. Detail of changes in landuse between 2001 and 2006.

S. No.	Landuse Change	Area (ha)	Area (%)
1	Shrubland to Soil	1497.0	1.9
2	Shrubland to Cropland	45.1	0.1
3	Shrubland to Rangeland	7002.4	9.0
4	Soil to Cropland	1302.8	1.7
5	Soil to Rangeland	10430.1	13.4
6	Cropland to Soil	333.7	0.4
7	Cropland to Rangeland	1287.6	1.7
8	Rangeland to Soil	9130.4	11.8
9	Rangeland to Cropland	2512.8	3.2
10	Others to Shrubland	5283.6	6.8
11	No change	38780.7	50.0
	Total	77606.2	100

DISCUSSION

The present study shows a decrease of about 11 percent in coverage of shrubland and conversion of 12 percent rangeland into bare soil area during drought year 2001 in the target area. This is in coherence with the findings of [19] that indicated a decline of 7 percent in seasonal vegetation of winter and 33 percent in seasonal vegetation of summer in Balochistan during drought year 2001 when compared with post drought period of 2005. Although in that study, coarse resolution NOAA data was used for landcover analysis but it provide general picture of vegetation behavior on regional scale in Balochistan during the drought period. According to Sajjad and Raza [11], the most severe drought had changed the productive lands into barren land. Although, water availability is one of the limiting factors for low vegetation growth in the dry condition, other factors like woodcutting, over grazing might had contributed to the change in vegetation cover. Rapid evaporation from the soils due to high temperatures coupled with prolonged dry seasons might have influenced the growth of plants [13]. Overall the drought 1998-2004 negatively affected the seasonal rainfall and vegetation cover in Balochistan. The improvement of seasonal vegetation in the post drought period from 2005 onward can be attributed to the reduction of livestock during the drought period as seven million heads of livestock in Balochistan had been severely affected and nearly two million perished during the dry period [20]. Similarly, other factors of vegetation recovery might include non-disturbance due to migration of people to other places and commencement of normal rainfalls after a prolonged drought period.

CONCLUSION

The results of the study show a distinct change in vegetation cover and landuse of Rarkan Rod-kohi area in response to the last drought occurred during 1998-2004. New scientific technologies of satellite remote sensing and geographical information systems (GIS) can be effectively utilized not only for forecasting and monitoring drought but also its impacts in the country. Regular monitoring and reliable prediction of climate are essential for development of better water management and risk mitigation strategies in order to cope with future impacts of drought in Balochistan. The option of developing storage ponds and reservoirs can provide solution to store excessive runoff/rainwater for irrigation and domestic use during the drier period. Although groundwater resource is limited in Balochistan area, yet effective use of this commodity through adopting high efficiency irrigation techniques like drip-irrigation system can prove helpful in raising vegetables and orchards on profitable basis. Spatial analysis in GIS can lead to a decision support system for the concerned government departments and NGOs to help reduce drought risk in the drought prone areas.

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Original Article

Biocontrol of Diamondback Moth *Plutella xylostella* L. (Lepidoptera: Plutellidae) Larvae – A Serious Pest of Cruciferous Vegetables

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Abstract: Entomopathogenic bacteria *Xenorhabdus nematophila* (*Enterobactriaceae*) from *Steinernema carpocapsae* (Rhabditida: Steinernematidae) and its toxic metabolites in broth and water suspensions were mixed in artificial diet and applied on Chinese cabbage leaves to control the larvae of diamondback moth (DBM) *Plutella xylostella*. All treatments were found effective but bacterial cells suspended in broth were slightly more lethal to DBM larvae. All the different DBM larval instars tested were susceptible to the lethal effects of the bacteria and their metabolites. Higher mortality was found in artificial diet than on Chinese cabbage leaves. Dried cells and their metabolites were less effective in dry conditions as compared to high moisture conditions. Fresh metabolites were found more effective as compared to stored metabolites when mixed in artificial diet than on Chinese cabbage leaf foliage. Their effectiveness degraded after few weeks in storage. These results suggest that DBM larvae can be controlled with bacterial cells or their toxic metabolites. The lethal potential of these bacteria can be integrated in integrated pest management programs (IPM) against diamondback moth larvae.

Keywords: Artificial diet, broth, cells, Chinese cabbage, diamondback moth, entomopathogenic bacteria, larvae, toxins, water

INTRODUCTION

Diamondback moth (DBM), *P. xylostella* L. (Lepidoptera: Plutellidae), is a notorious pest of cruciferous vegetables in many parts of the world. To control DBM, large amounts of insecticides are applied. However, DBM has developed resistance to many insecticides especially in intensive vegetable growing areas and only a few products remain that can control this insect pest. The resistance covers all major classes of chemicals [1] including the microbial insecticide *Bacillus thuringiensis* [2]. Products such as new insect growth regulators (IGRs) and *B. thuringiensis* used in this IPM package were safe for natural enemies, but within few years, resistance to these products was reported in DBM. The demise of these products puts at risk an as yet-unrealised IPM solution to DBM management [3]. Tanaka [4] has mentioned that diamondback moth is becoming increasingly difficult to control in the tropics.

The main drawbacks in insecticidal control of DBM, are development of insecticide resistance, resurgence (of the insect pest population after applications of insecticides) and non-selective killing of harmless and beneficial species [5]. The emergence of DBM as a major pest has also led to more concerted efforts for controlling this insect.

In recent years progress in biocontrol of DBM has been accelerated due to the development of resistance to chemical sprays and the growing concerns about risks to farmers, consumers and the environment.

Entomopathogenic nematodes of the genera *Steinernema* and *Heterorhabditis* are applied commercially as biological control agents against insect pests [6]. These nematodes are symbiotically associated with bacteria of the genera *Xenorhabdus* and *Photorhabdus* (*Enterobacteriaceae*), respectively [7]. The characteristics of both genera were reviewed by Forst and Nealson [8].

When the nematodes find a suitable host, such as insect larvae, they are able to penetrate it directly through the cuticle or through their natural openings. In a short period of time the symbiotic bacteria present in the nematodes are

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released into the insect haemocoele. The simultaneous presence of bacteria and nematodes cause the successful parasitization of the insect host [9]. It has been reported that *Xenorhabdus* and *Photorhabdus* alone display virulence against insects only upon injection of bacterial cells preparations directly into the insect haemocoele [10, 11].

Akhurst & Boemare [9] has described that *Xenorhabdus nematophila* is a GRAM- negative facultatively anaerobic bacterium with numerous peritrichous flagella mutualistically associated with the nematode *Steinernema carpocapsae* and is normally only found inside infective juveniles or infected hosts. However *X. nematophila* has been shown to control the fire ant, *Solenopsis invicta* Burer (Hymenoptera: Formicidae) [12] and the beet army worm, *Spodoptera exigua* Hübner [13].

Entomopathogenic nematodes and their symbiotic bacteria are used to control harmful insects. For some insects, application methods had already been studied. However, modern techniques are being developed constantly that are easier to apply and less costly and more effective than insecticides. In this present study cell suspensions of *X. nematophila* and its solutions containing their toxic metabolites were applied using artificial diet and cabbage leaf foliage to control DBM larvae.

The purpose of present experiments was to demonstrate that formulation of either cells or cell-free solutions, obtained from the entomopathogenic bacteria, *X. nematophila*, could be used by direct application to plants for the control of DBM larvae.

MATERIAL AND METHODS

Chinese Cabbage (*Brassica chinensis* L.) Plant Culture

Chinese cabbage cv. Wong Bok was grown in plastic pots containing a loam based compost (John Innes No: 2). Potted plants were kept in a glasshouse maintained at 20-25ºC and 70% RH with 16 hours light and watered daily. The plants were fertilised with soluble Phostrogen every two weeks. All other normal agronomical and cultural practices were used to maintain healthy plants as food for the diamondback moth culture.

DBM Culture on Chinese Cabbage (*B. Chinensis)*

The culture of diamond back moth was regularly maintained in a growth room using temperature at 20-25°C on Chinese cabbage cv. Wong Bok and 70% relative humidity with 14-16 hours of photoperiod was maintained.

DBM larvae were reared on fresh one month old Chinese cabbage (var. Wong Bok) plants, raised in plastic pots, kept in 50×50×45 cm wooden cages, fitted with muslin net. Potted cabbage plants were placed for oviposition and replaced with new plants daily. The duration of the lifecycle was dependant on various factors, the most important being temperature and availability of the food. The hatched larvae were provided with more plants in different cages for mass culturing. The lifecycle of DBM under the insectary conditions at 25°C showed that incubation period of eggs ranged from 2 to 5 days. In the present study the length of larval period was 12-17 days, the pre-pupal period was 1-1.5 days, the duration of the pupal stage was 6-10 days and the longevity of adults ranged from 12-18 days. Different instars of DBM larvae were selected for different experiments.

DBM Culture on Artificial Diet

DBM larvae were fed on artificial diet .The ingredients for one litre were as follows: A 19.8 g agar (agar-agar; Gum agar Sigma Ltd UK) and 161.6 g dry mixed diet (Bioserv. F 9219 B) already stored in fridge which contains casein, wheat germ (stabilized), ascorbic acid methyl paraben, aureomycin, cellulose, vitamin mix wesson fructose, linseed oil (raw) and sunflower oil. Agar was placed in a 2-litre plastic container to which 820 ml of cold sterilized water was added and mixed thoroughly. The container was placed in a microwave and heated on full power until the agar boiled for approximately 6 minutes, the liquid stirred for 15 seconds and again micro-waved for a further 6 minutes and stirred again. Then dry diet was added slowly whilst stirring the mixture. Once the diet was thoroughly mixed, it was dispensed into a plastic tray in a laminar airflow cabinet. For experiments fresh diet was prepared and used. Feeding behavioural response of DBM larvae was observed before using in the experiments.

Mass Culture of Bacterial Cells and Their Metabolites

The Steinernematid nematode *S. carpocapsae* was cultured in the larvae of wax moth *Galleria mellonella* as described by Woodring and Kaya [14]. *G. mellonella* larvae infected on filter paper with 200 infective juveniles died after 48h when they were sterilised and samples taken from the haemolymph and cultured on NBTA agar. A pure colony was cultured in nutrient broth (30 gL^{-1}) for 24h on shaker at 28°C. The number of cells in the broth suspension was estimated on a spectrophotometer and the concentration of cells was adjusted to 4×10^7 cells ml^{-1}, a concentration known to be effective against larvae of *Spodoptera exigua* [13]. To obtain cell suspensions in water the broth suspension was prepared in the same manner and centrifuged at 4100 rpm for 20 min to form a bacterial pellet at the bottom of the centrifuge tube. The supernatant broth solution was drawn off and replaced by distilled water, the process being repeated three times to remove all the broth material. To obtain cell-free solutions containing only metabolites, the cell suspension was filtered through a bacterial filter (pore size 0.2 μm). The efficiency of the filter was tested by inoculating the filtrate onto agar plates. Control treatments were either broth or water alone with 3% Tween-80.

For cabbage leaf foliage treatments single detached leaves of Chinese cabbage were placed in small water reservoirs so that leaves remained fresh throughout the experiments. Both top and bottom leaf surfaces were sprayed with 1 ml of either a broth suspension of cells or a cell-free solution containing only metabolites with 3% Tween-80 (Polyoxyethylene (20) sorbitan mono-oleate) applied as emulsifier. Ten 3rd instar DBM larvae were sprayed and placed either on each leaf or on mixed diet, with the suspension of bacterial cells or metabolite solution. The leaf carrying treated larvae was then placed in a larger sealed container to conserve moisture at 25°C until mortality was determined. When experiments were completed, all DBM larvae, whether alive or dead, were immersed in 70% alcohol to kill bacteria on the surface of larvae. To determine whether bacterial cells had penetrated into the haemocoele a small sample taken from the haemolymph was streaked onto agar plates and growth compared with that of the original culture. Identification was done by examining the colour, morphology and the margin of the colonies of the developing bacteria on NBTA plates.

Mortality of DBM Larval Instars on Artificial Diet

The objective of this experiment was to test the mortality of different DBM larval instars when bacterial suspensions and their toxins were mixed in artificial diet. Fresh bacterial suspensions of *X. nematophila* cells in broth and water were prepared at concentrations of 4×10^7 ml with 3% Tween-80. Toxic metabolites were prepared as described in material and methods already. Second, 3rd and 4th instar larvae of DBM were selected for this experiment. Fresh artificial diet was prepared as described already. Feeding efficiency on fresh diet was tested before conducting the proper experiment. One ml of bacterial suspension and their secretion was mixed in 50 g of artificial diet. Ten larvae of each instar were placed in each sterilised Petri-dish of size 9×4×5 cm. The control treatment was broth or water. The experiment was designed with four treatments and four replications. Containers containing diets with different instars were kept at 25°C. Mortality was recorded after 48 h.

Mortality of DBM Larvae on Leaf and Artificial Diet

Chinese cabbage leaves were spayed with 1 ml of either broth or water suspensions containing 4×10^7 cells ml^{-1} or a metabolite solution obtained from the cell suspension in the same manner as described in experiment 1. Single leaf of Chinese cabbage (approx. 10×7) was sprayed with 1 ml of suspension or toxic metabolite solution. Ten larvae of each instar were placed on sprayed Chinese cabbage leaf (leaves) and placed in the plastic container. For other treatments 1 ml of each bacterial suspension was mixed in 50 g of artificial diet. Ten 3rd instar larvae were placed in each sterilised Petri-dish of size 9×4×5 cm. The control treatment was water or broth. All containers containing the sprayed leaves and diet with (DBM) larvae were covered and kept at 25°C. Replication was 4-fold. Mortality was recorded after 48 h.

Mortality of DBM Larvae on Leaf and Artificial Diet under Dry Conditions

Fresh bacterial suspensions in water and broth were prepared at concentrations of 4×10^7/ml with 3% Tween-80. Toxic metabolites were prepared as already mentioned. Sprayed leaves and mixed diet were prepared in the same manner as mentioned in previous experiment. In this experiment sprayed Chinese cabbage leaves and mixed diet were allowed to dry in a laminar air-flow cabinet for 4 hours before DBM larvae were placed in containers on the leaves or diet. Ten third instar larvae were placed in each treatment. Plastic containers were covered with muslin to maintain a relatively dry condition. Mortality of insects was determined after 48 h.

Effect of Stored Metabolites on Leaf and on Diet against BDM Larvae

Fresh bacterial toxic solutions from bacterial suspensions in broth and water were produced as described already. The bacterial toxic solutions were stored at the 25°C for four weeks. Tween-80 was mixed in each solution at the rate of 3%. Every week stored solutions were tested using leaf and diet substrate in the same manner as described above. Containers were placed at 25°C. Replications were four fold. Mortality of DBM larvae was assessed after 48 h of application of stored toxin solution in all weeks.

Statistical Procedures

Data was analysed with the statistical package GenStat (VSN International Ltd, 2005) [15] using general ANOVA. Duncan's Multiple Range test was used to separate significant means. Tables of significant means were prepared with Microsoft Word.

RESULTS

Mortality of DBM Larval Instars on Artificial Diet

Mortality percentage of different DBM larval instars is shown in Table 1. Bacterial suspensions (cells) and their metabolites (toxins) were found significantly effective ($P < 0.001$) and caused mortality of DBM instars on artificial diet. The highest mortality (100%) was found in 2nd instar when treated with *X. nematophila* bacterial suspension in broth on artificial diet. In case of 3rd and 4th instar of DBM larvae 82.50 to 95.00% mortality was observed when treated with cell or cell-free suspension. Suspension or solution suspended in water caused lower mortality than suspension or solution suspended in broth. However, both treatments were significant ($P = 0.014$). Cause of death of DBM larvae was confirmed by streaking the haemolymph sample on agar plates treated with bacterial suspension.

Table 1. Mortality percentage of Diamondback moth larval instars fed on artificial diet treated with *Xenorhabdus nematophila* cells and toxins in broth and water.

	% Mortality					
	Broth			Water		
Bacterial Treatment	2nd Instar	3rd Instar	4th Instar	2nd Instar	3rd Instar	4th Instar
Cells	100.00 a*	92.50 abc	95.00 ab	95.00 ab	87.50 bc	87.50 bc
Toxins	90.00 abc	87.50 bc	87.50 bc	85.00 bc	82.50 c	82.50 c
Control	10.00 d	7.50 d	7.50 d	7.50 d	5.00 d	7.50 d

*Means not followed by the same letter/s are significantly different at $P = 0.05$; with S.E.M. = 3.48 and df = 51.

Mortality of DBM Larvae on Leaf and Artificial Diet

Mortality percentage of DBM larvae with *X. nematophila* bacterium (cells) and its metabolites (toxins) was highly significant ($P < 0.001$), when fed on Chinese cabbage leaf and artificial diet (Table 2). *X. nematophila* suspension in broth caused the maximum 97.50% mortality of DBM larvae on artificial diet, followed by *X. nematophila* in broth (87.50%) on cabbage leaves. There was a significant interaction ($P = 0.004$) between bacterial treatment and broth/water. Cell suspension in water caused 75.00% on diet and 70.00% mortality on leaves. *X. nematophila* toxic secretions in broth caused the maximum 82.50% mortality of DBM larvae on artificial diet, followed by 75.00% mortality of larvae on leaves by metabolites in broth. Mortality percentage of cell suspensions and its metabolites were higher ($P < 0.001$) in broth than in water. Metabolites in water caused slightly low mortality of 70.00% on diet, and 62.50% on leaves.

Table 2. Mortality percentage of Diamondback moth larvae fed on Chinese cabbage leaves and artificial diet treated with *Xenorhabdus nematophila* cells ad toxins in broth and water.

	% Mortality			
	Broth		Water	
Bacterial Treatment	Cabbage Leaves	Artificial Diet	Cabbage Leaves	Artificial Diet
Cells	87.50 ab*	97.50 a	70.00 de	75.00 cd
Toxins	s75.00 cd	82.50 bc	70.00 de	62.50 e
Control	7.50 f	10.00 f	5.00 f	10.00 f

*Means not followed by the same letter/s are significantly different at *P* = 0.05; with S.E.M. = 3.64 and df = 33.

Mortality of DBM Larvae on Leaf and Artificial Diet under Dry Conditions

Bacterial suspension (cells) or their metabolites (toxins) had significant effect (P < 0.001) on the mortality percentage of DBM larvae (Table 3). Bacterial suspension or their metabolites were less effective (P = 0.002) when exposed in water as compared to broth. Effects of cabbage leaves and artificial diet were highly significant (P < 0.001). *X. nematophila* cells in broth caused 62.50% mortality of DBM larvae was caused when exposed to dried diet, but only 52.50% was observed on dried leaves. Cells suspension in water caused 50.00% mortality when larvae were exposed to artificial diet and it was 45.00% on Chinese cabbage leaves. Metabolites in broth were found less effective in dry conditions and caused 65.00% mortality on dry diet and 50.00% mortality on dried leaves. Metabolites in water resulted only 55.00% mortality on dried diet and 42.50% on dried leaf substrate. Confirmation of cause of death of larvae by *X. nematophila* was done the same way as mentioned previously.

