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EFFECT OF IMPROVING BOTH THE AUXILIARY AND VARIABLE OF INTEREST IN RATIO AND PRODUCT ESTIMATORS

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Abstract: Ratio and product estimators that made use of information on auxiliary variable and variable of interest not yet drawn were compared with the conventional ones that made use of information on auxiliary variable and variable of interest already drawn when sampling without replacement from a finite population with known mean. From the two numerical examples used (when the population correlation coefficient is positive and negative), it was observed that the ratio and product estimators that made use of information on auxiliary variable and variable of interest not yet drawn always performed better once all of their conditions were met.

Keywords: Estimation, sample, mean square error, bias

Introduction

Several researchers have established many classes of ratio and product –type estimators in the past that reduce the bias and the mean square error (mse) by improving the auxiliary variable, x, (Srivenkataramana and Srinath [8], Shabbir and Gupta [5], Chakrabarty [2,3], Singh [6] and Srivastara [7]), and also the variable of interest ,y, (i.e. Shabbir [4], Sodipo [9]). On rare occasions, both have been improved upon [10] but all in different directions.

This paper aims at improving the conventional ratio and product estimators, $\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$ and $\bar{y}_p = \frac{\overline{yx}}{\bar{X}}$, and these improved ratio and product estimators are $\bar{y}_{mr} = \frac{\bar{y}^*}{\bar{x}^*}\bar{X}$ and $\bar{y}_{mp} = \frac{\bar{y}^*\bar{x}^*}{\bar{X}}$.

Here, N and n are the population and sample sizes respectively, $\bar{Y}$ and $\bar{X}$ are the population means for the auxiliary variable (X) and variable of interest (Y), and $\bar{x}$ and $\bar{y}$ are the sample means based on the sample drawn, $\bar{x}^*$ and $\bar{y}^*$ are the means of the auxiliary variables and variable of interest yet to be drawn [8] (that is the means corresponding to the *(N-n)* population units) and ρ is the population correlation coefficient between y and x.

The relationship between $\bar{X}, \bar{x}$ and $\bar{x}^*$ is $\bar{X} = f\bar{x} + (1-f)\bar{x}^*$ while that between $\bar{Y}, \bar{y}$ and $\bar{y}^*$ is $\bar{Y} = f\bar{y} + (1-f)\bar{y}^*$ [8], where $f = \frac{n}{N}$ is the sampling fraction, $\bar{x}^* = \bar{X}(1-(\frac{n}{N-n})\Delta_{\bar{x}})$, $\bar{y}^* = \bar{Y}(1-(\frac{n}{N-n})\Delta_{\bar{y}}), \bar{x} = \bar{X}(1+\Delta_{\bar{x}})$ and $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{x}} = e_0$ and $\Delta_{\bar{y}} = e_1$ [4], $E(\Delta_{\bar{x}}) = E(\Delta_{\bar{y}}) = 0$, $E(\Delta^2{}_{\bar{x}}) = \frac{C^2{}_x}{n}$, $E(\Delta^2{}_{\bar{y}}) = \frac{C^2{}_y}{n}$, $E(\Delta_{\bar{x}}\Delta_{\bar{y}}) = \frac{\rho C_x C_y}{n}$, $C_x = \frac{S_x}{\bar{X}}$ and $C_y = \frac{S_y}{\bar{Y}}$.

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Bias and MSE of conventional Estimators

Conventionally, $\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$ and $\bar{y}_p = \frac{\overline{yx}}{\bar{X}}$.

Then the biases and MSE of $\bar{y}, \bar{y}_r$ and $\bar{y}_p$ to the first degree of approximation are given below.

$Bias(\bar{y}) = 0$.

$$Bias(\bar{y}_r) = \bar{Y}(\frac{N-n}{Nn})(C^2{}_x - \rho C_x C_y).$$

$$Bias(\bar{y}_p) = \bar{Y}(\frac{N-n}{Nn})\rho C_x C_y.$$

$$MSE(\bar{y}) = \bar{Y}^2(\frac{N-n}{Nn})C^2{}_y.$$

$$MSE(\bar{y}_r) = \bar{Y}^2(\frac{N-n}{Nn})(C^2{}_y - 2\rho C_x C_y + C^2{}_x).$$

$$MSE(\bar{y}_p) = \bar{Y}^2(\frac{N-n}{Nn})(C^2{}_y + 2\rho C_x C_y + C^2{}_x).$$

Bias and MSE of proposed ratio and product estimators

Case (a): When ρ > 0

$$\bar{y}_{mr} = \frac{\bar{y}^*}{\bar{x}^*}\bar{X} = \frac{\bar{Y}(1-(\frac{n}{N-n})\Delta_{\bar{y}})}{\bar{X}(1-(\frac{n}{N-n})\Delta_{\bar{x}})}\bar{X}$$

$$= \bar{Y}(1-(\frac{n}{N-n})\Delta_{\bar{y}})(1-(\frac{n}{N-n})\Delta_{\bar{x}})^{-1}.$$

then,

$$Bias(\bar{y}_{mr}) = \bar{Y}(\frac{N-n}{Nn})(\frac{n}{N-n})^2(C^2{}_x - \rho C_x C_y).$$

$$MSE(\bar{y}_{mr}) = \bar{Y}^2(\frac{N-n}{Nn})(\frac{n}{N-n})^2(C^2{}_y - 2\rho C_x C_y + C^2{}_x).$$

