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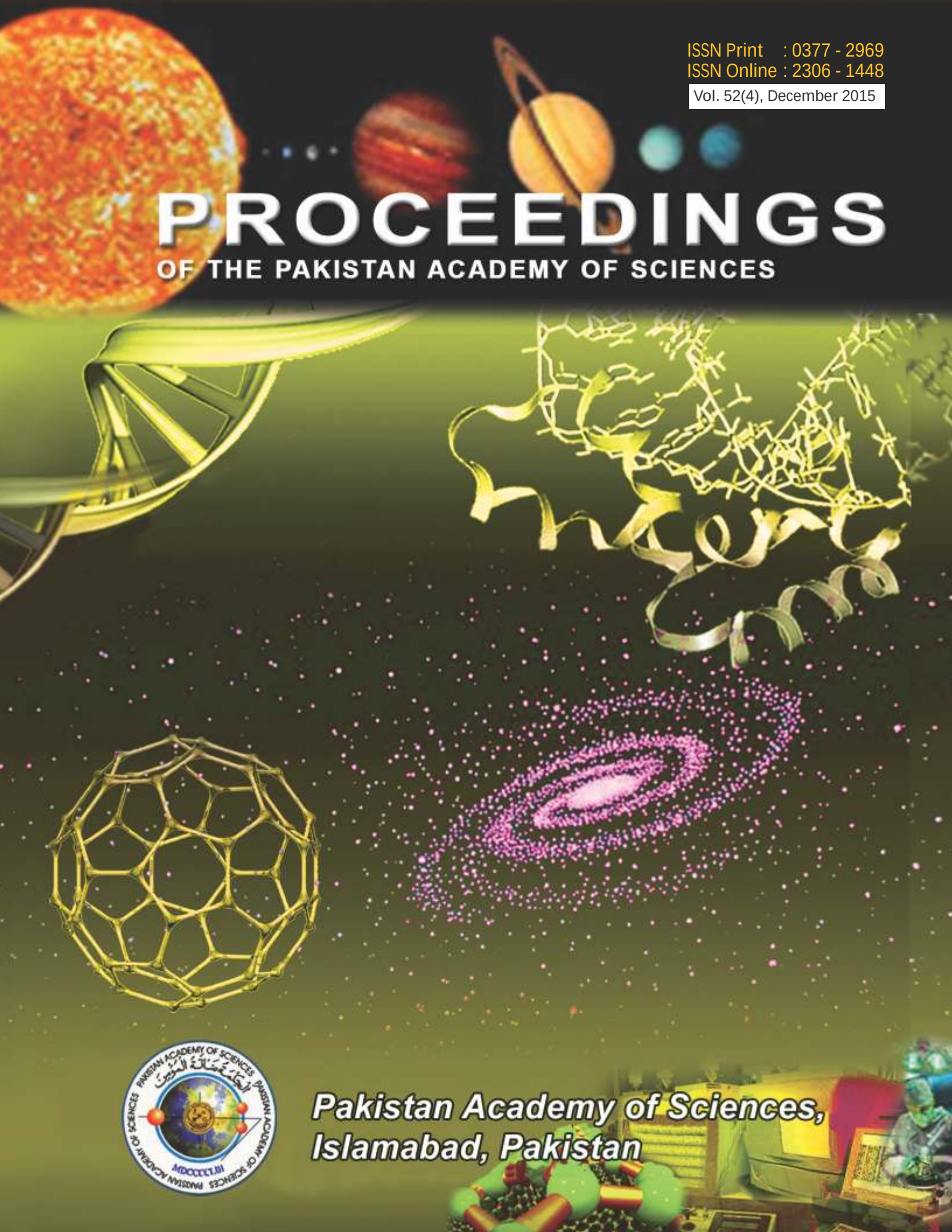

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Research Article

Compact Fractal Ground Based UWB Band Notch Antenna

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Abstract: A compact micro-strip fed planar UWB monopole antenna with band notch features is proposed in this study. The proposed design consists of rectangular radiating patch with impedance steps and fractal slots in the partial ground plane. Wide-band matching is obtained by using the stair cased radiating patch and fractal slots in the partial ground plane. A slot is inserted in the radiating patch to reject 5.8 GHz WLAN band.The design antenna has a compact size of (30×36 mm^2).The proposed antenna is modelled on FR4 substrate and is simulated in CST Microwave studio. The results have also been verified using Ansoft (HFSS).The antenna due to its compact size and appreciable properties can be used in portable UWB systems.

Keywords: UWB antenna, bandnotch, fractal

1. INTRODUCTION

The frequency band from 3.1 GHz to 10.6 GHz has been allocated by the Federal Communications Commission (FCC) for Ultra wideband (UWB) wireless communication applications. As UWB is the most promising technology for future short range wireless communication [1]. The advantages of UWB communication are that they offer more resistance to multipath phenomenon, high data rate short range wireless communication, low complexity and low emission power. Antenna is the important part of UWB system. The antenna required must have an omnidirectional and stable radiation pattern and high radiation efficiency [2].