Table 3. Mortality percentage of Diamondback month larvae fed on dried Chinese cabbage leaves and dried artificial diet treated with *Xenorhabdus nematophila* cells and toxins in broth and water.

	% Mortality			
	Broth		Water	
Bacterial Treatment	Cabbage Leaves	Artificial Diet	Cabbage Leaves	Artificial Diet
Cells	52.50 bcd*	62.50 ab	45.00 cd	50.00 cd
Toxins	50.00 cd	65.00 a	42.50 d	55.00 abc
Control	7.50 e	10.00 e	5.00 e	7.50 e

*Means not followed by the same letter/s are significantly different at *P* = 0.05; with S.E.M. = 3.70 and df = 37.

Effect of Stored Metabolites on Leaf and on Diet Against BDM Larvae

Table 4 shows the effect of fresh and stored bacterial toxins in broth and water against DBM larvae when exposed on leaf surface and artificial diet. Fresh toxins were significantly more effective (P < 0.001) as compared to stored toxins. Storage time had significant effect (P < 0.001) on the effectivity of the toxins. Mortality effect degraded from 92.50% to 52.50% of DBM larvae within four weeks when *X. nematophila* metabolites (stored secretions at 25°C) were applied on cabbage leaves. The percent mortality rate on artificial diet decreased in *X. nematophila* secretions in broth from 97.50% to 62.50% in four weeks' time. Secretion metabolites in water showed less effectiveness and mortality was degraded from 87.50% to 50.00% within four weeks when applied against DBM larvae when tested on leaf. The mortality rate on artificial diet decreased in metabolites in water from 87.50% to 55.00% in four weeks' time. Diet had significant effect (P < 0.001) on mortality percentage of BM larvae. Toxins in broth were significantly more effective (P < 0.001) than toxins in water. Statistical analysis showed a significant toxins x storage period interaction effect (P < 0.001).

Table 4. Mortality percentage of Diamondback moth larvae fed on Chinese cabbage leaves and artificial diet treated with *Xenorhabdus nematophila* toxins stored for 0-4 weeks and mixed in broth and water.

		% Mortality			
		Broth		Water	
Bacterial Storage	Bacterial Treatment	Cabbage Leaves	Artificial Diet	Cabbage Leaves	Artificial Diet
0 Weeks	Toxins	92.50 ab*	97.50 a	87.50 abc	87.50 abc
	Control	10.00 lmn	12.50 klmn	5.00 n	10.00 lmn
1 Week	Toxins	82.05 bcd	90.00 ab	77.50 cde	77.50 cde
	Control	10.00 lmn	15.00 klmn	7.50 mn	12.50 klmn
2 Weeks	Toxins	72.50 bcd	82.50 ab	67.50 cde	72.50 cde
	Control	12.50 lmn	17.50 klmn	10.00 mn	15.00 klmn
3 Weeks	Toxins	62.50 fghi	70.00 efg	60.00 ghij	65.00 fgh
	Control	12.50 klmn	20.00 kl	10.00 lmn	15.00 klmn
4 Weeks	Toxins	52.50 fghi	62.50 efg	50.00 ghij	55.00 fgh
	Control	17.50 klmn	22.50 kl	12.50 lmn	15.00 klmn

*Means not followed by the same letter/s are significantly different at $P = 0.05$; with S.E.M. = 3.53 and df = 117.

DISCUSSION

Entomopathogenic nematodes are very widely distributed in most soils [16] and where an insect larva or pupa is infected a large number of bacteria will be produced at the site of infection. As these symbiotic bacteria have a free-living existence, they may be able to reduce insect populations, particularly as toxic metabolites from bacteria may be produced in large quantities around a cadavar and hence exert a direct effect on larvae and pupae.

Results showed that the mortality of *P. xylostella* larvae was significantly greater when exposed to artificial diet than cabbage foliage in a moist condition. Suspensions of *X. nematophila* cells in broth and cell-free solutions containing metabolites were equally effective indicating that the lethal effects obtained were almost certainly due to toxins in the metabolites. When cell suspension was applied, the same species of bacteria were recovered from all DBM larvae for confirmation. It is thus clear that cells of *X. nematophila* can enter insects in the absence of the nematode vector but the mechanisms by which the bacteria are able to enter the insect is unclear.

Recently ffrench-Constant & Bowen [17] have shown that novel insecticidal genes from *X. nematophila* will control insects when applied in larval food or onto plants. Furthermore, they have reported for the insecticidal toxic genes from *X. nematophila* fed to or injected into the haemolymph of several species of insects [17]. The results here confirm the findings of Elawad et al. [13] concerning the effectiveness of *P. putida* cells and metabolites against the larvae and pupae of *Spodoptera exigua*.

In another study reported by Clarke and Dowds [18] it was concluded that secreted lipase enzyme from *X. nematophila* bacteria is responsible for insecticidal activity observed in *G. mellonella* larvae. In addition these extra cellular secretions produced by *X. nematophila* bacterium, another group of intracellular inclusion bodies similar to those produced by Bt. But unlike Bt, these intracellular seems to have no any insecticidal activity against insects. It has always been assumed that the association between entomopathogenic nematodes and their symbiotic bacteria is mutualistic and that the bacterium cannot survive without the nematode [19].

In all the treatments where cell and cell-free applications of bacteria were done there was no significant difference in the toxicity of the treatments indicating that it is toxic metabolites from *X. nematophila*, which is the responsible for the lethal effects observed. Same results were obtained when cell and cell free suspensions from *X. nematophila* and *X. bovienii* caused mortality of black vine weevil larvae, *Otiorhynchus sulcatus* when applied in potting soil or on growing strawberry plants [20]. The reason for these observations are clear-cut, the metabolites are always present in the cell suspensions and so for we have been unable to produce cell suspension free of the toxic metabolites. Even continuing washing of the cells of *X. nematophila* and their reconstruction in water resulted in only slightly reduced toxicity after 10 washings [21, 22]. In another experiment, cells of *X. nematophila* and *P. luminescens* caused high mortality to the larvae of *G. mellonella* larvae [23].

Cell suspensions of *X. nematophila* have also been reported to be lethal to the fire ant, *Solenopsis invicta* [12] and the beet armyworm *Spodoptera exigua* [13]. Treatments with cell suspensions or their metabolites from *X. nematophila* have also been shown to be toxic to DBM larvae, locust nymphs and *G. mellonella* larvae and pupae [24].

When cells of *X. nematophila* were applied to Chinese cabbage leaves, it was observed that cells could be recovered from either the abdomen of DBM larvae feeding on treated leaves or on artificial diet so it is clear that cells of *X. nematophila* can enter insects in the absence of nematode vector, *S. carpocapsae*. The method by which *X. nematophila* gains entry is unclear but entomopathogenic bacteria of the genera *Xenorhabdus* do show swarming motility when grown on suitable solid media [7, 8, 25]), so it is possible that they can move some distance under suitable environmental conditions.

The present experiments demonstrated that formulation of either cells or cell-free solutions obtained from the entomopathogenic bacteria, *X. nematophila*, could be used by direct application to plants for the control of DBM larvae. In order to use these treatments containing toxic secretions from bacteria in the field, it would be necessary to carry out toxicology tests, before application on large scale, against *P. xylostella* larvae. It is relevant to point out that *X. nematophila* carried by its nematode vector *S. carpocapsae* is exempt from normal registration procedures world-wide.

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Original Article

Traditional Knowledge and Use of Medicinal Plants by the *Patra* Tribe Community in the North-Eastern Region of Bangladesh

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Abstract: An exploratory study was conducted on the traditional medical ethno-botany perception of the *Patra* tribe community living in and around the Khadimnagar National Park of North-eastern Bangladesh, to focus on the implications of healthcare using medicinal plants. The study indicates that the traditional medical system is well functioning in the area. A total of 33 households were assessed using different participatory appraisals through direct observation and semi-structured questionnaire. A total of 31 medicinal plant species belonging to 23 families were recorded with the help of tribal people of the area as mostly collected from the forest and were found to use for curing 55 ailments; ranging from simple headaches to highly complicated eye and heart diseases, including diabetes, jaundice, chickenpox, dysentery, constipation, teeth ache, cut and wounds, diarrhea etc. Study also revealed that majority of the species used by the local inhabitants was herbs (6 species) followed by trees (14 species), shrubs (8 species), and climbers (3 species). For curing ailments, the use of aboveground plant parts was higher (76%) than the underground plant parts (17%) among mainly used leaves, either taken orally or used externally. Homesteads (32%) are the primary source followed by forest (27%) and few medicinal plants are cultivated. Since this knowledge is handed down orally from generation to generation, there is a great possibility that the knowledge will be lost over time. The findings of the study conclude that the conservation of the indigenous knowledge of the *Patra* tribe regarding medicinal plants can help conserve the forest.

Keywords: Medicinal plants, indigenous knowledge, healthcare, sustainable use, conservation, Bangladesh

INTRODUCTION

The life of tribal's always centered on the forests. The religious, cultural and economic activities of the tribal people depend on forests. It's their main life supporting system, and therefore, it plays a vital role in their economy [1]. Tribal people are the ecosystem people who live in harmony with the nature and maintain a close link between man and environment [2]. It is well acknowledged in literature that their age-old practices of using plants to cure numerous ailments have paved the way to further discovery of many life-saving drugs [3, 4]. The livelihoods of tribal and forest dwellers are mainly dependent on the forests, which have built up their socio-economic and cultural life [5-6].

Indigenous knowledge (IK) plays a central role in disease diagnosis and healthcare practices in traditional medication systems throughout the world [7]. The latest record of indigenous communities in Bangladesh gives the number as 45 distributed in the various regions of the country and recorded by solidarity [8]. In Bangladesh, there are many marginalized tribal communities of different lifestyles and cultures. From time immemorial, tribal people have traditionally and culturally used medicinal plants [9]. The greater Sylhet region, comprising of Sylhet, Moulvibazar, Habiganj and Sunamganj districts, is inhabited by a number of tribes. *Patra* tribe community is found in Sylhet district and their ethno-botanical knowledge is rich. Trees are considered sacred or given very high magi co-religious value by the local people and their use as medicinal plants is preferred over allopathic and homeopathic treatments for their

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health care.

The present study was conducted in Khadimnagar National Park (KNP), situated in North-eastern corner of Bangladesh. The study explored the local data on forest of *Patra* tribe community and its impacts on the KNP. There is no recognized and institutional record how and when the *Patra* people settled in Sylhet permanently. Their history of pricing is also completely unknown and elusive. But, there are a few traditional and popular stories regarding the background of the community. Finding no other government and non government recognized and institutional record; the researcher has to depend on the oral history of the aged people of the *patra* community. The *patra* consider themselves as the descendants of Raja Gour Gubind (an ancient king), the last Hindus king of Gour of Sylhet. These people are also known as *Laleng* in their language [10]. At present, only 3562 people of 32 villages live under 6 unions of Khadimnagar, Khadimpara, Chicknagul, Jaintapur, Fatepur, and East Jaflong in Sylhet [11]. But they are different in physique and complexion and are not as sturdy as *Khasia, Manipuri* and *Hajong* tribes of Sylhet region. Considerably large numbers of *Patra* people are also living in India: Assam, Khachar, Bikhara, and Patrakhandi.

Local Bengali people and *Patra* inhabit both in our study villages. They used forests for various purposes: subsistence, livestock rearing, fuelwood collection, medicinal plants and as a source of goods to sell in the market. These communities place various and different pressures on forests for maintaining their livelihoods, depending on the nature of the forest area and the economic resources available to them. Tribal communities use surrounding plants for their primary healthcare along with other necessities, which are based on their traditional knowledge and dynamic cultural heritage. But in Bangladesh, the knowledge of this *Patra* community remains un-documented and is handed down orally. The younger generation has different ambitions due to changing circumstances and, therefore, the traditional knowledge is feared to get lost. However, since no study has so far been carried out on the indigenous medicinal knowledge, the present study was undertaken to ascertain the same in Sylhet region of Bangladesh. Also, no study has been conducted on indigenous healthcare practice of *Patra*. It was, therefore, desirable to explore their perception and indigenous healthcare practice about medicinal plants. Its aim was to assess plant-based ethno-medicinal practice and document indigenous knowledge of this vulnerable community people associated with it. The present report provides in-depth information on the plant species used by the *Patra* tribe community and document their traditional knowledge and cultural practices which may be under threat due to the pressure of modernization.

METHODOLOGY

The Study Area

The study was conducted during August 2010 to October 2010 at Khadimnagar National Park; the area was purposely selected for the study considering that this is the most important tropical forests and protected area in the Northeastern region of Bangladesh, its unique geo-physical features and richness in biological diversity and dependency of people on it. The KNP is located at North Sylhet Range-1 (sub-division) in Sylhet Forest Division under tropical evergreen and semi-evergreen bio-geographic zone. Formerly known as Khadimnagar Reserve Forest, it was declared as KNP in 2006 [12]. The soil of this area is generally sandy-loam soil. One of the richest semi-evergreen hill forests of Bangladesh is located in this region. The area is situated between 23° 55′ and 25° 02′ north latitude and between 90°55′ and 92° 30′east longitude [13]. Total area of the site is 679 ha, surrounded by three tea gardens and is submerged with several watersheds locally known as “*chara*”. The hills are generally low and gently sloping. Soil ranges from clay loams to pale brown (acidic) clay loams on the hills [12]. Climate conditions are warm and humid. April and May are the warmest, and December and January are the coolest months. The tropical monsoon climate prevails in the area with average maximum temperature of 30.7˚C and average minimum temperature of 18.9˚C. The average annual rainfall is 3931mm, most of which falls between June-September [14].

Research Methods

KNP was selected randomly because this area was inhabited mostly by the *Patra* community. We selected 5 villages (situated about 0-2 km away from the park area), namely Paikpara, Faringura, Bararhat, Darabazar and Kushirgul by multi-stage random sampling inhabited by both local and *Patra* community. The study began by preparing community profiles to learn details of the community. After that collected primary information from key informants, drew community maps, conducted transect walks, and engaged in focus group interviews. For properly gathering information seven focus group discussions were conducted to learn about the livelihoods and social conditions of the community. Out of five villages there are about 55 households and about 33 households were selected for interviews randomly. These samples may not be representative of the whole area, but do represent the selected villages. A semi-structured questionnaire was used in household interviews. We interviewed mainly older men, women and experienced members from these households using a local guide in each site. Respondents were interviewed using a structured questionnaire to ascertain the plant species and the parts used, for what diseases, the sources they prefer, the reasons for cultivating any plant and the involvement of younger generation in this regard. The plant species used for medicine were firstly identified by local names. The scientific names were obtained by consulting the literature [15-18]. A final list of the species used for medicinal purposes was prepared based on the study by Dey [19]. The collected information was analyzed, and correlation was made between different species of the medicinal plants in order to understand the pattern in medicinal plant uses and occurrences.

RESULTS AND DISCUSSION

All the *Patra* communities (i.e., 100%) are medium to poor; primarily due to their main occupation, i.e., tea labor, daily labor, drivers, etc. Due to an excess of manpower (mainly unskilled labor), the local labor market is highly competitive, resulting in a lower wage rate for these communities. Their monthly income is below Tk. 5000 (US$ 71.5), which sometimes exceeds their family expenditure. Average household size in the study villages is about six-eight persons per family. The illiteracy rate is around 45%, mainly because parents are not willing to send their children to school during the day or in working periods, so that they can help with household activities (mainly tea labor, daily labor and fuel wood collection) and earn money. They also use the adjoining KNP illegally to sustain their livelihoods. In the absence of modern health facilities, they depend extensively on plants to treat their ailments. Most of the older men and women have knowledge of medicinal plant use and preparation, and claim that this knowledge is becoming vulnerable due to the local disappearance of many medicinal plants.

Status of Medicinal Plant Resources

A total 31 ethno-medicinal plant species, including herb, shrub, tree and climber distributed across 23 families were documented in the study to be used by the *Patra* community for curing ailments (Table 1). It was found that 14 tree species (45%) found from 10 families followed by 8 shrubs (26%) and 6 herbs (19%) species were found belonging to 7 and 6 families also it was recorded that 3 species of climbers (10%) belonging to 3 families respectively. A similar trend was also reported by Uddin et al [20], Mukul et al [21] and Miah and Chowdhury [22] but Halim et al [23] and Ghani [24] found herbs were dominated in the observed plant species. Of the 23 plant families with medicinal properties, two families (Araceae, Combretaceae) each had three species, four families (Anacardiaceae, Compositae, Moraceae, Verbenaceae) had two species each, and the other families had one species only. The uses of some medicinal plants families, recorded in the study areas, was also reported in other communities throughout the world by the researchers such as Apocynaceae and Asclepiadaceae among the *Jaintia* in India [2]; Fabaceae, Moraceae, Cucurbitaceae, Apocynaceae and Euphorbiaceae among the *Tharus* in Nepal [25]; and Umbelliferae among the *Tibetans* of Yunnan province in China [26]. Most of the recorded species were collected by the respondents from their surrounding homestead (32%) followed by forest (27%), roadsides (22%), other places (e.g. fallow land, local markets) (12%) and agricultural land (7%) (Fig. 1). According to Chowdhury et al [27] and Chowdhury and Koike [28], rural homestead or surrounding home garden is the main source of medicinal plants in Bangladesh.

Table 1. Medicinal plants with their traditional uses by the *Patra* tribe community in Bangladesh.