Case (b): When ρ < 0.

$$\bar{y}_{mp} = \frac{\bar{y}^*\bar{x}^*}{\bar{X}}$$

$$= \bar{Y}(1-(\frac{n}{N-n})\Delta_{\bar{y}})(1-(\frac{n}{N-n})\Delta_{\bar{x}}).$$

then

$$Bias(\bar{y}_{mp}) = \bar{Y}(\frac{N-n}{Nn})(\frac{n}{N-n})^2\rho C_x C_y.$$

$$MSE(\bar{y}_{mp}) = \bar{Y}^2(\frac{N-n}{Nn})(\frac{n}{N-n})^2(C^2{}_y + 2\rho C_x C_y + C^2{}_x).$$

Comparison of MSE's

(a). $\bar{y}_{mr}$ is said to be better than $\bar{y}_r$ whenever
 (i) $\rho > 0$ and
 (ii) $MSE(\bar{y}_{mr}) < MSE(\bar{y}_r)$.

That is,

$$\bar{Y}^2(\frac{N-n}{Nn})(\frac{n}{N-n})^2(C^2{}_y - 2\rho C_x C_y + C^2{}_x)$$

$$< \bar{Y}^2(\frac{N-n}{Nn})(C^2{}_y - 2\rho C_x C_y + C^2{}_x).$$

This implies that

$$MSE(\bar{y}_{mr}) < MSE(\bar{y}_r)$$

whenever $(\frac{n}{N-n})^2 \leq 1$.

(b). $\bar{y}_{mp}$ is said to be better than $\bar{y}_p$ whenever

(i) $\rho < 0$ and

(ii) $MSE(\bar{y}_{mp}) < MSE(\bar{y}_p)$.

That is,

$$\bar{Y}^2 \left(\frac{N-n}{Nn}\right)\left(\frac{n}{N-n}\right)^2 (C^2{}_y + 2\rho C_x C_y + C^2{}_x)$$

$$< \bar{Y}^2 \left(\frac{N-n}{Nn}\right)(C^2{}_y + 2\rho C_x C_y + C^2{}_x).$$

This implies that

$$MSE(\bar{y}_{mp}) < MSE(\bar{y}_p)$$

whenever $\left(\frac{n}{N-n}\right)^2 \leq 1$.

Results

To confirm these conditions shown above, two data sets were used which are:

Population 1:- Cochran [1], p.171 - 172

S^2_x =7619, S^2_y = 620, $\bar{X}$ =117.28, $\bar{Y}$ =26.30, ρ = 0.67, n = 100, N = 2010

Population 2:- (Hypothetical population)

S^2_x =0.0036 , S^2_y =0.0036 , $\bar{X}$ =0.2 , $\bar{Y}$ =0.3, ρ = -0.05, n =30, N =100

The results obtained are shown in Table 1.

Table 1. The empirical results obtained on the two data sets used.

	Population 1	population 2
ρ	0.67	-0.05
$Bias(\bar{y})$	0	0
$Bias(\bar{y}_r)$	0.0204	-
$Bias(\bar{y}_{mr})$	-0.0007	-
$Bias(\bar{y}_p)$	-	-0.00002
$Bias(\bar{y}_p)$	-	-0.000004
$Bias(\bar{y}_{mp})$	5.8915	0.000084
$MSE(\bar{y})$	3.3258	-
$MSE(\bar{y}_{mr})$	0.0091	-
$MSE(\bar{y}_p)$	-	0.00026
$MSE(\bar{y}_{mp})$	-	0.00005
$\frac{MSE(\bar{y}_r)}{MSE(\bar{y})}$	0.5645 < 1	-
$\frac{MSE(\bar{y}_{mr})}{MSE(\bar{y})}$	0.0015 < 1	-
$\frac{MSE(\bar{y}_p)}{MSE(\bar{y})}$	-	3.0952 > 1
$\frac{MSE(\bar{y}_{mp})}{MSE(\bar{y})}$	-	0.5952 < 1
$\frac{n}{N-n}$	0.0524 <1	0.4286<1
$\left(\frac{n}{N-n}\right)^2$	0.0027<1	0.1837<1

Discussion

From Table 1, we can conclude the following. Firstly, for population 1, the modified ratio estimator, $\bar{y}_{mr}$ is better than $\bar{y}$ and $\bar{y}_r$ since it has (i) the correlation of 0.67 which is greater than zero, (ii) the least estimated bias of –0.0007, (iii) the least estimated mean square error of 0.0091 which results in the highest percentage relative efficiency of 64,741.76, and (iv) the least estimated mean square error ratio of 0.0015 which is less than one. Hence, the modified ratio estimator, $\bar{y}_{mr}$ is preferred to $\bar{y}$ and $\bar{y}_r$. Secondly, for population 2, the modified product estimator, $\bar{y}_{mp}$ is better than $\bar{y}$ and $\bar{y}_p$ since it has (i) the correlation of -0.05 which is less than zero, (ii) the least estimated bias of –0.000004, (iii) the least estimated mean square error of 0.00005 which results in the highest percentage relative efficiency of 168.00, and (iv) the least estimated mean square error ratio of 0.5952 which is less than one. Hence, the modified product estimator is preferred to $\bar{y}$ and $\bar{y}_p$.

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