The problem is that the IEEE 802.11a WLAN system operates in 5.15 to 5.825 GHz band which generate potential interference with the UWB communication. This interference can be avoided by using good filtering techniques. But the filtering technique is much expensive and increases the system complexity. So by designing antenna having band notch features is the most simple and economical solution [3-5]. Various band-notched UWB antennas have been developed for UWB wireless communication. There are various techniques to design band notch antennas such as etching L-shaped, E-shaped, C-shaped, arc shaped and U-shaped slots on the radiating patch [6-10]. Also there is another technique which uses parasitic strips in the printed monopole [11-12].

In this paper, a novel compact planar UWB antenna is analyzed and simulated. The proposed rectangular patch antenna parameters are calculated based on transmission line modal analysis [13] and the detailed antenna geometry and parameters are given. The antenna with non-uniform impedance steps and fractal slots in the ground plane can cover the entire UWB frequency band without rejecting WLAN band. First the antenna results have been analyzed with and without fractal slots [14] in the partial ground plane. Then we have analyzed the antenna results with and without notch by introducing slot in the radiating patch. A slot in the radiating patch is inserted to notch the 5.8 GHz WLAN band without affecting its gain. The antenna designed has high gain, stable radiation pattern and best matching in the desired frequency band.

2. ANTENNA GEOMETRY

The configuration of the proposed UWB antenna having band notch characteristics is shown in Fig. 1.

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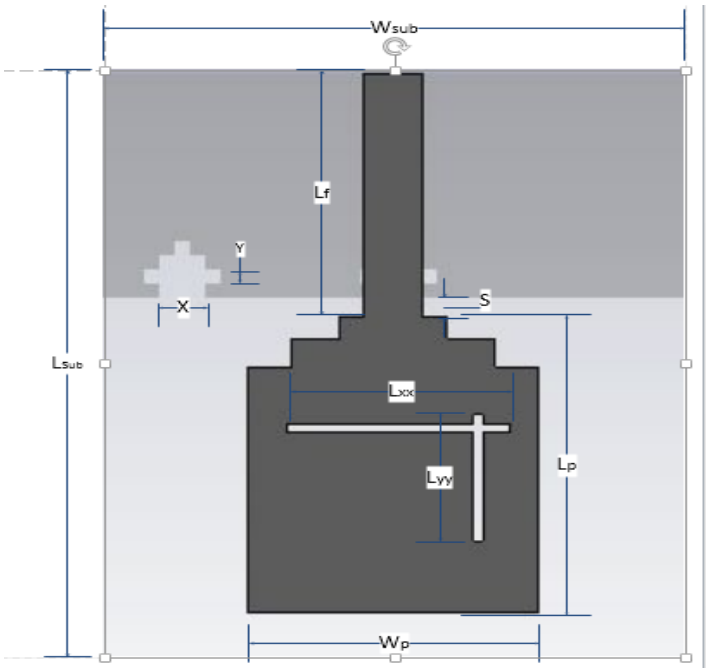

Fig. 1. Front view of proposed band notch UWB antenna.

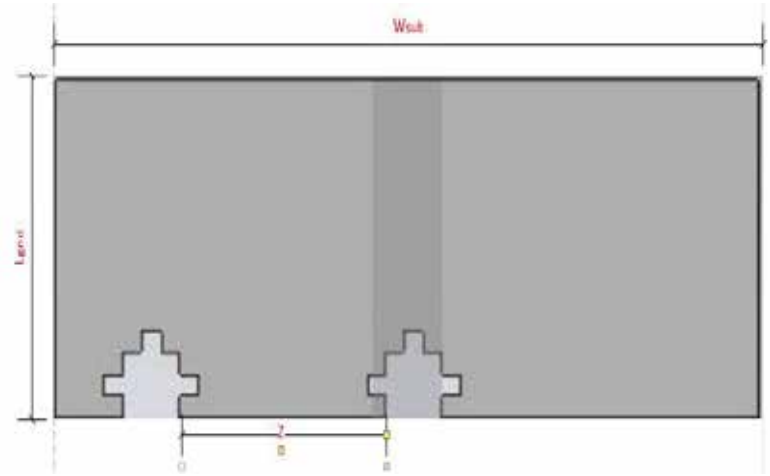

Fig. 2. Bottom view of proposed UWB antenna with fractal slots.