Family	Local name	Scientific name	1Habit	2Incidence	3Key sources	Parts used	Diseases treated	4Use rate (%)
Acanthaceae	Bashok	*Adhatoda vasica* Nees.	Sh	++	F, H	Leaves	Cough, asthma, cold ailments, malaria, bleeding of piles	54
Anacardiaceae	Aam	*Mangifera indica* L.	Tr	+++	F, H, R	Fruit, seed, bark and latex	Diarrhea, diabetes, asthma, cough, piles, dysentery, constipation and diphtheria	41
	Amra	*Spondias pinnata* (L.f.) Kurz.	Tr	+	H	Fruits, leaves	Dysentery, pain at joints	28
Apiaceae	Adamoni	*Centella asiatica* (L.) Urban	H	++	R, H, O	Fresh green leaves	Flatulence, dysentery and bleeding of piles	51
Araceae	Kacu	*Colacasia esculenta*	Sh	+++	H, F, O	Green leaves, roots also whole plant	Constipation, joint pain, ear disease and mouth disease	58
	Man Kacu	*Colocasia affinis* Schott.	Sh	+++	H, F, O	Roots and leaves	Nyctalopic, constipation, pain rheumatism	39
	Narikel	*Cocos nucifera* L.	Tr	++	H, F	Root and fruit	Cholera, diarrhea, dysentery, constipation, stimulate	43
Combretaceae	Arjun	*Terminalia arjuna* Bedd.	Tr	++	H, F, R	Bark	Burning, dysentery, hypertension, heart disease, diarrhea, piles, bone fracture and cough	54
	Horitoki	*Terminalia chebula* Retz.	Tr	+	H, F, R	Fruits	Dysentery, asthma, cough, constipation, stomach trouble, dysentery, rheumatism, and eye disease	46
	Bohera	*Terminalia belerica* (Gaertn) Roxb.	Tr	+	H, F, R	Fruits, Bark	Asthma, heart disease, Dysentery, headache, painful menstruation, jaundice, constipation and fever	38
Compositae	Assam pata	*Chromolaena odorata (L.) King & Robinson*	Sh	+++	H, F, R, Ag	Green leaves Flowers	Anti-hemorrhoid, narcotic, influenza, fever, cough and diabetes	42
	Assam lata	*Mikania cordata* (Burm. F.) Roxb.	Cl	+++	H, F, O, R	Green leaves	Anti-hemorrhoid	29
Convolvulaceae	Voi Kumra	*Ipomea digita* L.	Cl	+	F	Root	Tonic, alterative, demulcent, aphrodisiac and purgative	33
Crassulaceae	Pathor kuchi	*Bryophyllum pinnatum* (Lamk.) Oken	H	+	R, H, O	Leaves	Cough, flatulence	49
Euphorbiaceae	Amloki	*Emblica officinalis* L.	Tr	+	H, F	Fruits	Blood dysentery, vomiting, loss of appetite, jaundice, dyspepsia, skin diseases, hair falls, digestive problem	41
Gramineae	Durba grass	*Cynodon dactylon* L. Pers.	H	+++	H, Ag, R, O	Tender leaves	Tooth ache, cut and wounds	68
Labiatae	Tulsi	*Ocimum sanctum* L.	Sh	++	H, R	Leaves	Asthma, cough, cold ailment, stomachache and dysentery	64
Leguminoseae	Tentul	*Tamarindus indica* L.	Tr	++	H, F, R	Fruits, leaves	Loss of appetite	45
Magnoliaceae	Champa	*Michelia champaca* L.	Tr	++	F, R	Bark, bark of roots,	Fever and rheumatism	30
Moraceae	Kanthal	*Artocarpus heterophyllus* Lamk.	Tr	++	H, F, R	Green and ripen fruit, root and latex	Skin disease, diarrhea, dysentery and constipation	36
	Chapalish	*Artocarpus chaplasha* Roxb.	Tr	++	F, R	Fruit	Constipation, diarrhea and stimulation	27
Mimosoidea	Lazzabati	*Mimosa pudica* L.	H	+++	H, Ag, R, O	Whole plant	Curing sexual problem, pox, kidney problem, strengthen body, scabies, piles and jaundice	44
Meliaceae	Neem	*Azadirachta indica* A. Juss.	Tr	++	H, F, Ag	Bark, leaves	Fever, skin disease, diarrhea, insect biting, chicken pox, antiseptic, eczema, ulcer, dysentery, diabetes	58
Pandanaceae	Pan	*Piper betel* L.	Cl	+	H	Fresh green leaves	Dysentery, loss of appetite, digestive problem, and belly ache	50
Rutaceae	Lebu	*Citrus limon* (Linn.) Burm. f.	Sh	++	H	Fruit, root	Indigestion and dysentery	34
Sterculiaceae	Ulatkambal	*Abroma augusta* L.	H	+	F	Bark, root	Dysmenorrheal	29
Theaceae	Cha	*Camellia sinensis* O.Kuntze.	Sh	+	R, O	Tender leaves	Heart disease, cold ailments and cough, stimulate	55
Thymelaeaceae	Agar	*Aquilaria agallocha* Roxb.	Tr	+	F, O	Bark	Rheumatism	27
Verbenaceae	Shegon	*Tectona grandis* L.f.	Tr	+++	F	Roots, flowers, seeds	Hair growth, urinary problems	31

Contd….

Table 1. (Contd.)

Family	Local name	Scientific name	1Habit	2Incidence	3Key sources	Parts used	Diseases treated	4Use rate (%)
	Bhat pata	*Clerodendrom viscosum* Vent.	Sh	+++	F, R, H	Roots, barks, leaves and young plants.	Vomiting, ear ache, fever, skin disease and worms	42
Zingiberaceae	Ada	*Zinziber officinale* Rosc.	H	++	H, F, Ag	Rhizome	Possesses stimulant, cough, cold, tonsil, typhoid, vomiting, diarrhea and teeth ache	63

1 **Incidence:** +++- Very frequent, ++- Fairly frequent and +- Rare;
2 **Key sources:** F-Forest, H-Homestead, R-Roadside, Ag-Agricultural land, and O-Others;
3 **Habit:** Tr-Tree, H- Herb, Sh- Shrub, and Cl-Climber;
4 **Use rate:** number of *Patra* people reported medicinal plants use in relation to the total number of times that particular species was cited.

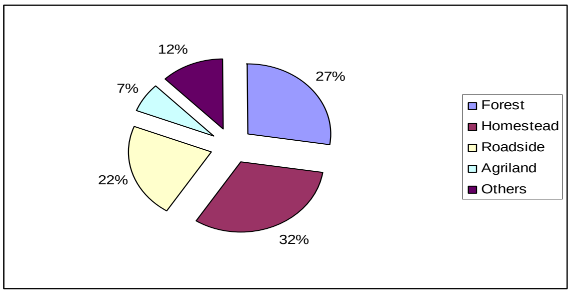

Fig. 1. Sources of recorded medicinal plants.

Due to inaccessibility to modern medical facilities, *Patra* community is highly dependent on herbal medicine. The medicine is generally prepared by an indigenous medical practitioner, called *Kabiraj/hakime/baiddya*, from the plants available in the forest and the homestead forest and fallow lands. Then he will give various prescriptions to the community members. It is noteworthy that the homestead forest is common in Bangladesh, containing a mixture of natural and planted species in a complex structure and being a source of economic-benefit of the rural poor [29]. Due to predominant dependence on herbal treatment and long-term uses, not only the practitioners but also elder community members have become to have a good knowledge of the medicinal value of some plants, which those species are usually used to treat common diseases such as diarrhea, dysentery, cough, cold ailments, cut and wounds, skin diseases, etc. In case of medicinal plant sources, they preferred wild sources like natural forest, homestead, around homestead jungles, graveyards, roadside area and pond bank rather than planted sources (plantation in any site where regeneration is artificial) for collection of medicinal plants. They believe that wildly collected medical plants are more effective than planted sources.

Medicinal Plant Part Utilization

For medicinal preparations, people mostly use above-ground plant parts (76%), followed by belowground parts (17%) and whole plants (7%). Of the above ground plant parts, leaves are used most frequently (25%), followed by roots and fruits (20% each), bark (16%), whole plants (9%), flowers (4%), latex (4%) and seed (2%). This finding is also reported in the literature [e.g., 2, 30-33]. In most cases, the paste and juice made from leaves and barks are used in medicines, while fruits are eaten raw. Different belowground plant parts, such as roots and rhizomes, are also used to treat ailments (Fig. 2). In this study, the *Patra* people used mostly leaves. This ensures sustainable harvesting of medicinal plants. It provides an incentive to protect and maintain wild populations and their habitats and the genetic diversity of medicinal plants [34]. The whole plants of the five species, viz. *Colacasia esculenta*, *Kalanchoe pinnata*, *Mimisa pudica*, *Clerodendrom viscosum*, *Cynodon dactylon* were used as medicine in the study area.

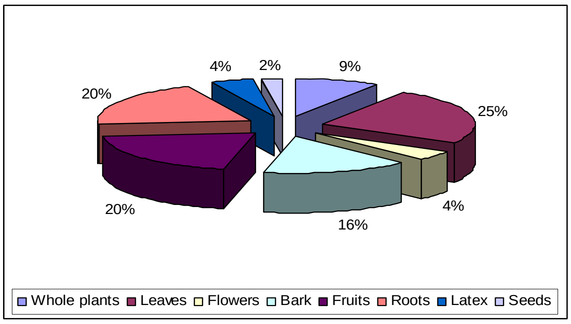

Fig. 2. Medicinal plants break-up by parts used.

The major reasons for uses of medicinal plants mentioned by the respondents are summarized in Fig. 3. Medicinal plants obtained freely from forest (143 respondents) followed by does not have any side effect (116 respondents), more effective (94 respondents), etc. are the major reasons for using medicinal plants statement by the respondents. Plants identified in the area are used to treat 55 conditions, ranging from simple headaches to highly complicated eye and heart diseases, including diabetes, jaundice and chickenpox. Medicinal plants are generally used to treat fever, coughs, cuts and wounds, cold ailments, tooth disease, hair loss, skin diseases and weakness.

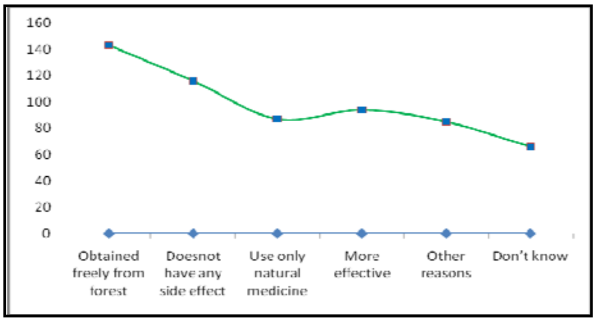

Fig. 3. Reasons for using medicinal plants by the *Patra* tribe community.

Ailments

The largest number of medicinal plant species are used to treat dysentery (12 species), cough (10 species), constipation (8 species), diarrhea (7 species) followed by asthma, fever and piles equally (5 species), rheumatism, skin diseases, stomach problem and cold ailments together (4 species), diabetes, heart disease, jaundice, joint pain, loss of appetite, teeth ache, cut and wounds, eye disease, scabies, etc. More than one species is used for treating some common conditions: dysentery, constipation, joint pain, cold ailments, hair loss, vomiting, etc. In most cases, only one species is used to treat an ailment at a time, but in some cases a mixture of several species is also used. In some cases, different parts of an individual plant were found to be used to cure different ailments in different ways (Fig. 4). The use of *Adhatoda vasica* for treatment of cough, asthma, cold ailments, malaria, bleeding of piles was found in the study area. In addition, *Centella asiatica*, is used against flatulence, dysentery and bleeding of piles and *Mimosa pudica* is also reported for curing sexual problem, pox, kidney problem, strengthen body, scabies, piles and jaundice in the study area. The use of *Ocimum sanctum* against asthma, cough, cold ailment, stomachache and dysentery were recorded in the study area. *Ocimum sanctum* has a long Bangladeshi history of bearing an antitussive property but its analgesic use has never been reported earlier. Some other species were used frequently against various ailments by the *Patra* community. The present results are also supported by Rahman et al [11, 35] where described that forest dependent peoples living in and around the KNP uses various plant parts for curing ailments.

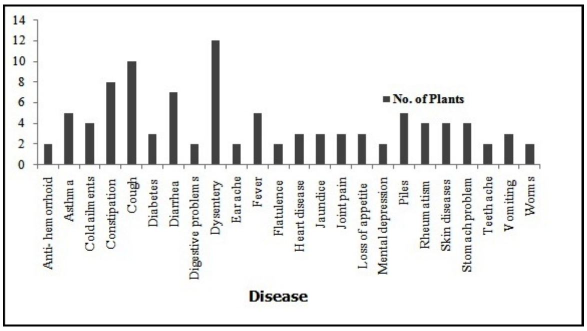

Fig. 4. Number of medicinal plants used for treating major ailments.

Cultivation of Medicinal Plants

More or less all the medicinal plant species in the study area were found to be collected wild from the surrounding forests; although the *Patra* people prefer wild sources for medicinal plants collection, they also cultivate a number of plants that are needed by them frequently time and have spiritual and cultural values. For example, *Mangifera indica, Colacasia esculenta, Cocos nucifera*, *Terminalia arjuna*, *Ocimum sanctum*, *Artocarpus heterophyllus*, *Piper betel*, *Citrus limon* and *Azadirachta indica*. Plants like; *Ipomea digita*, *Abroma augusta*, *Emblica officinalis*, *Terminalia belerica*, *Bryophyllum pinnatum*, etc. have become endangered because of overexploitation for potential medicinal importance. The *Patras* of the area should be encouraged for the cultivation of economically important plants and they should be provided with lands and seedlings by the government. There should be a strong market infrastructure for the selling of cultivated plants and plant products without involving any middle-man keeping in view the economic up keeping of the tribal's and checking their exploitation [11]. The forest management programs should also involve the tribal people of the area which may also be a tool both for economic up keeping of the area and maintenance of biodiversity. The detailed assessment of the productive capacity, sustainable use of plant resources and cultivation of valuable medicinal and economically important plants is required.

Traditional Medicinal Knowledge

Our survey indicated that the *Patra* people are willing to share their traditional medicinal knowledge with their own people, but not with outsiders. For this reason, this study takes a longer time to convince the community people so that they mention their traditional uses. Older people possess more knowledge regarding the uses and identification of medicinal plants; in most cases, this knowledge is orally transferred from one generation to the next, this is also supported by Rahman et al [11]. Throughout the study, the older people claimed that they have less knowledge about preparation and the use of medicinal plants than their grandparents and that the present/younger generation knows much less than them. This perception indicates a gap or erosion in the traditional knowledge system regarding medicinal plants that should be investigated for the wellbeing of future generations, as well as for science. We also found that local people are aware of the degradation or disappearance of medicinal plants growing in the forest and surrounding habitats.

The present study indicates that the *Patra* people depend heavily on plant parts for curing a variety of ailments. Not only older people, but also younger people, possess some knowledge of medicinal use of plants. However, while older people have a vast knowledge on the treatment of disorders ranging from simple cuts to incurable diabetes, younger people are knowledgeable only about plants used to treat common ailments such as cuts, wounds, scabies, aching joints, stomach pain, cold, coughs, diarrhea and dysentery. Our fieldwork suggested that older people generally suffer from more complex ailments, which makes them more interested in looking for the curative agents, while younger people usually do not care much about these issues. Whenever children get sick, their parents or other older members of the family take care of them with medicinal plants. However, if anybody suffers severely from a disease, they go to the Health Complex, Family Planning Centers, Satellite Clinics and village 'physicians' for conventional medical treatments.

CONCLUSION

The traditional medicinal practice and culture is alive and functioning well in the study area. In many of the tribal villages, the herbal drugs are the cheapest option and the only way for treatmenting different ailments. The study area is floristically rich with strong ethno-botanical traditions among the *Patra* community. Lack of sustainable use pattern, inadequate knowledge about forest management and constraint of financial resources are the main causes of over exploitation. Therefore, there is an utmost need for conserving and sustaining the use of these plants, which are part of traditional medical system practiced by local tribes residing in different areas of Bangladesh. Within the study area, most the medicinal plants are edible fruit bearers, planted for the seasonal fruits and not for medicinal purposes. Yet, the local communities depend on these plants for medicinal purposes and, thus, are extremely concerned about their extinction, as now they need to travel long distances to collect these plants. The medicinal plant used by the *Patra* tribes merit thorough phyto-chemical investigation regarding alkaloid extraction and isolation along with associated clinical trials. This could help in creating sound awareness regarding the need for conserving such plants and also in promoting the ethno-medico-botany knowledge within the region, and preservation and enrichment of the gene bank of such economically important species before they are lost forever. Thus investigation, coupled with clinical trials would increase confidence of traditional users and healers. The development of roads and enhancement of tourism in tourist these areas has allured the younger generations towards market economy; this certainly will have positive implications. Thus, *Patras* of the area are encouraged to plant medicinal species in their fallow land and uncultivable hilly terrains.

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Original Article

Matrix Representation of the Elasticity Tensor

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Abstract: A simple method is described for finding matrix representation of the elasticity tensor belonging to a trigonal, tetragonal or a hexagonal material. Suitable tensors of rank two are formed and their symmetry properties lead to the vanishing of several components and certain relations are obtained among the components of the elasticity tensor belonging to these class.

Keywords: Elasticity tensor, anisotropic material, 6×6 matrix representation, tetragonal, trigonal and hexagonal materials, simple method.

1. INTRODUCTION

An elastic material is called isotropic if its properties are same in every direction. On the other hand, properties of an *anisotropic* material are direction dependent. For example the velocity of a wave in a certain direction may be much greater than the corresponding velocity of a similar kind of wave in other directions. Anisotropy is a consequence of crystalline structure.

Let $\mathbf{T}$ and $\mathbf{E}$ represent respectively the stress and strain tensors at a point of an elastic material. The generalized Hooke's law states

$$T_{ij} = C_{ijkl} E_{kl}$$

where free indices take the values $1,2,3$ and repeated indices imply summation. The tensor C of rank 4, with 81 components C_{ijkl}, is called the elasticity tensor. It has the following symmetries

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij}. \tag{1.1}$$

As a result the number of independent components of C reduces from 81 to 21 [4]. It is usual to introduce the following two index representation in which a pair of indices corresponds to a single index in the following manner

$$(11)\leftrightarrow 1,\ (22)\leftrightarrow 2,\ (33)\leftrightarrow 3,\ (23) = (32)\leftrightarrow 4,\ (13) = (31)\leftrightarrow 5,\ (12) = (21)\leftrightarrow 6.$$

Thus, $C_{1123} = C_{14}$ and $C_{2312} = C_{46}$ etc. Now the elasticity tensor can be represented by a 6×6 *symmetric* matrix,

$$(\mathrm{C}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \tag{1.2}$$

A vector $\mathbf{p}$ is called an $n-$ fold axis of symmetry, A_n, of an elastic material if a rotation of the material about $\mathbf{p}$ through an angle $2\pi/n$ leaves the material invariant. A point is called a centre of symmetry if the material is invariant with respect to an inversion through the point and a plane is called a plane of symmetry if the material has invariance with respect to a reflection in that plane i.e. the plane acts like a mirror.

Classes of Elastic Materials

The crystalline structure in materials leads to anisotropy and the elastic materials can be divided into eight classes on the basis of their symmetry.

1) Triclinic system. It has only a center of symmetry, in common with all other systems, and has no axis or plane of

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symmetry. The matrix representation of C_{ijkl} for this system is given by (1.2). It has 21 independent components.

2) Monoclinic system. It has a single A_2 axis. It can be easily shown that, if x_3 is chosen parallel to the symmetry axis, any component C_{ijkl} with an odd number of 3'*s* in the subscript must banish [5]. Thus in the matrix representation of a monoclinic system *all components in the fourth and fifth rows and columns vanish with the exception of* C_{44}, C_{45}, C_{54} and C_{55}. (1.3)

3) Orthorhombic system. It has three mutually perpendicular two-fold axes.

4) Cubic system. The unit cell is a cube. In addition to three mutually perpendicular A_2 axes corresponding to the three edges meeting at a point, it has four A_3 axes corresponding to the diagonals joining opposite vertices.

5) Isotropic system. A tensor is called isotropic if it has same components in every Cartesian coordinate system. There are only three independent isotropic tensors of rank 4 in three dimensions i.e. $\delta_{ij}\delta_{kl}$, $\delta_{ik}\delta_{jl}$, $\delta_{il}\delta_{jk}$, therefore the elasticity tensor for an isotropic material must be a linear combination of these three tensors.

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu_1\delta_{ik}\delta_{jl} + \mu_2\delta_{il}\delta_{jk}$$

Because of the symmetries (1.1), $\mu_1 = \mu_2 = \mu$ and the elasticity tensor for an isotropic material has only two independent components.

6) Trigonal system possessing an A_3 axis. Its tensor has seven independent components.

7) Tetragonal system with an A_4 axis. Its tensor has seven independent components.

8) Hexagonal system with an A_6 axis. Its tensor has five independent components.

Symmetry properties of elastic materials are mathematically described by the invariance of the elasticity tensor under the special orthogonal group *SO*(3) [7,8]. The number of independent components of the elasticity tensor steadily decreases from 21 to 2 as we move from the least symmetry of the triclinic system to highest symmetry of the isotropic system.

Whereas matrices representing each of the first five classes are found in a straightforward manner, the same task for the last three classes requires some ingenuity. Some standard text-books discuss the problem only partially and leave unanswered some questions related to matrix representation, especially of the trigonal system [1, 2]. Hearmon [3] wrote down transformation equations for the components $C_{11}, C_{12}, C_{22}, C_{16}, C_{26}$ and C_{66} then equated "*only the rational terms*" in the equations for C_{16} and C_{26} to conclude that these components vanish. Such an argument could be justified only if the components of the elasticity tensor were *rational*. In [4] the problem has been tackled by diagonalizing the rotation matrix followed by a rotation of the coordinate axes where the transformed axes are now *complex*. Ting [5] has dealt with the trigonal, tetragonal and hexagonal symmetries by selecting two planes of symmetry and considering invariance with respect to reflections in them.

In a classroom, it would be natural to first consider the simpler case of the effect of an axis of symmetry on a tensor of rank 2. In the next section, we shall show that much of the information related to the representation of the classes listed at 6-8 can be gained by simply reducing the problem to a consideration of tensors of rank 2.

2. MATRIX REPRESENTATION OF A TENSOR WITH AN $\mathbf{A}_3, \mathbf{A}_4$ OR $\mathbf{A}_6$ AXIS

Let us consider a material possessing an axis of symmetry A_n with $n = 3, 4$ or 6. We choose a rectangular coordinate system so that $x_3 -$ axis is aligned with the axis of symmetry. Let $\mathbf{B}$ be a tensor of rank 2 associated with the material. It is easily shown that, the matrix representation of $\mathbf{B}$ will be [4]

$$(\mathbf{B}) = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{11} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \qquad (2.1)$$

Now define a tensor $\mathbf{G}$ such that $G_{ij} = \frac{1}{2}\varepsilon_{ijk}\varepsilon_{lmn}C_{iljm}$, where ε_{ijk} are the components of the alternating tensor. The matrix representation of $\mathbf{G}$ is

$$(\mathbf{G}) = \begin{pmatrix} C_{23}-C_{44} & C_{45}-C_{36} & C_{46}-C_{25} \\ C_{45}-C_{36} & C_{13}-C_{55} & C_{56}-C_{14} \\ C_{46}-C_{25} & C_{56}-C_{14} & C_{12}-C_{66} \end{pmatrix} \quad (2.2)$$

This tensor was used by Ahmad [6] in connection with quadratic invariants of the elasticity tensor. On comparison with (2.1), we find:

$$C_{36} = C_{45},\ C_{25} = C_{46},\ C_{14} = C_{56},\ C_{23} - C_{44} = C_{13} - C_{55}. \quad (2.3)$$

Same results can be obtained by a consideration of $U_{ij} = C_{ijkk}$ and $V_{ij} = C_{ikjk}$. For example $U_{12} = 0 = V_{12}$ will give the first relation of (2.3).

Let $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$ be two tensors of rank two which are independent of each other and the elasticity tensor. They might, for example, be tensors based on measurement of physical data such as the permittivity tensor and the conductivity tensor. But for our purpose, *any* pair of hypothetical tensors will suffice. These tensors will exhibit the symmetry due to A_n and will have matrix representations of the form (2.1). Also the tensors $M_{ij} = \tau_{pq} C_{ijpq}$ and $N_{ij} = \sigma_{pq} C_{ijpq}$ will be similarly represented. Equating M_{12} and N_{12} to zero, we get

$$\tau_{11}(C_{16} + C_{26}) + \tau_{33} C_{36} = 0, \quad (2.4a)$$

and

$$\sigma_{11}(C_{16} + C_{26}) + \sigma_{33} C_{36} = 0. \quad (2.4b)$$

Since tensors $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$ are independent, the above equations imply

$$C_{36} = 0,\ C_{16} + C_{26} = 0. \quad (2.5)$$

Similarly $M_{13} = 0 = N_{13}$ and $M_{23} = 0 = N_{23}$ yield

$$C_{35} = 0,\ C_{15} + C_{25} = 0,\ C_{34} = 0,\ C_{14} + C_{24} = 0. \quad (2.6)$$

Equations $M_{11} = M_{22}$ and $N_{11} = N_{22}$ respectively give

$$\tau_{11}(C_{11} - C_{22}) + \tau_{33}(C_{13} - C_{23}) = 0,\ \sigma_{11}(C_{11} - C_{22}) + \sigma_{33}(C_{13} - C_{23}) = 0,$$

which imply

$$C_{11} = C_{22},\ C_{13} = C_{23} \quad (2.7)$$

The last equality of (2.7) compared with the last equality of (2.3) gives

$$C_{44} = C_{55}. \quad (2.8)$$

Conditions (2.3), (2.4)-(2.8) are common to all materials possessing an $n-$ fold axis of symmetry with $n = 3, 4$ or 6. To proceed further, we have to consider them separately. The component $C_{11} = C_{1111}$ transforms, after a rotation about x_3- axis through θ, $(\theta = 2\pi/n, n = 3, 4 \text{ or } 6)$, as

$$C_{11}\cos^4\theta + 4C_{16}\cos^3\theta\sin\theta + 2C_{12}\cos^2\theta\sin^2\theta + 4C_{66}\cos^2\theta\sin^2\theta + 4C_{26}\cos\theta\sin^3\theta + C_{22}\sin^4\theta.$$

Setting $C_{22} = C_{11}$ in the above expression, (see (2.7)), and equating it to C_{11}, we obtain

$$(C_{11} - C_{12} - 2C_{66})\sin^2\theta\cos^2\theta = 2\cos\theta\sin\theta(C_{16}\cos^2\theta + C_{26}\sin^2\theta). \quad (2.9)$$

A similar calculation with C_{16} gives

$$(C_{11} - C_{12} - 2C_{66})(\sin^3\theta\cos\theta - \cos^3\theta\sin\theta) = 8\sin^2\theta\cos^2\theta C_{16}. \quad (2.10)$$

For a tetragonal material, $\theta = \pi/2$, the above equations are identically satisfied and (2.9) and (2.10) impose no further conditions on the components of the elasticity tensor for this class. However an A_4 axis also behaves as an A_2 axis hence, in addition to (2.3), (2.5)-(2.8), we also have $0 = C_{14} = C_{15} = C_{24} = C_{25} = C_{46} = C_{56}$ [see (1.3)]. Thus the matrix representation of the elasticity tensor for a tetragonal material is the following

$$(\mathrm{C})_{\text{tetragonal}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{11} & C_{13} & 0 & 0 & -C_{16} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ C_{16} & -C_{16} & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad (2.11)$$

For a trigonal system, $\sin\theta\cos\theta \neq 0$. Using the condition $C_{16} = -C_{26}$, (see (2.5)), Eqs. (2.9) and (2.10) become

$$C_{11} - C_{12} - 2C_{66} = 2(\cot\theta - \tan\theta)C_{16},$$
$$(\tan\theta - \cot\theta)(C_{11} - C_{12} - 2C_{66}) = 8C_{16}.$$

The determinant of the above system fails to vanish if $\theta = 2\pi/3$. Hence, for a trigonal system

$$C_{16} = 0,\ C_{11} - C_{12} - 2C_{66} = 0. \qquad (2.12)$$

The matrix representation of such a system follows

$$(\mathrm{C})_{\text{trigonal}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & 0 \\ C_{12} & C_{11} & C_{13} & -C_{14} & -C_{15} & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & -C_{15} \\ C_{15} & -C_{15} & 0 & 0 & C_{44} & C_{14} \\ 0 & 0 & 0 & -C_{15} & C_{14} & (C_{11}-C_{12})/2 \end{bmatrix}. \qquad (2.13)$$

Finally since an A_6 axis is simultaneously an A_2 axis as well as an A_3 axis, the matrix representing a hexagonal material should incorporate properties of both (1.3) and (2.13). Thus we get

$$(\mathrm{C})_{\text{hexagonal}} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (C_{11}-C_{12})/2 \end{bmatrix}.$$

The elasticity tensor for a hexagonal material has five independent components.

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Original Article

Almost Unbiased Ratio Type Estimators of Population Mean using Two Auxiliary Attributes in Systematic Sampling

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Abstract: In this paper, we propose a class of almost unbiased ratio-type estimators for estimating the finite population mean using systematic sampling when information on two auxiliary attributes is available. The proposed class of estimators is always more efficient than the usual mean estimator, i.e., Niak and Gupta (1996) estimator and Sahai and Ray (1980) estimator. Two real data sets are used for numerical comparison.

Keywords: Attributes, bias, mean square error (*MSE*), efficiency

INTRODUCTION

In survey sampling, the use of auxiliary information increases the precision of the estimators when the study variable (y) is correlated with the auxiliary variable (x). But in several practical situations, the information is available in the form of auxiliary attributes. For example, season (summer or winter) is a good auxiliary attribute while estimating the amount of gas bill. Similarly, type of feed (mother feed or not) can be used as an auxiliary attribute while dealing with the weight of the infants. Niak and Gupta [3] utilized the information on the auxiliary attribute and introduced the ratio estimator. Here, we extend the work of Niak and Gupta [3], and introduce a class of almost unbiased ratio-type estimators for population mean when using two auxiliary attributes. We assume that both auxiliary attributes has significant point bi-serial correlation with the study variable. Two examples of such situations are:

(i) The study variable (y) is the prices of real-estate and the auxiliary attribute (ϕ) is the type of land (commercial or noncommercial) and another auxiliary attribute (ψ) is the status of the location of the place (front or back)

(ii) The study variable (y) is the production of wheat and the auxiliary attribute (ϕ) is the variety (say HYV or other) and another auxiliary attribute (ψ) is the type of land (irrigated or non-irrigated).

In the above examples we expect a point bi-serial correlation between the study variable and the auxiliary attributes. There also exists correlation between the auxiliary attributes called phi-correlation.

Consider a finite population $U = (U_1, U_2, ..., U_N)$ consisting of N units. It is assumed that $N = nk$, where n and k are the integers. Thus we have k samples each having n units. We randomly select one sample from these k samples and observe each value of the study variable y and auxiliary attributes (ϕ and ψ). Let y_{ij}, ϕ_{ij} and ψ_{ij} denote the observations on variables y, ϕ and ψ respectively for the i^{th} unit of the j^{th} sample. It is assume that ϕ_{ij} =1, if the i^{th} unit of the j^{th} sample possess attribute ϕ and $\phi_{ij} = 0$, otherwise. Similarly ψ_{ij} =1, if the i^{th} unit of the j^{th} sample possess attribute ψ and $\psi_{ij} = 0$, otherwise. Let $A_1 = \sum_{j=1}^{k}\sum_{i=1}^{n}\phi_{ij}$ and

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$A_2 = \sum_{j=1}^{k}\sum_{i=1}^{n}\psi_{ij}$ denote the total number of units in the population possess attributes ϕ and ψ respectively. Also let $a_1 = \sum_{i=1}^{n}\phi_i$ and $a_2 = \sum_{i=1}^{n}\psi_i$ denote the total number of units in the sample possess attributes ϕ and ψ respectively. Let $P_\phi = \frac{A_1}{N}$ and $P_\psi = \frac{A_2}{N}$ denote the proportion of units in the population possess attributes ϕ and ψ respectively. Also let $p_\phi = \frac{a_1}{n}$ and $p_\psi = \frac{a_2}{n}$ denote the proportion of units in the sample possess attributes ϕ and ψ respectively.

Now we discuss different estimators available in the literature.

To obtain the bias and *MSE* of different estimators, the error terms and their expectations are given below.

Let $e_y = \frac{(\bar{y}-\bar{Y})}{\bar{Y}}$, $e_\phi = \frac{(p_\phi - P_\phi)}{P_\phi}$ and $e_\psi = \frac{(p_\psi - P_\psi)}{P_\psi}$ such that

$E(e_i) = 0$, $i = y,\ \phi,\ \psi$.

To first order of approximation, we have

$E(e_y^2) = \theta g_y C_y^2$, $E(e_\phi^2) = \theta g_\phi C_\phi^2$,

$E(e_\psi^2) = \theta g_\psi C_\psi^2$,

$E(e_y e_\phi) = \theta\sqrt{g_y}\sqrt{g_\phi}\rho_{y\phi}C_y C_\phi$,

$E(e_y e_\psi) = \theta\sqrt{g_y}\sqrt{g_\psi}\rho_{y\psi}C_y C_\psi$ and

$E(e_\phi e_\psi) = \theta\sqrt{g_\phi}\sqrt{g_\psi}\rho_{\phi\psi}C_\phi C_\psi$,

where $\theta = \frac{1}{n}\left(1-\frac{1}{N}\right)$, $g_y = 1+(n-1)\rho_y$,

$g_\phi = 1+(n-1)\rho_\phi$, $g_\psi = 1+(n-1)\rho_\psi$. Here

$$\rho_y = \frac{E(y_{ij}-\bar{Y})(y_{iu}-\bar{Y})}{E(y_{ij}-\bar{Y})^2},$$

$$\rho_\phi = \frac{E(\phi_{ij}-P_\phi)(\phi_{iu}-P_\phi)}{E(\phi_{ij}-P_\phi)^2} \text{ and}$$

$$\rho_\psi = \frac{E(\psi_{ij}-P_\psi)(\psi_{iu}-P_\psi)}{E(\psi_{ij}-P_\psi)^2}$$

be the correlation coefficients between pairs of the units that are within the same systematic sample. Also $\rho_{y\phi} = \frac{S_{y\phi}}{S_y S_\phi}$ and $\rho_{y\psi} = \frac{S_{y\psi}}{S_y S_\psi}$ are the point bi-serial correlation coefficients between (y,ϕ) and (y,ψ) respectively. Similarly $\rho_{\phi\psi} = \frac{S_{\phi\psi}}{S_\phi S_\psi}$ is the phi-correlation coefficient between the two auxiliary attributes.

The variances, covariances and coefficient of variations are given below:

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2,$$

$$S_\phi^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\phi_i-P_\phi)^2,$$

$$S_\psi^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\psi_i-P_\psi)^2$$

$$S_{y\phi} = \frac{1}{N-1}\left(\sum_{i=1}^{N}y_i\phi_i - NP_\phi\bar{Y}\right),$$

$$S_{y\psi} = \frac{1}{N-1}\left(\sum_{i=1}^{N}y_i\psi_i - NP_\psi\bar{Y}\right),$$

$$S_{\phi\psi} = \frac{1}{N-1}\left(\sum_{i=1}^{N}\phi_i\psi_i - NP_\phi P_\psi\right),$$

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2},\ C_\phi^2 = \frac{S_\phi^2}{P_\phi^2} \text{ and } C_\psi^2 = \frac{S_\psi^2}{P_\psi^2}.$$

Now we discuss the different estimators.

(i) *Usual sample mean estimator*

The variance of the usual mean estimator $(\bar{y})$ in systematic sampling is given by

$$Var(\bar{y}) = \theta\bar{Y}^2 g_y C_y^2. \quad (1)$$

(ii) *Naik and Gupta estimator*

Naik and Gupta [3] proposed a ratio estimator for population mean which is given by

$$\bar{y}_{NG} = \bar{y}\left(\frac{P_\phi}{p_\phi}\right). \quad (2)$$

The bias and *MSE* of $\bar{y}_{NG}$, to first order of approximation, in systematic sampling is given by

$$B(\bar{y}_{NG}) \cong \theta\bar{Y}g_y\left(\frac{C_\phi^2}{\rho_\phi^{*2}} - \frac{1}{\rho_\phi^*}\rho_{y\phi}C_yC_\phi\right), \quad (3)$$

and

$$MSE(\bar{y}_{NG}) \cong \theta\bar{Y}^2 g_y C_y^2\left[1+\frac{C_\phi^2}{\rho_\phi^{*2}C_y^2} - 2\frac{\rho_{y\phi}}{\rho_\phi^*}\frac{C_\phi}{C_y}\right], \quad (4)$$

where $\rho_\phi^* = \dfrac{\sqrt{g_y}}{\sqrt{g_\phi}}$.

(iii) Sahai and Ray [4] estimator in systematic sampling when using the auxiliary attribute is given by

$$\bar{y}_{SR} = \bar{y}\left(2-\frac{p_\phi}{P_\phi}\right). \quad (5)$$

The bias of $\bar{y}_{SR}$, to first order of approximation, is given by

$$B(\bar{y}_{SR}) = -\theta\bar{Y}g_y\frac{\rho_{y\phi}}{\rho_\phi^*}C_yC_\phi. \quad (6)$$

From (3) and (6), we see that

$$|B(\bar{y}_{NG})| \cong |B(\bar{y}_{SR})| + \theta\bar{Y}g_y\frac{C_\phi^2}{\rho_\phi^{*2}}.$$

This shows that

$$|B(\bar{y}_{SR})| < |B(\bar{y}_{NG})|.$$

The *MSE* of $\bar{y}_{SR}$ to first order of approximation is given by

$$MSE(\bar{y}_{SR}) = \theta\bar{Y}^2 g_y C_y^2\left[1+\frac{C_\phi^2}{\rho_\phi^{*2}C_y^2} - 2\frac{\rho_{y\phi}}{\rho_\phi^*}\frac{C_\phi}{C_y}\right], \quad (7)$$

which is equal to the *MSE* of the Niak and Gupta (1996) estimator $\bar{y}_{NG}$.