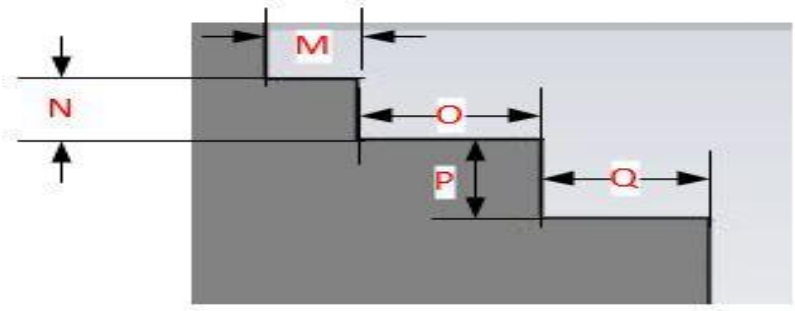

Fig. 3. Stair case steps for impedance matching.

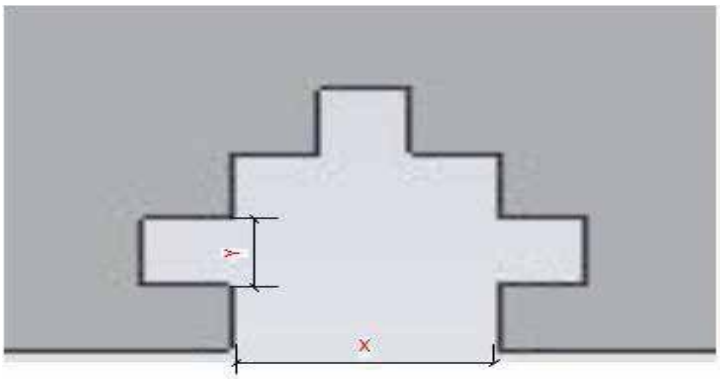

Fig. 4. Fractal slot in ground plane.

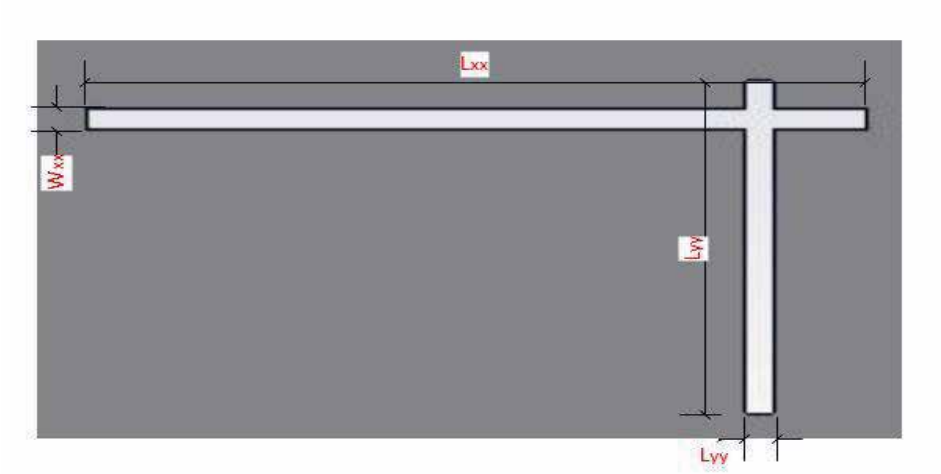

Fig. 5. L-shaped slot inserted in the patch for introducing band notch.

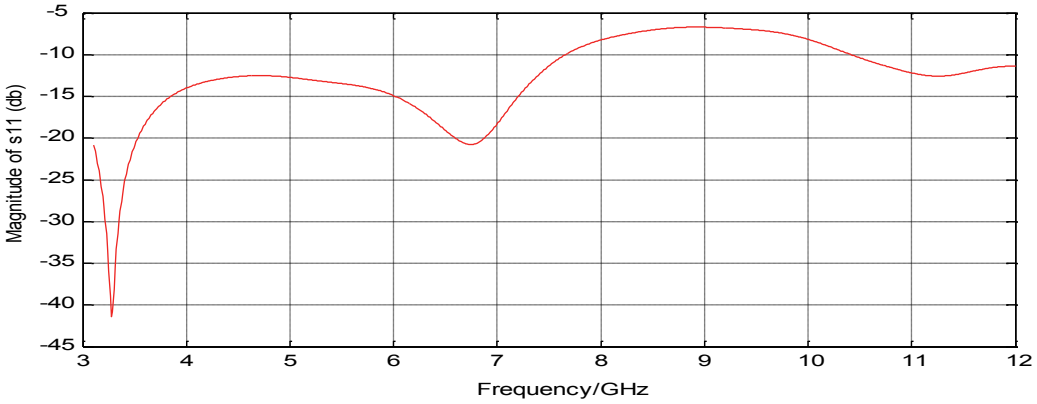

Fig. 6. S_{11} of UWB antenna without slot.

Table 1. Dimensions of substrate, patch, ground plane and feed line.