Proposed Estimators

We propose a following class of estimators for population mean $\bar{Y}$ in systematic sampling which is given by

$$\bar{y}_P = \sum_{r=1}^{4}\alpha_r d_r, \quad (8)$$

where

$$d_1 = \bar{y},\ d_2 = \bar{y}\left(\frac{P_\phi^*}{p_\phi^*}\right), \quad d_3 = \bar{y}\left(2-\frac{p_\psi^*}{P_\psi^*}\right)$$

and

$$d_4 = \bar{y}\left(2-\frac{p_\phi^*}{P_\phi^*}\right)\left(\frac{P_\psi^*}{p_\psi^*}\right)$$

such that $\sum_{r=1}^{4}\alpha_r = 1, \alpha_r \in R$ and $\alpha_r(r=1,2,3,4)$ denote the unknown constants to be determined.

Let $p_\phi^* = p_\phi + \tau$, $P_\phi^* = P_\phi + \tau$, $p_\psi^* = p_\psi + \tau$ and $P_\psi^* = P_\psi + \tau$,

where $\tau = \dfrac{1+\theta(C_{y\phi}+C_{y\psi})}{1+\theta(C_\phi^2+C_\psi^2)} > 0$ (see Shabbir and Yaab [5]).

Rewriting (8), we have

$$\bar{y}_P = \bar{y}\left[\alpha_1 + \alpha_2\left(\frac{P_\phi^*}{p_\phi^*}\right) + \alpha_3\left(2-\frac{p_\psi^*}{P_\psi^*}\right) + \alpha_4\left(2-\frac{p_\phi^*}{P_\phi^*}\right)\left(\frac{P_\psi^*}{p_\psi^*}\right)\right] \quad (9)$$

Expressing (9) in terms of *e*'s, to first order of approximation, we have

$$\bar{y}_P - \bar{Y} = \bar{Y}\left[e_y - \gamma_1 q_\phi e_\phi - \gamma_2 q_\psi e_\phi + \alpha_2 q_\phi^2 e_\phi^2 + \alpha_4 q_\phi^2 e_\psi^2 + \alpha_4 q_\phi q_\psi e_\phi e_\psi + \gamma_1 q_\phi e_y e_\phi + \gamma_2 q_\psi e_y e_\psi\right], \quad (10)$$

where $\gamma_1 = \alpha_2 + \alpha_4$, $\gamma_2 = \alpha_3 + \alpha_4$,

$$q_\phi = \left(\frac{P_\phi}{P_\phi + \tau}\right) \text{ and } q_\psi = \left(\frac{P_\psi}{P_\psi + \tau}\right).$$

Solving (10), the *MSE* of $\bar{y}_P$, to first order of approximation, is given by

$$MSE(\bar{y}_P) = \theta\bar{Y}^2\left[g_yC_y^2 + \gamma_1^2 q_\phi^2 g_\phi C_\phi^2 + \gamma_2^2 q_\psi^2 g_\psi C_\psi^2 - 2\gamma_1 q_\phi\sqrt{g_y}\sqrt{g_\phi}\rho_{y\phi}C_yC_\phi - 2\gamma_2 q_\psi\sqrt{g_y}\sqrt{g_\psi}\rho_{y\psi}C_yC_\psi + 2\gamma_1\gamma_2 q_\phi q_\psi g_\phi g_\psi \rho_{\phi\psi}C_\phi C_\psi\right]. \quad (11)$$

From (11), the optimum values of γ_1 and γ_2 are

$$\gamma_1 = \frac{1}{\left(1-\rho_{\phi\psi}^2\right)}\left(\frac{1}{q_\phi}\right)\left(\frac{C_y}{C_\phi}\right)\rho_\phi^*\left(\rho_{y\phi} - \rho_{y\psi}\rho_{\phi\psi}\right) \quad (12)$$

and

$$\gamma_2 = \frac{1}{\left(1-\rho_{\phi\psi}^2\right)}\left(\frac{1}{q_\psi}\right)\left(\frac{C_y}{C_\psi}\right)\rho_\psi^*\left(\rho_{y\psi} - \rho_{y\phi}\rho_{\phi\psi}\right). \quad (13)$$

Substituting the optimum values of γ_1 and γ_2 in (11), we get the minimum *MSE* of $\bar{y}_P$, which is given by

$$MSE(\bar{y}_P)_{\min} \cong \theta\bar{Y}^2 g_y C_y^2(1-R_{y.\phi\psi}^2), \quad (14)$$

where $R_{y.\phi\psi}^2 = \dfrac{\rho_{y\phi}^2 + \rho_{y\psi}^2 - 2\rho_{y\phi}\rho_{y\psi}\rho_{\phi\psi}}{1-\rho_{\phi\psi}^2}$ is the multiple correlation coefficient of y on ϕ and ψ.

Biases of Different Estimators

The bias of $d_r\ (r = 1,2,3,4)$, to first order of approximation, are given by

$$B(d_1) = 0$$

$$B(d_2) \cong \theta\bar{Y}g_y q_\phi\left(\frac{C_\phi^2}{\rho_\phi^{*2}}q_\psi - \frac{1}{\rho_\phi^*}\rho_{y\phi}C_yC_\phi\right)$$

$$B(d_3) \cong -\theta\bar{Y}g_y q_\psi \frac{1}{\rho_\psi^*}\rho_{y\psi}C_yC_\psi$$

and

$$B(d_4) \cong \theta\bar{Y}g_y\left[\frac{1}{\rho_\psi^{*2}}q_\psi^2C_\psi^2 - \frac{1}{\rho_\psi^*}q_\psi\rho_{y\psi}C_yC_\psi - \frac{1}{\rho_\phi^*}q_\phi\rho_{y\phi}C_yC_\phi + q_\phi q_\psi\frac{1}{\rho_\phi^*}\frac{1}{\rho_\psi^*}\rho_{\phi\psi}C_\phi C_\psi\right].$$

From (10), (12) and (13), we have

$$\alpha_2 + \alpha_4 = \frac{1}{\left(1-\rho_{\phi\psi}^2\right)}\left(\frac{1}{q_\phi}\right)\left(\frac{C_y}{C_\phi}\right)\rho_\phi^*\left(\rho_{y\phi} - \rho_{y\psi}\rho_{\phi\psi}\right) \quad (15)$$

and

$$\alpha_3 + \alpha_4 = \frac{1}{(1-\rho_{\phi\psi}^2)}\left(\frac{1}{q_\psi}\right)\left(\frac{C_y}{C_\psi}\right)\rho_\psi^*\left(\rho_{y\psi} - \rho_{y\phi}\rho_{\phi\psi}\right), \quad (16)$$

also we have

$$\sum_{r=1}^{4}\alpha_r = 1\ . \quad (17)$$

Note: From (15)-(17), there are four unknown quantities to be determined from only three equations. It is therefore not possible to obtain the unique values for the constants α_r's (r =1,2,3,4) to be used for bias reduction. To get the unique values for these constants α_r's we shall impose the additional linear restriction as

$$\sum_{r=1}^{4}\alpha_r B(d_r) = 0, \quad (18)$$

where $B(d_r)$ stands for bias in r^{th} (r =1, 2, 3,4) estimator.

Equations (10), (17) and (18) can be written as

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ B(d_1) & B(d_2) & B(d_3) & B(d_4) \end{bmatrix}\begin{bmatrix}\alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4\end{bmatrix} = \begin{bmatrix}\gamma_1 \\ \gamma_2 \\ 1 \\ 0\end{bmatrix} \quad (19)$$

Solving for α_r's $(r = 1,2,3,4)$, we have

$$\alpha_1 = \frac{\gamma_1\{B(d_3)-B(d_4)\} + \gamma_2\{B(d_2)-B(d_4)\} + B(d_4) - B(d_3) - B(d_2)}{B(d_1)-B(d_2)-B(d_3)+B(d_4)} = \alpha_1^*$$

$$\alpha_2 = \frac{\gamma_1\{B(d_4)-B(d_3)\} + \gamma_2\{B(d_3)-B(d_1)\} + B(d_1)}{B(d_1)-B(d_2)-B(d_3)+B(d_4)} = \alpha_2^*$$

$$\alpha_3 = \frac{\gamma_1\{B(d_2)-B(d_4)\} + \gamma_2\{B(d_4)-B(d_2)\} + B(d_1)}{B(d_1)-B(d_2)-B(d_3)+B(d_4)} = \alpha_3^*$$

$$\alpha_4 = \frac{\gamma_1\{B(d_1)-B(d_2)\} - \gamma_2\{B(d_3)-B(d_1)\} - B(d_1)}{B(d_1)-B(d_2)-B(d_3)+B(d_4)} = \alpha_4^*$$

Substituting the optimum values of α_r's $(r = 1,2,3,4)$ in (9), we get an almost unbiased estimator for $\bar{Y}$ which is given by

$$\bar{y}_P^* = \bar{y}\left[\alpha_1^* + \alpha_2^*\left(\frac{P_\phi^*}{p_\phi^*}\right) + \alpha_3^*\left(2-\frac{p_\psi^*}{P_\psi^*}\right) + \alpha_4^*\left(2-\frac{p_\phi^*}{P_\phi^*}\right)\left(\frac{P_\psi^*}{p_\psi^*}\right)\right]. \quad (20)$$

Solving (20), we have

$$Var(\bar{y}_P^*) \cong \theta\bar{Y}^2 g_y C_y^2(1-R_{y.\phi\psi}^2),$$

which is equal to minimum *MSE* of $\bar{y}_P$ given in (14).

Efficiency Comparison

Now we compare different estimators considered here with the proposed estimator.

Condition (i)

$Var(\bar{y}_P^*) < Var(\bar{y})$ if

$Var(\bar{y}) - Var(\bar{y})(1 - R^2_{y.\phi\psi}) > 0$, or if

$Var(\bar{y}) R^2_{y.\phi\psi} > 0$,

which is always true

Condition (ii)

$Var(\bar{y}_P^*) < \{MSE(\bar{y}_{NG})$ or $MSE(\bar{y}_{SR})\}$ if

$\{MSE(\bar{y}_{NG})$or $MSE(\bar{y}_{SR})\} - Var(\bar{y}_P^*) > 0$ or if

$$Var(\bar{y})\left[1 + \frac{C_\phi^2}{\rho_\phi^{*2} C_y^2} - 2\frac{\rho_{y\phi}}{\rho_\phi^*}\frac{C_\phi}{C_y}\right] - Var(\bar{y})(1 - R^2_{y.\phi\psi}) > 0,$$

or if

$$1 + \frac{C_\phi^2}{\rho_\phi^{*2} C_y^2} - 2\frac{\rho_{y\phi}}{\rho_\phi^*}\frac{C_\phi}{C_y} - 1 + R^2_{y.\phi\psi} > 0, \text{ or if}$$

$$\frac{C_\phi^2}{\rho_\phi^{*2} C_y^2} - 2\frac{\rho_{y\phi}}{\rho_\phi^*}\frac{C_\phi}{C_y} + \rho_{y\phi}^2 - \rho_{y\phi}^2 + R^2_{y.\phi\psi} > 0, \text{ or}$$

if

$$\left(\frac{C_\phi}{\rho_\phi^* C_y} - \rho_{y\phi}\right)^2 \left(1 - \rho_{\phi\psi}^2\right) - \rho_{y\phi}^2\left(1 - \rho_{\phi\psi}^2\right)$$

$$+ \frac{\rho_{y\phi}^2 + \rho_{y\psi}^2 - 2\rho_{y\phi}\rho_{y\psi}\rho_{\phi\psi}}{(1 - \rho_{\phi\psi}^2)} > 0, \text{ or if}$$

$$\left(\frac{C_\phi}{\rho_\phi^* C_y} - \rho_{y\phi}\right)^2 (1 - \rho_{\phi\psi}^2) + (\rho_{y\phi}\rho_{\phi\psi} - \rho_{y\psi})^2 > 0,$$

which is always true.

Empirical Study

We use the following two populations for comparison of estimators

.

Population 1: Source: (Government of Pakistan [1])

$y =$ the production of all vegetables in tonnes for year 2004-05

$\phi = 1$, if the cultivation vegetables area is more than 2500 hectares in 2004-05

= 0, otherwise

$\psi = 1$, if the cultivation vegetables area is more than 2500 hectares in 2003-04

=0, otherwise.

$N = 112$, $n = 16$, $k = 7$, $\bar{Y} = 27217.787$,

$P_\phi = 0.27678$, $P_\psi = 0.28571$, $C_y = 1.50736$,

$C_\phi = 1.62371$, $C_\psi = 1.58824$,

$\rho_{y\phi} = 0.69060$, $\rho_{y\psi} = 0.69018$,

$\rho_{\phi\psi} = 0.93398$, $\rho_y = -0.02502$,

$\rho_\phi = -0.01994$, $\rho_\psi = -0.01500$,

$\tau = 0.91532$.

Population 2: (Source: Kadilar and Cingi [2])

$y =$ Production of apple in black sea region of Turkey in 1999.

$\phi = 1$, if number of apple trees are less than 50000 in black sea region of Turkey in1999

$= 0$, otherwise

$\psi = 1$, if production of apples is less than 1000 in black sea region of Turkey in 1998

$= 0$ otherwise.

$N = 204$, $n = 17$, $k = 12$, $\bar{Y} = 966.955$,

$P_\phi = 0.10784$, $P_\psi = 0.21078$, $C_y = 2.47143$,

$C_\phi = 2.88331$, $C_\psi = 1.93975$, $\rho_{y\phi} = 0.604$,

$\rho_{y\psi} = 0.522$, $\rho_{\phi\psi} = 0.518$, $\rho_y = -0.014$,

$\rho_\phi = -0.038$, $\rho_\psi = -0.002$, $\tau = 0.81983$

Table 1. $|Bias|$ of estimators.

Estimator	Population 1		Population 2	
	$\tau = 0.91532$	$\tau = 0$	$\tau = 0.81983$	$\tau = 0$
d_2	267.468	1218.771	13.032	49.534
d_3	457.006	1921.080	24.996	122.220
d_4	542.237	2442.667	29.515	50 .364

Table 1 shows that when τ is used the amount of bias of different estimators are reduced considerably corresponding to the bias when $\tau = 0$. So τ plays an important role in reducing the bias in different estimators.

Table 2. Percent relative efficiency of different estimators with respect to $\bar{y}$.

Estimator	**Population 1**	**Population 2**
$\bar{y}$	100.000	100.000
$\bar{y}_{NG}$ or $\bar{y}_{SR}$	137.759	145.878
$\bar{y}_{P}^{*}$	197.206	173.834

In Table 2, it is observed that the proposed estimator $\bar{y}_{P}^{*}$ is more efficient than the usual mean estimator $(\bar{y})$, Naik and Gupta [3] estimator $(\bar{y}_{NG})$ as well as Sahai and Ray [4] estimator $(\bar{y}_{SR})$ in both data sets.

CONCLUSIONS

We propose almost unbiased ratio-type estimators and comparisons are made both theoretically and numerically using two auxiliary attributes. It is observed that the proposed class of estimators always perform better than the all other considered estimators which are expected results. Although Naik and Gupta [3] estimator $(\bar{y}_{NG})$ and Sahai and Ray [4] estimator $(\bar{y}_{SR})$ are equally efficient but Sahai and Ray [4] estimator is preferable because of least bias and *MSE*. Overall the proposed estimator $\bar{y}_{P}^{*}$ when using two auxiliary attributes is preferable.

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Original Article

On Some Expansion Formulae Involving a Basic Analogue of Generalized Hypergeometric Function

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Abstract: In the present paper an expansion formulae for a basic analogue I-function have been derived by the applications of the q-Leibniz rule for the type q-derivatives of a product of two functions. Expansion formulae involving a basic analogue of Fox's H-function, Meijer's G-function and MacRobert's E-function have been derived as special cases of the main results.

Keywords: q-Leibniz rule, Weyl fractional, q-integral operator, Fox's H-function, I-function
(AMS SUBJECT CLASSFICATION: 33D60 and 26A33)

INTRODUCTION

Yadav and Purohit [1] introduced a new q-extension of the lebniz rule for the derivatives of a product of two basic functions in terms of a finite q-series involving Weyl type q-derivatives of the functions in the following manner:

$$ {}_zD^{\alpha}_{\infty,q}\left\{U(z)V(z)\right\} = \sum_{r=0}^{\alpha}\frac{(-1)^r q^{r(r+1)/2}\left(q^{-\alpha};q\right)_r}{(q;q)_r}\, {}_zD^{\alpha-r}_{\infty,q}\left\{U(z)\right\}\, {}_zD^{\alpha}_{\infty,q}\left\{V\left(zq^{\alpha-r}\right)\right\}, \qquad (1.1)$$

$U(z)$ and $V(z)$ are two functions and the fractional q-differential operator ${}_zD^{\alpha}_{\infty,q}(.)$ of Weyl type is given by

$$ {}_zD^{\alpha}_{\infty,q}\left\{f(z)\right\} = \frac{q^{-\alpha(1+\alpha)/2}}{\Gamma_q(-\alpha)}\int_z^{\infty}(t-z)_{-\alpha-1} f(tq^{1+\alpha})d(t;q), \qquad (1.2)$$

where $\mathrm{Re}(\alpha)<0$ and

$$ (x-y)_v = x^v\prod_{n=0}^{\infty}\left[\frac{1-\left(y/x\right)q^n}{1-\left(y/x\right)q^{v+n}}\right], \qquad (1.3)$$

The basic integration cf. Gasper and Rehman [2], is defined as:

$$\int_z^{\infty} f(t)d(t;q) = z(1-q)\sum_{k=1}^{\infty}q^{-k}f\left(zq^{-k}\right). \qquad (1.4)$$

In view of the relation (1.4), operator (1.2) can be expressed as:

$$ {}_zD^{\alpha}_{\infty,q}\left\{f(z)\right\} = \frac{q^{\alpha(1-\alpha)/2}z^{-\alpha}(1-q)}{\Gamma_q(-\alpha)}\sum_{k=0}^{\infty}q^{\alpha k}(1-q^{k+1})_{-\alpha-1}f\left(zq^{\alpha-k}\right), \qquad (1.5)$$

where $\mathrm{Re}(\alpha)<0$.

In particular, for $f(z)=z^{-p}$, the equation (1.5) yields to

$$ {}_zD^{\alpha}_{\infty,q}\left\{z^{-p}\right\} = \frac{\Gamma_q(p+\alpha)}{\Gamma_q(p)}q^{-\alpha p+\alpha(1-\alpha)/2}z^{-p-\alpha}, \qquad (1.6)$$

where $\mathrm{Re}(\alpha)<0$.

We shall make use of the following notations and definitions in the sequel:
For real or complex a and $|q|<1$, the q-shifted factorial is defined as:

$$\left(a;q\right)_n = \begin{cases}1, & if\ n=0\\ (1-a)(1-aq)...(1-aq^{n-1}), & if\ n\in N\end{cases} \qquad (1.7)$$

In terms of the q-gamma function, (1.7) can be expressed as

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$$(a;q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)}, n>0 \qquad (1.8)$$

where the q-gamma function cf. Gasper and Rahman , is given by

$$\Gamma_q(a) = \frac{(q;q)_\infty}{\left(q^a;q\right)_\infty (1-q)^{a-1}}, \qquad (1.9)$$

where $a \neq 0,-1,-2,\ldots$.

The basic analogue of the I-function Saxena [3] in terms of Mellin-Barnes type basic contour integral is in the following manner:

$$I^{m,n}_{p_i,q_i;r}\left[z;q\left|^{(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p}}_{(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q}}\right.\right] =$$

$$\frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m} G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n} G\left(q^{1-a_j+\alpha_j s}\right)\pi z^s}{\sum_{i=1}^{r}\left\{\prod_{j=n+1}^{p_i} G\left(q^{b_{ji}-\beta_{ji}s}\right)\prod_{j=m+1}^{q_i} G\left(q^{1-a_{ji}+\alpha_{ji}s}\right)\right\}} ds \qquad (1.10)$$

where

$$G(q^\alpha) = \left\{\prod_{n=0}^{\infty}\left(1-q^{\alpha+n}\right)\right\}^{-1} = \frac{1}{\left(q^\alpha;q\right)_\infty} \qquad (1.11)$$

and $0 \le m \le q_i, 0 \le n \le p_i; \alpha_j$ and β_j are all positive integers. The contour C is a line parallel to $\mathrm{Re}(ws)=0$, with indentations, if necessary, in such a manner that all the poles of $G\left(q^{b_j-\beta_j s}\right), 1 \le j \le m$, are to the right, and those of $G\left(q^{1-a_j+\alpha_j s}\right), 1 \le j \le n$ to the left of C. The integral converges if $\mathrm{Re}[s\log(z) - \log\sin\pi s] < 0$ for large values of $|s|$ on the contour C. That is, if $|\left\{\arg(z) - w_2 w_1^{-1}\log|z|\right\}| < \pi$ where $|q|<1, \log q = -w = -(w_1+iw_2), w, w_1, w_2$ are definite quantities. w_1 and w_2 being real.

We shall use the following notations:

$A^* = (a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}$ and

$B^* = (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}$

For $r=1$, the I-function reduces to Fox's H-function and eq. (1.10) reduces to the q-analogue of the Fox's H-function due to Saxena et. al. [4], namely

$$H^{m,n}_{p,q}\left[z;q\left|^{(a,\alpha)}_{(b,\beta)}\right.\right] =$$

$$\frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m} G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n} G\left(q^{1-a_j-\alpha_j s}\right)\pi z^s}{\prod_{j=m+1}^{q} G\left(q^{1-b_j-\beta_j s}\right)\prod_{j=n+1}^{p} G\left(q^{a_j+\alpha_j s}\right)\sin\pi s} ds, \qquad (1.12)$$

where $0 \le m \le q, 0 \le n \le p$ and $\mathrm{Re}[s\log(z) - \log\sin\pi s] < 0$.

For $\alpha_j = \beta_j = 1, j = 1,\ldots,q$ the definition (1.12) reduces to the q-analogue of the Meijer's G-function due to Saxena et al. namely

$$H^{m,n}_{p,q}\left[z;q\left|^{(a,1)}_{(b,1)}\right.\right] = G^{m,n}_{p,q}\left[z;q\left|^{a_1,\ldots,a_p}_{b_1,\ldots,b_q}\right.\right] =$$

$$\frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{m} G\left(q^{b_j-s}\right)\prod_{j=1}^{n} G\left(q^{1-a_j-s}\right)\pi z^s}{\prod_{j=m+1}^{q} G\left(q^{1-b_j-s}\right)\prod_{j=n+1}^{p} G\left(q^{a_j+s}\right)\sin\pi s} ds \qquad (1.13)$$

where $0 \le m \le q, 0 \le n \le p$ and $\mathrm{Re}[a\log(z) - \log\sin\pi s] < 0$.

Further, if we set $n=0$ and $m=q$ in the equation (1.13), we get the basic analogue of MacRobert's E-function due to Agarwal [5], namely,

$$G^{m,0}_{p,q}\left[z;q\left|^{a_1,\ldots,a_A}_{b_1,\ldots,b_B}\right.\right] = E_q\left[q;b_j:p;a_j:z\right] =$$

$$\frac{1}{2\pi i}\int_C \frac{\prod_{j=1}^{q} G\left(q^{b_j-s}\right)\pi z^s}{\prod_{j=1}^{p} G\left(q^{a_j-s}\right)G\left(q^{b_j-s}\right)\sin\pi s} ds, \qquad (1.14)$$

where $\mathrm{Re}[s\log(z) - \log\sin\pi s] < 0$.

The Fox's H-function and Meijer's G-function have been studied in detail by several mathematicians for their theoretical and applications point of view. These functions have found wide ranging applications in mathematical, physical, biological and statistical sciences. It would be interesting to observe that almost all the classical special functions are the particular cases of the Fox's H-function. A detailed account of various classical special functions expressible in terms of Meijer's G-function or Fox's H-function along with their applications to the aforementioned field can be found in the research monographs by Mathai and Saxena [6,7].

A new generalization was considered by Saxena et. al. in the form of the q-extansions of the Fox's H-function and Meijer's G-function by means of the Mellin-Bernes type of basic integral. The advantage of these new extansions of the Fox's H-function and Meijer's G-functions lies in the fact that a number of q-special functions including the basic hypergeometric functions, happens to be the particular cases of the $H_q(.)$ and $G_q(.)$ functions, thus widening the scope for further applications. In a paper, Saxena and Kumar [8], besides proving some interesting relations, have established an important limit formula for the $H_q(.)$ functions when q tends to 1. Various basic functions expressible in terms of the basic analogue of Fox's H-function or basic Meijer's G-function with their applications can be found in the research papers due to Saxena et. al. [9] and Yadav and Purohit [10].

In the present paper, we shall explore the possibility for derivation of some expansion formulae involving the basic analogue of the Fox's H-function by the applications of the q-Leibniz rule for the Weyl type q-derivatives of a product of two functions. We also investigate the expansion formulae involving the basic analogues of Meijer's G-function and MacRobert's E-function.

MAIN RESULTS

In this section, the author will establish certain results associated with the basic analogue of I-function by assigning suitable values to the function $U(z), V(z)$ and α in the q-Leibniz rule (1.1). The main results to be established are as under:

$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[\rho\left(zq^{\mu}\right)^k;q\left|{}^{A^*,(\lambda,k)}_{(\mu+\lambda,k),B^*}\right.\right]=$$

$$\sum_{R=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$

$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[\rho\left(zq^{\mu}\right)^k;q\left|{}^{A^*,(0,k)}_{(R,k),B^*}\right.\right], \qquad (2.1)$$

where $0\le m\le q_i, 0\le n\le p_i, \mathrm{Re}[s\log(z)-\log\sin\pi s] < 0, k\ge 0$ and ρ being any complex quantity.

$$I^{m,n+1}_{p_i+1,q_i+1:r}\left[\rho\left(zq^{\mu}\right)^k;q\left|{}^{(1-\mu-\lambda,-k),A^*}_{B^*,(1-\lambda,-k)}\right.\right]=$$

$$\sum_{R=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$

$$I^{m,n+1}_{p_i+1,q_i+1:r}\left[\rho\left(zq^{\mu}\right)^k;q\left|{}^{(1-R,-k),A^*}_{B^*,(1,-k)}\right.\right] \qquad (2.2)$$

where $0\le m\le q_i, 0\le n\le p_i, \mathrm{Re}[s\log(z)-\log\sin\pi s] < 0, k<0$ and ρ being any complex quantity.

Proof of the Main Result

To prove the result (2.1) and (2.2), we begin with $U(z)=z^{-\lambda}$ and

$$V(z)=I^{m,n}_{p_i,q_i:r}\left[\rho z^k;q\left|{}^{A^*}_{B^*}\right.\right]$$

In equation (1.1.) to obtain

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda}I^{m,n}_{p_i,q_i:r}\left[\rho z^k;q\left|{}^{A^*}_{B^*}\right.\right]\right\}=$$

$$\sum_{r=0}^{\mu}\frac{(-1)^R q^{R(R+1)/2}\left(q^{-\mu};q\right)_R}{(q;q)_R}\,{}_zD^{\mu-R}_{\infty,q}\left\{z^{-\lambda}\right\}$$

$${}_zD^{\alpha}_{\infty,q}\left\{I^{m,n}_{p_i,q_i:r}\left[\rho(z^{\mu-R})^k;q\left|{}^{A^*}_{B^*}\right.\right]\right\} \qquad (2.3)$$

n view of the definition (1.10), the left hand side of equation (2.3) becomes

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda}I^{m,n}_{p_i,q_i:r}\left[\rho z^k;q\left|{}^{A^*}_{B^*}\right.\right]\right\}$$

$$=\frac{1}{2\pi i}\int_C\frac{\prod_{j=1}^{m}G\left(q^{b_j-\beta_j s}\right)\prod_{j=1}^{n}G\left(q^{1-a_j-\alpha_j s}\right)\pi\rho^s}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}G\left(q^{1-b_{ji}+\beta_{ji}s}\right)\prod_{j=n+1}^{p_i}G\left(q^{a_{ji}-\alpha_{ji}s}\right)G\left(q^{1-s}\right)\sin\pi s\right\}}.$$

$${}_zD^{\mu}_{\infty,q}\left\{z^{-(\lambda-ks)}\right\}ds \qquad (2.4)$$

On making use of fractional q-derivative formula (1.6) in the above equation (2.4), we obtain following interesting transformation for the $I_q(.)$ function after certain simplifications:

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda}I^{m,n}_{p_i,q_i:r}\left[\rho z^k;q\left|{}^{A^*}_{B^*}\right.\right]\right\}=\frac{z^{-\lambda-\mu}q^{-\mu\lambda+\mu(1-\mu)/2}}{(1-q)^{\mu}}$$

$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[\rho(zq^{\mu})^k q;\left|{}^{A^*,(\lambda,k)}_{(\mu+\lambda,k),B^*}\right.\right], \qquad (2.5)$$

where $k \geq 0$.

Again, if we take $k < 0$, we obtain the following fractional q-derivative formula for the $I_q(.)$ function, namely

$${}_zD^{\mu}_{\infty,q}\left\{z^{-\lambda} I^{m,n}_{p_i,q_i:r}\left[\rho z^k; q \middle| {}^{A*}_{B*}\right]\right\} = \frac{z^{-\lambda-\mu} q^{-\mu\lambda+\mu(1-\mu)/2}}{(1-q)^{\mu}}$$
$$I^{m,n+1}_{p_i+1,q_i+1:r}\left[\rho(zq^{\mu})^k; q \middle| {}^{(1-\mu-\lambda,-k),A*}_{B*,(1-\lambda,-k)}\right]. \quad (2.6)$$

We now substitute and replace μ by R and then z by $zq^{\mu-R}$ respectively, in equation (2.5) to obtain the following transformation for the $I_q(.)$ function:

$${}_zD^{R}_{\infty,q}\left\{I^{m,n}_{p_i,q_i:r}\left[\rho\left(zq^{\mu-R}\right)^k; q \middle| {}^{A*}_{B*}\right]\right\} = \frac{z^{-R} q^{\frac{R(R+1)}{2}-\mu R}}{(1-q)^{R}}$$
$$I^{m+1,n}_{p_i+1,q_i+1:r}\left[\rho(zq^{\mu})^k; q \middle| {}^{A*,(0,k)}_{(R,k),B*}\right]. \quad (2.7)$$

Further, in view of the result (1.6), one can easily obtain the following relation

$${}_zD^{\mu-R}_{\infty,q}\left\{z^{-\lambda}\right\} = \frac{\Gamma_q(\lambda+\mu-R)}{\Gamma_q(\lambda)} q^{(\mu-R)(1-\mu-R-2\lambda)/2} z^{-\lambda-\mu+R} \quad (2.8)$$

On substituting the values of various expressions involved in the equation (2.3), from equations (2.5), (2.7) and (2.8), we arrive at the main result (2.1).

The proof of the result (2.2) follows similarly when $k < 0$ and by the usages of the transformation formula (2.6) and the relation (2.8).

SPECIAL CASES

In this section, we shall consider some special cases of the main results and deduce certain expansion formulae involving the basic analogue of Fox's H-function , basic analogue of Meijer's G-function and basic analogue of MacRobert's E-function.

If we set $r = 1$, in the main result (2.1), we obtain the following expansion formula involving Fox's H-function, namely

$$H^{m+1,n}_{p+1,q+1}\left[\rho\left(zq^{\mu}\right)^k; q \middle| {}^{(a_j,\alpha_j)_{1,p},(\lambda,k)}_{(\mu+\lambda,k),(b_j,\beta_j)_{1,q}}\right] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}\left(q^{-\mu};q\right)_R\left(q^{\lambda};q\right)_{\mu-R}}{(q;q)_R}$$
$$H^{m+1,n}_{p+1,q+1}\left[\rho(zq^{\mu})^k; q \middle| {}^{(a_j,\alpha_j)_{1,p},(0,k)}_{(R,k),(b_j,\beta_j)_{1,q}}\right], \quad (3.1)$$

where $0 \leq m \leq q, 0 \leq n \leq p, \mathrm{Re}[s\log(z) - \log\sin \pi s] < 0, k \geq 0$ and ρ being any complex quantity.

Similarly, for $r = 1$ and $k = -1$, the main result (2.1) reduces to yet another expansion formula associated with the basic analogue of Fox's H-function, namely

$$H^{m,n+1}_{p+1,q+1}\left[\rho(zq^{\mu})^k; q \middle| {}^{(1-\mu-\lambda,-k),(a_j,\alpha_j)_{1,p}}_{(b_i,\beta_i)_{1,q},(1-\lambda,-k)}\right] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R(q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$H^{m,n+1}_{p+1,q+1}\left[\rho(zq^{\mu})^k; q \middle| {}^{(1-R,-k),(a_j,\alpha_j)_{1,p}}_{(b_i,\beta_i)_{1,q},(1,-k)}\right], \quad (3.2)$$

where $0 \leq m \leq q, 0 \leq n \leq p, \mathrm{Re}[s\log z - \log\sin \pi s] < 0, k < 0$ and ρ being any complex quantity.

If we set $\alpha_j = \beta_i = 1, j = 1,..., p; i = 1,..., q$ and $k = 1$, in (3.1), we obtain the following expansion formula involving Meijer's $G_q(.)$ function, namely

$$G^{m+1,n}_{p+1,q+1}\left[\rho zq^{\mu}; q \middle| {}^{(a_j,1)_{1,p},(\lambda,1)}_{(\mu+\lambda,1),(b_i,1)_{1,q}}\right] = \sum_{R=0}^{\mu} \frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R(q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$
$$G^{m+1,n}_{p+1,q+1}\left[\rho zq^{\mu}; q \middle| {}^{(a_j,1)_{1,p},(0,1)}_{(R,1),(b_i,1)_{1,q}}\right], \quad (3.3)$$

where $0 \leq m \leq q, 0 \leq n \leq p, \mathrm{Re}[s\log z - \log\sin \pi s] < 0$ and ρ being any complex quantity.

Similarly, for $\alpha_j = \beta_i = 1, j = 1,..., p; i = 1,..., q$ and $k = -1$, the result (3.2) reduces to yet another expansion formula associated with the basic analogue of Meijer's $G_q(.)$ function, namely

$$G_{p+1,q+1}^{m,n+1}\left[\rho/(zq^{\mu});q\left|\begin{matrix}(1-\mu-\lambda,1),(a_j,1)\\(b_i,1)_{1,q},(1-\lambda,1)\end{matrix}\right.\right]=$$

$$\sum_{R=0}^{\mu}\frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R(q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$

$$G_{p+1,q+1}^{m,n+1}\left[\rho/(zq^{\mu});q\left|\begin{matrix}(1-R,1),(a_j,1)_{1,p}\\(b_i,1)_{1,q},(1,1)\end{matrix}\right.\right], \qquad (3.4)$$

where $0\le m\le q, 0\le n\le p, \mathrm{Re}[s\log z-\log\sin \pi s]<0$ and ρ being any complex quantity.

Finally, if we set $n=0$ and $m=q$, the result (3.3), yields to an expansion formula involving MacRobert's $E_q(.)$ function, namely

$$E_q[q+1;b_j,\mu+\lambda:p+1;a_j,\lambda:\rho zq^{\mu}]=$$

$$\sum_{R=0}^{\mu}\frac{(-1)^R q^{\frac{R(R+1)}{2}+\lambda R}(q^{-\mu};q)_R(q^{\lambda};q)_{\mu-R}}{(q;q)_R}$$

$$E_q[q+1;b_j,R:p+1,a_j,0:\rho zq^{\mu}], \qquad (3.5)$$

where $\mathrm{Re}[s\log z-\log\sin \pi s]<0$ and ρ being any complex quantity.

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Original Article

Some Properties for Certain Class of Analytic Functions defined by Dziok-Srivastava Operator with Complex Order

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Abstract: In this paper we introduce a new class $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$ of analytic functions in the open unit disc $U=\{z\in C:\ |z|<1\}$ defined by Dziok-Srivastava operator with complex order. The object of the present paper is to determine coefficient estimates, extreme points, distortion theorems, the radii of close-to-convexity, starlikeness and convexity and integral means for functions belonging to the class $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$. We also obtain several results for the neighborhood of functions belonging to this class.

Keywords and phrases: Analytic functions, starlike functions, convex functions, β -uniformly convex functions, neighborhood, integral means, Hadamard products, Dziok- Srivastava operator.
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1. INTRODUCTION

Let A denote the class of functions of the form:

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n, \tag{1.1}$$

which are analytic in the open unit disc $U=\{z\in C:\ |z|<1\}$.

Let $K(\alpha)$ and $S^*(\alpha)$ denote the subclasses of A which are, respectively, convex and starlike functions of order α, $0\le\alpha<1$. For convenience, we write $K(0)=K$ and $S^*(0)=S^*$ (see [20]).

A function $f\in A$ is said to be in $UST(\alpha,\beta)$, the class of β-uniformly starlike functions of order $\alpha,-1\le\alpha<1$, if it satisfies the condition:

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}-\alpha\right\}>\beta\left|\frac{zf'(z)}{f(z)}-1\right| \quad (\beta\ge 0). \tag{1.2}$$

Replacing $f(z)$ in (1.2) by $zf'(z)$ we have the condition:

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}-\alpha\right\}>\beta\left|\frac{zf''(z)}{f'(z)}\right| \quad (\beta\ge 0),$$

required for the function f to be in the subclass $UCV\left(\alpha,\beta\right)$ of β-uniformly convex functions of order α (see [9], [10] and [15]) ·

The Hadamard product (or convolution) $(f*g)(z)$ of the functions $f(z)$ is given by (1.1) and $g(z)$ is given by $g(z)=z+\sum_{n=2}^{\infty}b_n z^n$, is defined by

$$(f*g)(z)=z+\sum_{n=2}^{\infty}a_n b_n z^n=(g*f)(z).$$

For positive real parameters $\alpha_1,\dots,\alpha_q$ *and* $\beta_1,\dots,\beta_s\ \beta_j\in C\setminus Z_0^-, Z_0^-=0,-1,-2,\dots;\ j=1,2,\dots,s)$, the generalized hypergeometric function ${}_qF_s(\alpha_1,\dots,\alpha_q;\beta_1,\dots,\beta_s;\ z)$ is defined by

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$${}_qF_s(\alpha_1,.....,\alpha_q;\beta_1,.....,\beta_s;z)=$$
$$\sum_{n=0}^{\infty}\frac{(\alpha_1)_n...(\alpha_q)_n}{(\beta_1)_n...(\beta_s)_n n!}z^n$$
$(q\le s+1; s,q\in \mathrm{N}_0=\mathrm{N}\cup\{0\}, \mathrm{N}=\{1,2,.........\};\ z\in U)$,

where $(\theta)_n$, is the Pochhammer symbol defined in terms of the Gamma function Γ, by

$$(\theta)_n=\frac{\Gamma(\theta+n)}{\Gamma(\theta)}=\begin{cases}1 & (n=0)\\ \theta(\theta+1)....(\theta+n-1) & (n\in\mathrm{N}).\end{cases}$$

For the function $h(\alpha_1,....,\alpha_q;\beta_1,...\beta_s;z)=z{}_qF_s(\alpha_1,.....,\alpha_q;\ \beta_1,.....,\beta_s;\ z)$, the Dziok-Srivastava linear operator (see [6] and [7]) $H_{q,s}(\alpha_1,...,\alpha_q;\ \beta_1,...,\beta_s): A\to A$, is defined by the Hadamard product as follows:

$$H_{q,s}(\alpha_1,.....,\alpha_q;\ \beta_1,.....,\beta_s)f(z)=h(\alpha_1,....,\alpha_q;\beta_1,...\beta_s;z)*f(z)=z+\sum_{n=2}^{\infty}\Psi_n(\alpha_1)a_nz^n\quad(z\in U),\tag{1.3}$$

where

$$\Psi_n(\alpha_1)=\frac{(\alpha_1)_{n-1}.......(\alpha_q)_{n-1}}{(\beta_1)_{n-1}...(\beta_s)_{n-1}\ (n-1)!}.\tag{1.4}$$

For brevity, we write

$H_{q,s}(\alpha_1,.....,\alpha_q;\ \beta_1,.....,\beta_s;z)f(z)=H_{q,s}(\alpha_1)f(z)$.

For $\beta\ge 0$, $b\in C^*=C\setminus\{0\}$ and for all $z\in U$, let $S_{q,s}([\alpha_1];b,\beta)$ denote the subclass of A consisting of functions $f(z)$ of the form (1.1) and satisfying the analytic criterion

$$\mathrm{Re}\left\{1+\frac{1}{b}\left(\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right\}\ge\beta\left|\frac{1}{b}\left(\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right|.\tag{1.5}$$

We note that:

(1) Putting $b=e^{-i\mu}\cos\mu\left(|\mu|<\frac{\pi}{2}\right)$ in (1.5), the class $S_{q,s}\left([\alpha_1];e^{-i\mu}\cos\mu,\beta\right)$ reduces to the class $G_{q,s}\left([\alpha_1];\mu,\beta\right)$

$$=\left\{f\in A:\ \mathrm{Re}\left(e^{i\mu}\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}\right)\ge\beta\left|\left(\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right|,\ \beta\ge0,|\mu|<\frac{\pi}{2}\right\};$$

(2) Putting $b=(1-\gamma)e^{-i\mu}\cos\mu\left(|\mu|<\frac{\pi}{2},0\le\gamma<1\right)$ in (1.5), the class $S_{q,s}\left([\alpha_1];(1-\gamma)e^{-i\mu}\cos\mu,\beta\right)$ reduces to the class $G_{q,s}\left([\alpha_1];\gamma,\mu,\beta\right)$

$$=\left\{f\in A:\ \mathrm{Re}\left(e^{i\mu}\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}-\gamma\cos\mu\right)\ge\beta\left|\left(\frac{z\left(H_{q,s}(\alpha_1)f(z)\right)'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right|,\ \beta\ge0,|\mu|<\frac{\pi}{2},0\le\gamma<1\right\};$$

(3) Putting $b=(1-\gamma)e^{-i\mu}\cos\mu\left(|\mu|<\frac{\pi}{2},0\le\gamma<1\right)$, $q=2,s=1,\alpha_1=\lambda+1\ (\lambda>-1)$ and $\alpha_2=\beta_1=1$ in (1.5), the class $S_{2,1}\left([\lambda+1];(1-\gamma)e^{-i\mu}\cos\mu,\beta\right)$ reduces to the class $W_\lambda(\gamma,\mu,\beta)$

$$=\left\{f\in A:\ \mathrm{Re}\left\{\left(e^{i\mu}\frac{z(D^\lambda f(z))'}{D^\lambda f(z)}-\gamma\cos\mu\right)\right\}\ge\beta\left|\frac{1}{b}\left(\frac{z(D^\lambda f(z))'}{D^\lambda f(z)}-1\right)\right|,\ \beta\ge0,\lambda>-1,|\mu|<\frac{\pi}{2},0\le\gamma<1,z\in U\right\},$$

where $D^{\lambda}(\lambda > -1)$ is the Ruscheweyh derivative operator (see [16]). Also the class $W_{\lambda}(\gamma,\mu,\beta)$ reduces to the class $T(\mu,\lambda)$ for $\gamma = 0$ and $\beta = 1$ (see [21]).

Denote by T the subclass of A consisting of functions of the form:

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \ (a_n \geq 0), \qquad (1.6)$$

which are analytic in U . We define the class $TS_{q,s}([\alpha_1];\alpha,\beta)$ by:

$$TS_{q,s}([\alpha_1];b,\beta) = S_{q,s}([\alpha_1];b,\beta) \cap T. \quad (1.7)$$

We note that for suitable choices of q,s,b and β , we obtain the following subclasses:

(i) Putting $b = 1-\gamma(0 \leq \gamma < 1), q = 2$ and $s = \alpha_1 = \alpha_2 = \beta_1 = 1$ in (1.5), the class $TS_{2,1}([1];1-\gamma,\beta)$ reduces to the class $UST(\gamma,\beta)$ (see [9], [10] and [15]) and for $b=1-\gamma(0\leq\gamma<1), q=2$ and $s=\beta=\alpha_1=\alpha_2=\beta_1=1$ in (1.5), the class $TS_{2,1}([1];1-\gamma,1)$ reduces to the class $S_p T(\gamma)$ (see [2]);

(ii) Putting $b = 1-\gamma(0 \leq \gamma < 1)$ in (1.5), the class $TS_{q,s}([\alpha_1];1-\gamma,\beta)$ reduces to the class $T_{q,s}([\alpha_1];\gamma,\beta)$ (see [1]).

Also, we note that:

(1) Putting $q = 2, s = 1, \alpha_1 = a(a > 0), \alpha_2 = 1$ *and* $\beta_1 = c(c > 0)$ in (1.5), the class $TS_{2,1}([a,1;c];b,\beta)$ reduces to the class $T\ell(a,c;b,\beta)$

$$= \left\{ f \in T : \operatorname{Re}\left\{1 + \frac{1}{b}\left(\frac{z(L(a,c)f(z))'}{L(a,c)f(z)} - 1\right)\right\} \geq \beta\left|\frac{1}{b}\left(\frac{z(L(a,c)f(z))'}{L(a,c)f(z)} - 1\right)\right|, \ b \in C^*, \beta \geq 0, a > 0, c > 0, z \in U \right\},$$

where $L(a,c)$ is the Carlson - Shaffer operator (see [3]);

(2) Putting $q = 2, s = 1, \alpha_1 = \lambda + 1(\lambda > -1)$ *and* $\alpha_2 = \beta_1 = 1$ in (1.5), the class $TS_{2,1}([\lambda+1];b,\beta)$ reduces to the class $TW_{\lambda}(b,\beta)$

$$= \left\{ f \in T : \operatorname{Re}\left\{1 + \frac{1}{b}\left(\frac{z(D^{\lambda}f(z))'}{D^{\lambda}f(z)} - 1\right)\right\} \geq \beta\left|\frac{1}{b}\left(\frac{z(D^{\lambda}f(z))'}{D^{\lambda}f(z)} - 1\right)\right|, \ b \in C^*, \beta \geq 0, \lambda > -1, z \in U \right\};$$

(3) Putting $q = 2, s = 1, \alpha_1 = v + 1(v > -1), \alpha_2 = 1$ *and* $\beta_1 = v + 2$ in (1.5), the class $TS_{2,1}([v+1,1;v+2];b,\beta)$ reduces to the class $T\zeta(v;b,\beta)$

$$= \left\{ f \in T : \operatorname{Re}\left\{1 + \frac{1}{b}\left(\frac{z(J_v f(z))'}{J_v f(z)} - 1\right)\right\} \geq \beta\left|\frac{1}{b}\left(\frac{z(J_v f(z))'}{J_v f(z)} - 1\right)\right|, \ b \in C^*, \beta \geq 0, v > -1, z \in U \right\},$$

where $J_v f(z)$ is the generalized Bernardi - Libera - Livingston operator (see [11]);

(4) Putting $q = 2, s = 1, \alpha_1 = 2, \alpha_2 = 1$ *and* $\beta_1 = 2-\mu(\mu \neq 2,3,....)$ in (1.5), the class $TS_{2,1}([2,1;2-\mu];b,\beta)$ reduces to the class $TF(\mu;b,\beta)$

$$= \left\{ f \in T : \operatorname{Re}\left\{1 + \frac{1}{b}\left(\frac{z(\Omega_z^{\mu}f(z))'}{\Omega_z^{\mu}f(z)} - 1\right)\right\} \geq \beta\left|\frac{1}{b}\left(\frac{z(\Omega_z^{\mu}f(z))'}{\Omega_z^{\mu}f(z)} - 1\right)\right|, \ b \in C^*, \beta \geq 0, \mu \neq 2,3,...., z \in U \right\},$$

where $\Omega_z^{\mu} f(z)$ is the Srivastava - Owa fractional derivative operator (see [14]);

(5) Putting $q = 2, s = 1, \alpha_1 = \mu(\mu > 0), \alpha_2 = 1$ *and* $\beta_1 = \lambda + 1(\lambda > -1)$ in (1.5), the class $TS_{2,1}([\mu,1;\lambda+1];b,\beta)$ reduces to the class $T\pounds(\mu,\lambda;b,\beta)$

$$=\left\{f\in T\ :\ \mathrm{Re}\left\{1+\frac{1}{b}\left(\frac{z(I_{\lambda,\mu}f(z))'}{I_{\lambda,\mu}f(z)}-1\right)\right\}\geq \beta\left|\frac{1}{b}\left(\frac{z(I_{\lambda,\mu}f(z))'}{I_{\lambda,\mu}f(z)}-1\right)\right|,\ b\in C^*,\beta\geq 0,\mu>0,\lambda>-1,z\in U\right\},$$

where $I_{\lambda,\mu}f(z)$ is the Choi-Saigo-Srivastava operator (see [5]);

(6) Putting $q=2, s=1, \alpha_1=2, \alpha_2=1$ *and* $\beta_1=k+1(k>-1)$ in (1.5), the class $TS_{2,1}([2,1;k+1];b,\beta)$ reduces to the class $TA(k;b,\beta)$

$$=\left\{f\in T\ :\ \mathrm{Re}\left\{1+\frac{1}{b}\left(\frac{z(I_k f(z))'}{I_k f(z)}-1\right)\right\}\geq \beta\left|\frac{1}{b}\left(\frac{z(I_k f(z))'}{I_k f(z)}-1\right)\right|,\ b\in C^*,\beta\geq 0,k>-1,z\in U\right\},$$

where $I_k f(z)$ is the Noor integral operator (see [13]);

(7) Putting $q=2, s=1, \alpha_1=c(c>0), \alpha_2=\lambda+1(\lambda>-1)$ *and* $\beta_1=a(a>0)$ in (1.5), the class $TS_{2,1}([c,\lambda+1;a];b,\beta)$ reduces to the class $T\Im(c,a,\lambda;b,\beta)$

$$=\left\{f\in T\ :\ \mathrm{Re}\left\{1+\frac{1}{b}\left(\frac{z(I^{\lambda}(a,c)f(z))'}{I^{\lambda}(a,c)f(z)}-1\right)\right\}\geq \beta\left|\frac{1}{b}\left(\frac{z(I^{\lambda}(a,c)f(z))'}{I^{\lambda}(a,c)f(z)}-1\right)\right|,\ b\in C^*,\beta\geq 0,c>0,\lambda>-1,a>0,z\in U\right\},$$

where $I^{\lambda}(a,c)f(z)$ is the Cho-Kwon-Srivastava operator (see [4]).

2. COEFFICIENT ESTIMATES

Unless otherwise mentioned, we shall assume in the reminder of this paper that, the parameters $\alpha_1,\ldots,\alpha_q$ and $\beta_1,\ldots,\beta_s$ are positive real numbers, $\beta\geq 0$, $n\geq 2, b\in \mathbb{C}^*$, $z\in U$ *and* $\Psi_n(\alpha_1)$ is defined by (1.4).

Theorem 1. *A function $f(z)$ of the form (1.1) is in the class $S_{q,s}([\alpha_1];b,\beta)$ if*

$$\sum_{n=2}^{\infty}\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)|a_n|\leq|b|. \quad (2.1)$$

Proof. It suffices to show that

$$\beta\left|\left(\frac{z\left[H_{q,s}(\alpha_1)f(z)\right]'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right|-\mathrm{Re}\left\{\left(\frac{z\left[H_{q,s}(\alpha_1)f(z)\right]'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right\}\leq|b|.$$

We have

$$\beta\left|\left(\frac{z\left[H_{q,s}(\alpha_1)f(z)\right]'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right|-\mathrm{Re}\left\{\left(\frac{z\left[H_{q,s}(\alpha_1)f(z)\right]'}{H_{q,s}(\alpha_1)f(z)}-1\right)\right\}$$

$$\leq(1+\beta)\left|\frac{z\left[H_{q,s}(\alpha_1)f(z)\right]'}{H_{q,s}(\alpha_1)f(z)}-1\right|$$

$$\leq\frac{(1+\beta)\sum_{n=2}^{\infty}(n-1)\Psi_n(\alpha_1)|a_n|}{1-\sum_{n=2}^{\infty}\Psi_n(\alpha_1)|a_n|}.$$

This last expression is bounded above by $|b|$ if

$$\sum_{n=2}^{\infty}[(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1)|a_n|\leq|b|.$$

This completes the proof of Theorem 1.

Theorem 2. *A necessary and sufficient condition for $f(z)$ of the form (1.6) to be in the class $TS_{q,s}([\alpha_1];b,\beta)$ is that*

$$\sum_{n=2}^{\infty}\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)a_n \le |b|. \quad (2.2)$$

Proof. In view of Theorem 1, we need only to prove that if $f(z)\in TS_{q,s}([\alpha_1];b,\beta)$ and z is real, then

$$\frac{|b|-\sum_{n=2}^{\infty}\left[|b|+(n-1)\right]\Psi_n(\alpha_1)a_n z^{n-1}}{1-\sum_{n=2}^{\infty}\Psi_n(\alpha_1)a_n z^{n-1}} \ge$$

$$\frac{\beta\sum_{n=2}^{\infty}(n-1)\Psi_n(\alpha_1)a_n z^{n-1}}{1-\sum_{n=2}^{\infty}\Psi_n(\alpha_1)a_n z^{n-1}}$$

Letting $z\to 1^-$ along the real axis, we obtain the inequality (2.2). This completes the proof of Theorem 2.

Corollary 1. *Let the function* $f(z)$ defined by (1.6) be in the class $TS_{q,s}([\alpha_1];b,\beta)$, *then*

$$a_n \le \frac{|b|}{\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}\,(n\ge 2). \quad (2.3)$$

The result is sharp for the function

$$f(z)=z-\frac{|b|}{\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}z^n\ (n\ge 2). \quad (2.4)$$

3. DISTORTION THEOREMS

Theorem 3. *Let the function* $f(z)$ *defined by (1.6) belong to the class* $TS_{q,s}([\alpha_1];b,\beta)$.
Then for $|z|=r<1$, *we have*

$$r-\frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r^2 \le |f(z)| \le r + \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r^2, \quad (3.1)$$

provided $\Psi_n(\alpha_1)\ge\Psi_2(\alpha_1)$ $(n\ge 2)$. *The result is sharp for the function* $f(z)$ *defined by*

$$f(z)=z-\frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}z^2 \quad (3.2)$$

at $z=r$ *and* $z=re^{i(2n+1)\pi}$ $(n\in \mathrm{N})$.

Proof. We have

$$|f(z)| \le r+\sum_{n=2}^{\infty}a_n r^n \le r + r^2\sum_{n=2}^{\infty}a_n. \quad (3.3)$$

Since for $n\ge 2$, we have

$$(1+\beta+|b|)\Psi_2(\alpha_1) \le [(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1),$$

then (2.2) yields

$$(1+\beta+|b|)\Psi_2(\alpha_1)\sum_{n=2}^{\infty}a_n \le \sum_{n=2}^{\infty}\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)a_n \le |b|$$

or

$$\sum_{n=2}^{\infty}a_n \le \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}. \quad (3.5)$$

From (3.5) and (3.3) we have

$$|f(z)| \le r + \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r^2$$

and similarly, we have

$$|f(z)| \ge r - \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r^2.$$

This completes the proof of Theorem 3.

Theorem 4. *Let the function* $f(z)$ *defined by (1.6) belong to the class* $TS_{q,s}([\alpha_1];b,\beta)$.
Then for $|z|=r<1$, *we have*

$$1-\frac{2|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r \le |f'(z)| \le 1+\frac{2|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}r, \quad (3.6)$$

provided $\Psi_n(\alpha_1)\ge\Psi_2(\alpha_1)$ $(n\ge 2)$. *The result is sharp for the function* $f(z)$ *given by (3.2) at* $z=r$ *and* $z=re^{i(2n+1)\pi}$ $(n\in \mathrm{N})$.

Proof. For a function $f(z)\in TS_{q,s}([\alpha_1];b,\beta)$, it follows from (2.2) and (3.5) that

$$\sum_{n=2}^{\infty}na_n \le \frac{2|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}.$$

Since the remaining part of the proof is similar to the proof of Theorem 3, we omit the details.

4. EXTREME POINTS

Theorem 5. *The class* $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$ *is closed under convex linear combinations.*

Proof. Let $f_j\left(z\right)\in TS_{q,s}(\left[\alpha_1\right];b,\beta)\left(j=1,2\right)$, where

$$f_j\left(z\right)=z-\sum_{n=2}^{\infty}a_{n,j}z^n \quad \left(a_{n,j}\geq 0;\ j=1,2\right). \tag{4.1}$$

Then it is sufficient to prove that the function $h\left(z\right)$ given by

$$h\left(z\right)=\mu f_1(z)+(1-\mu)f_2(z) \quad (0\leq\mu\leq 1)$$

$$=z-\sum_{n=2}^{\infty}[\mu a_{n,1}+(1-\mu)a_{n,2}]z^n,$$

is also in the class $TS_{q,s}([\alpha_1];b,\beta)$. For $0\leq\mu\leq 1$, using Theorem 2, we have

$$\sum_{n=2}^{\infty}\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)\cdot$$

$$[\mu a_{n,1}+(1-\mu)a_{n,2}]$$

$$\leq \mu|b|+(1-\mu)|b|=|b|,$$

which implies that $h\left(z\right)\in TS_{q,s}(\left[\alpha_1\right];b,\beta)$. This completes the proof of Theorem 5.

As a consequence of Theorem 5, there exist extreme points of the class $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$, which are given by:

Theorem 6. *Let* $f_1(z)=z$ *and*

$$f_n(z)=z-\frac{|b|}{[(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1)}z^n.$$

Then $f\left(z\right)$ *is in the class* $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$ *if and only if it can be expressed in the form*

$$f\left(z\right)=\sum_{n=1}^{\infty}\mu_n f_n(z), \tag{4.2}$$

where $\mu_n\geq 0\ (n\geq 1)$ *and* $\sum_{n=1}^{\infty}\mu_n=1$.

Proof. Assume that

$$f\left(z\right)=\sum_{n=1}^{\infty}\mu_n f_n(z)$$

$$=z-\sum_{n=2}^{\infty}\frac{|b|}{\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)}\mu_n z^n.$$

Then it follows that

$$\sum_{n=2}^{\infty}\frac{\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)}{|b|}$$

$$\frac{|b|}{\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)}\mu_n$$

$$=\sum_{n=2}^{\infty}\mu_n=(1-\mu_1)\leq 1. \tag{4.3}$$

So, by Theorem 2, we have $f\left(z\right)\in TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$.

Conversely, assume that the function $f\left(z\right)$ defined by (1.6) belongs to the class $TS_{q,s}(\left[\alpha_1\right];b,\beta)$. Then a_n are given by (2.3). Setting

$$\mu_n=\frac{\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)}{|b|}a_n \tag{4.4}$$

and

$$\mu_1=1-\sum_{n=2}^{\infty}\mu_n,$$

we see that $f\left(z\right)$ can be expressed in the form (4.2). This completes the proof of Theorem 6.

Corollary 2. *The extreme points of the class* $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$ *are the functions* $f_1(z)=z$ *and*

$$f_n(z)=z-\frac{|b|}{\left[(n-1)(1+\beta)+|b|\right]\Psi_n\left(\alpha_1\right)}z^n\ (n\geq 2).$$

5. RADII OF CLOSE-TO-CONVEXITY, STARLIKENESS AND CONVEXITY

Theorem 7. *Let the function* $f\left(z\right)$ *defined by (1.6) be in the class* $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$.

Then $f\left(z\right)$ *is close-to-convex of order* $\rho\ (0\leq\rho<1)$ *in* $|z|<r_1$, *where*

$$r_1=\inf_{n\geq 2}\left\{\frac{(1-\rho)\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}{n|b|}\right\}^{\frac{1}{n-1}}. \quad (5.1)$$

The result is sharp with the extremal function given by (2.4).

Proof. We must show that

$$|f'(z)-1|\leq 1-\rho \qquad \text{for } |z|<r_1,$$

where r_1 is given by (5.1). Indeed we find from the (1.6) that

$$|f'(z)-1|\leq \sum_{n=2}^{\infty} na_n|z|^{n-1}.$$

Thus

$$|f'(z)-1|\leq 1-\rho,$$

if

$$\sum_{n=2}^{\infty}\left(\frac{n}{1-\rho}\right)a_n|z|^{n-1}\leq 1. \quad (5.2)$$

But, by Theorem 2, (5.2) will be true if

$$\left(\frac{n}{1-\rho}\right)|z|^{n-1}\leq \frac{[(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1)}{|b|},$$

that is, if

$$|z|\leq\left\{\frac{(1-\rho)\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}{n|b|}\right\}^{\frac{1}{n-1}} \ (n\geq 2). \quad (5.3)$$

Theorem 7 follows easily from (5.3).

Theorem 8. *Let the function* $f(z)$ *defined by (1.6) be in the class* $TS_{q,s}([\alpha_1];b,\beta)$.

Then $f(z)$ *is starlike of order* ρ $(0\leq\rho<1)$ *in* $|z|<r_2$, *where*

$$r_2=\inf_{n\geq 2}\left\{\frac{(1-\rho)\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}{(n-\rho)|b|}\right\}^{\frac{1}{n-1}}. \quad (5.4)$$

The result is sharp, with the extremal function $f(z)$ *given by (2.4).*

Proof. It is sufficient to show that

$$\left|\frac{zf'(z)}{f(z)}-1\right|\leq 1-\rho \quad (|z|<r_2),$$

where r_2 is given by (5.4). Indeed we find, again from the (1.6) that

$$\left|\frac{zf'(z)}{f(z)}-1\right|\leq\frac{\sum_{n=2}^{\infty}(n-1)a_n|z|^{n-1}}{1-\sum_{n=2}^{\infty}a_n|z|^{n-1}}.$$

Thus

$$\left|\frac{zf'(z)}{f(z)}-1\right|\leq 1-\rho,$$

if

$$\sum_{n=2}^{\infty}\left(\frac{n-\rho}{1-\rho}\right)a_n|z|^{n-1}\leq 1. \quad (5.5)$$

But, by Theorem 2, (5.5) will be true if

$$\left(\frac{n-\rho}{1-\rho}\right)|z|^{n-1}\leq\frac{[(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1)}{|b|}$$

that is, if

$$|z|\leq\left\{\frac{(1-\rho)\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}{(n-\rho)|b|}\right\}^{\frac{1}{n-1}} \ (n\geq 2). \quad (5.6)$$

Theorem 8 follows easily from (5.6).

Similarly, we can prove the following theorem.

Theorem 9. *Let the functions* $f(z)$ *defined by (1.6) be in the class* $TS_{q,s}([\alpha_1];b,\beta)$.

Then $f(z)$ *is convex of order* ρ $(0\leq\rho<1)$ *in* $|z|<r_3$, *where*

$$r_3=\inf_{n\geq 2}\left\{\frac{(1-\rho)\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)}{n(n-\rho)|b|}\right\}^{\frac{1}{n-1}}. \quad (5.7)$$

The result is sharp, with the extremal function $f(z)$ *given by (2.4).*

6. INTEGRAL MEANS

In [18] Silverman found that the function $f_2=z-\frac{z^2}{2}$ is often extremal over the family T. He applied this function to resolve his integral means inequality, conjectured and settled in [19]:

$$\int_0^{2\pi}\left|f(re^{i\theta})\right|^{\delta}d\theta \le \int_0^{2\pi}\left|f_2(re^{i\theta})\right|^{\delta}d\theta,$$

for all $f \in T$, $\delta > 0$ and $0 < r < 1$. In [18], he also proved his conjecture for the subclasses $T^*(\alpha)$ and $C(\alpha)$ of T, where $C(\alpha)$ *and* $T^*(\alpha)$ are the classes of convex and starlike functions of order α, $0 \le \alpha < 1$, respectively.

In this section, we prove Silverman's conjecture for functions in the class $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$.

Lemma 1 [12]. *If the functions* f *and* g *are analytic in* U *with* $g \prec f$, *then for* $\delta > 0$ *and* $0 < r < 1$,

$$\int_0^{2\pi}\left|g(re^{i\theta})\right|^{\delta}d\theta \le \int_0^{2\pi}\left|f(re^{i\theta})\right|^{\delta}d\theta.$$

Applying Theorems 1, 2 and Lemma 1 we prove the following theorem.

Theorem 10. *Suppose* $f(z)\in TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right), \delta>0$, *the sequence* $\{\Psi_n(\alpha_1)\}$ $(n \ge 2)$ is non-decrecing *and* $f_2(z)$ *is defined by:*

$$f_2(z) = z - \frac{|b|}{(1+\beta+|b|)\Psi_2\left(\alpha_1\right)}z^2, \qquad (6.1)$$

then for $z = re^{i\theta}$, $0 < r < 1$, *we have*

$$\int_0^{2\pi}\left|f(re^{i\theta})\right|^{\delta}d\theta \le \int_0^{2\pi}\left|f_2(re^{i\theta})\right|^{\delta}d\theta. \qquad (6.2)$$

Proof. For $f(z)$ of the form (1.10), (6.2) is equivalent to proving that

$$\int_0^{2\pi}\left|1-\sum_{n=2}^{\infty}a_n z^{n-1}\right|^{\delta}d\theta \le$$

$$\int_0^{2\pi}\left|1-\frac{|b|}{(1+\beta+|b|)\Psi_2\left(\alpha_1\right)}z\right|^{\delta}d\theta.$$

By using Lemma 1, it suffices to show that

$$1-\sum_{n=2}^{\infty}a_n z^{n-1} \prec 1-\frac{|b|}{(1+\beta+|b|)\Psi_2\left(\alpha_1\right)}z. \quad (6.3)$$

Setting

$$1-\sum_{n=2}^{\infty}a_n z^{n-1} = 1-\frac{|b|}{(1+\beta+|b|)\Psi_2\left(\alpha_1\right)}w(z), \quad (6.4)$$

and using (2.2) and the hypothesis $\left\{\Psi_n\left(\alpha_1\right)\right\}$ $\left(n\ge 2\right)$ is non-decrecing, we obtain

$$\begin{aligned} |w(z)| &= \left|\frac{(1+\beta+|b|)\Psi_2(\alpha_1)}{|b|}\sum_{n=2}^{\infty}a_n z^{n-1}\right| \\ &\le |z|\sum_{n=2}^{\infty}\frac{(1+\beta+|b|)\Psi_2(\alpha_1)}{|b|}a_n \\ &\le |z|\sum_{n=2}^{\infty}\frac{[(n-1)(1+\beta)+|b|]\Psi_n(\alpha_1)}{|b|}a_n \\ &\le |z|. \end{aligned}$$

This completes the proof of Theorem 10.

7. NEIGHBORHOOD FOR THE CLASS $TS_{q,s}\left(\left[\alpha_1\right];b,\beta\right)$

In this section, we define the δ - neighborhood of functions of T ([8] and [17]) by

$$N_\delta(f) = \left\{\begin{matrix} h\in T \ : \ h(z) = z \\ -\sum_{n=2}^{\infty}c_n z^n, \ \sum_{n=2}^{\infty}n\left|a_n - c_n\right| \le \delta \end{matrix}\right\}. \quad (7.1)$$

In particular, if

$$e(z) = z, \qquad (7.2)$$

we immediately have

$$N_\delta(e) = \left\{\begin{matrix} h\in T \ : \ h(z) = \\ z-\sum_{n=2}^{\infty}c_n z^n, \ \sum_{n=2}^{\infty}n\left|c_n\right| \le \delta \end{matrix}\right\}. \qquad (7.3)$$

Theorem 11. *If* $\Psi_n\left(\alpha_1\right)\ge\Psi_2\left(\alpha_1\right)$ $(n\ge 2)$ *and*

$$\delta = \frac{2|b|}{(1+\beta+|b|)\Psi_2\left(\alpha_1\right)}, \qquad (7.4)$$

then

$$TS_{q,s}(\left[\alpha_1\right];b,\beta) \subset N_\delta(e) \qquad (7.5)$$

Proof. Let $f \in TS_{q,s}([\alpha_1]; b, \beta)$. Then, in view of the assertion (2.2) of Theorem 2 and the given condition that $\Psi_n(\alpha_1) \geq \Psi_2(\alpha_1)$ $(n \geq 2)$, we have

$$(1+\beta+|b|)\Psi_2(\alpha_1)\sum_{n=2}^{\infty} a_n \leq \sum_{n=2}^{\infty}\left[(n-1)(1+\beta)+|b|\right]\Psi_n(\alpha_1)a_n \leq |b|, \tag{7.6}$$

which readily yields

$$\sum_{n=2}^{\infty} a_n \leq \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}. \tag{7.7}$$

Making use of (2.2) again, in conjunction with (7.7), we get

$$\Psi_2(\alpha_1)\sum_{n=2}^{\infty} na_n \leq |b| + (1-|b|-\beta)\Psi_2(\alpha_1)\sum_{n=2}^{\infty} a_n$$
$$\leq |b| + (1-|b|-\beta)\Psi_2(\alpha_1)\frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}$$
$$\leq \frac{2|b|}{(1+\beta+|b|)}.$$

Hence

$$\sum_{n=2}^{\infty} na_n \leq \frac{2|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)} = \delta, \tag{7.8}$$

which, by means of (7.3), completes the proof of Theorem 11.

Now we determine the neighborhood for the class $TS_{q,s}^{(\alpha)}([\alpha_1]; b, \beta)$, which we define as follows. A function $f(z) \in T$ is said to be in the class $TS_{q,s}^{(\alpha)}([\alpha_1]; b, \beta)$ if there exists a function $\zeta(z) \in TS_{q,s}([\alpha_1]; b, \beta)$ such that

$$\left|\frac{f(z)}{\zeta(z)} - 1\right| < 1-\alpha \quad (0 \leq \alpha < 1) \tag{7.9}$$

Theorem 12. *If* $\zeta(z) \in TS_{q,s}([\alpha_1]; b, \beta)$ *and*

$$\alpha = 1 - \frac{\delta(1+\beta+|b|)\Psi_2(\alpha_1)}{2\left[(1+\beta+|b|)\Psi_2(\alpha_1) - |b|\right]} \tag{7.10}$$

then

$$N_\delta(\zeta) \subset TS_{q,s}^{(\alpha)}([\alpha_1]; b, \beta) \tag{7.11}$$

where

$$\delta \leq 2 - 2|b|\left[(1+\beta+|b|)\Psi_2\right]^{-1}. \tag{7.12}$$

Proof. Suppose that $\zeta(z) \in N_\delta(\zeta)$. We find from (7.1) that

$$\sum_{n=2}^{\infty} n|a_n - c_n| \leq \delta, \tag{7.13}$$

which readily implies that

$$\sum_{n=2}^{\infty} |a_n - c_n| \leq \frac{\delta}{2}. \tag{7.14}$$

Next, since $\zeta(z) \in TS_{q,s}([\alpha_1]; b, \beta)$, we have [cf. equation (7.7)] that

$$\sum_{n=2}^{\infty} c_n \leq \frac{|b|}{(1+\beta+|b|)\Psi_2(\alpha_1)}, \tag{7.15}$$

so that

$$\left|\frac{f(z)}{\zeta(z)} - 1\right| \leq \frac{\sum_{n=2}^{\infty}|a_n - c_n|}{1 - \sum_{n=2}^{\infty} c_n}$$
$$\leq \frac{\delta}{2}\frac{(1+\beta+|b|)\Psi_2(\alpha_1)}{\left[(1+\beta+|b|)\Psi_2(\alpha_1) - |b|\right]}$$
$$= 1-\alpha,$$

provided that α is given by (7.10). Thus, by the above definition, $f(z) \in TS_{q,s}^{(\alpha)}([\alpha_1]; b, \beta)$ for α given by (7.10). This completes the proof of Theorem 12.

REMARK

Specializing b, q, s and β, in the above results, we obtain the corresponding results for the corresponding classes defined in the introduction.

THE MOTIVATION

The motivation of this paper is introducing the complex order $b \in C^* = C/\{0\}$ in the definition of the uniformly starlike functions associated with the Dziok-Srivastava operator. From the class $TS_{q,s}([\alpha_1]; b, \beta)$ of $\beta-$uniformly starlike functions, we can obtain many subclasses of β-uniformly $\lambda-$spiral functions which are not handled before and have applications in fractional calculus.

CONCLUSION

In this paper we determine coefficient estimates, extreme points, distortion theorems, the radii of close-to-convexity, starlikeness and convexity and integral means for functions belonging to the class $TS_{q,s}\left([\alpha_1];b,\beta\right)$ of $\beta-$ uniformly starlike functions of complex order $b\in C^* = C/\{0\}$. Consequently, specializing $b,q;s$ and β in the obtained results, we obtain the corresponding results for the corresponding new subclasses.

OPEN PROBLEMS

Generalizing the obtained results for multivalent functions, the results can be obtained by using another operators.

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a. **Journal Articles** *(Name of the journal to be stated in full)*

1. Golding, I. Real time kinetics of gene activity in individual bacteria. *Cell*, 123**:** 1025–1036 (2005).
2. Bialek, W. & S. Setayeshgar. Cooperative sensitivity, and noise in biochemical signaling. *Physical Review Letters*, 100**:** 258–263 (2008).
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b. **Books**

4. Luellen, W.R. *Fine-Tuning Your Writing*. Wise Owl Publishing Company, Madison, WI, USA (2001).
5. Alon, U. & D.N. Wegner (Eds.). *An Introduction to Systems Biology: Design Principles of Biological Circuits*. Chapman & Hall/CRC, Boca Raton, FL, USA (2006).

c. **Book Chapters**

6. Sarnthein, M.S., & J.D. Stanford. Basal Sauropodomorpha: historical and recent phylogenetic developments. In: *The Northern North Atlantic: A Changing Environment.* Schafer, P.R. & W. Schluter (Eds.), Springer, Berlin, Germany, p. 365–410 (2000).
7. Smolen, J.E. & L.A. Boxer. Functions of Europhiles. In: *Hematology, 4th ed*. Williams, W.J., E. Butler, and M.A. Litchman (Eds.), McGraw Hill, New York, USA, p. 103–101 (1991).

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