Substrate Width (Wsub)	Substrate Length (Lsub)	Patch Width (Wp)	Patch Length (Lp)	Ground Length (Lgnd)	Feed line length (Lf)	Feed line Width (Wf)	Gap between patch and gnd (S)
30 mm	36 mm	15 mm	16.5 mm	12.5 mm	13.5 mm	3 mm	1 mm

Table 2. Dimensions of stair cased impedance steps.

M	N	O	P	Q
1.25 mm	1.25 mm	2.5 mm	1.6 mm	2.5 mm

Table 3. Dimensions of inserted slot in the patch.

Horizontal length (Lxx)	Vertical length (Lyy)	Horizontal Width (Wxx)	Vertical Width (Wyy)
11.5 mm	7 mm	0.5 mm	0.5 mm

The top view of antenna consists of rectangular patch with L shaped slots as shown in Fig. 1. This antenna covers the entire UWB range while rejecting the WLAN band. The antenna is fed with a 50 Ω microstrip line modeled on FR4 substrate having thickness 1.6 mm, relative permittivity of 4.4 and tanδ = 0.0025. The bottom side of antenna consists of partial ground plane with two fractal shaped slots as shown in Fig. 2. The overall size of antenna is 30×36 mm2 (Wsub × Lsub) which is quite compact. The distance between ground plane and radiating patch is kept at S = 1mm.

The optimized dimensions of the proposed design are listed in Table 1 to Table 3. Fig. 3 shows the stair case configuration at the edge of antenna for impedance matching. Fig. 4 shows fractal slot inserted in the ground plane. There are two fractal slots in the ground plane and the distance between them is Z =11.2 mm while X = 2.4 mm and Y = 0.8 mm. Fig. 5 shows the L shaped slots inserted in the antenna for introducing the required band notch at 5.8 GHz WLAN band.

3. RESULTS AND DISCUSSION

3.1 UWB Antenna without Slot

First the antenna has been designed without fractal slots in the ground plane. The S_{11} plot shows that the antenna cannot cover the entire UWB band and is matched to the 50 Ωtransmission line only from 3.1 to 7.7 GHz.

So we must enhance the impedance bandwidth of the antenna. This is achieved by increasing electrical path length for the surface current. To increase the electrical path length for surface current distribution two similar fractal slots are etched on top edge of the ground plane. So by increasing the electrical path length for surface current the impedance bandwidth in turn enhances [15, 16]. The fractal geometry has been introduced in the ground plane as shown in Fig. 4. The distance between these two slots(Z) is optimized to achieve the required UWB frequency range. Good Impedance matching has been found at Z= 11.2 mm and S = 1 mm which is the gap between the radiating patch and the ground plane. The S_{11} curve in Fig. 7 shows that the antenna now covers the entire UWB frequency band and has a maximum value of -25db at 6.65 GHz.

3.2. UWB Antenna with Slot

By inserting slot in the radiating patch, the antenna operates in the entire UWB band while rejecting WLAN signal. The slot geometry is shown in the Fig. 5. Now there is no more potential interference of the UWB and WLAN signals. The length of the notch band is calculated from the equation below:

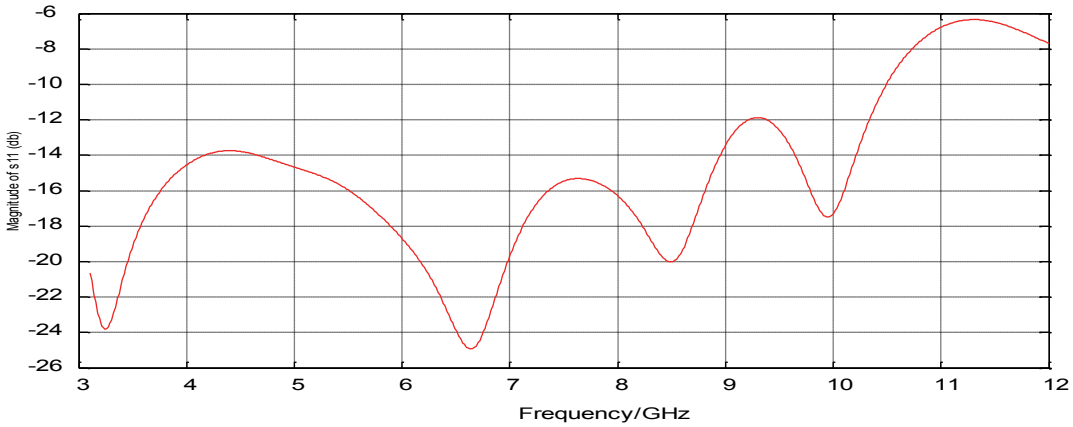

Fig. 7. S_{11} for UWB antenna with fractal slots.

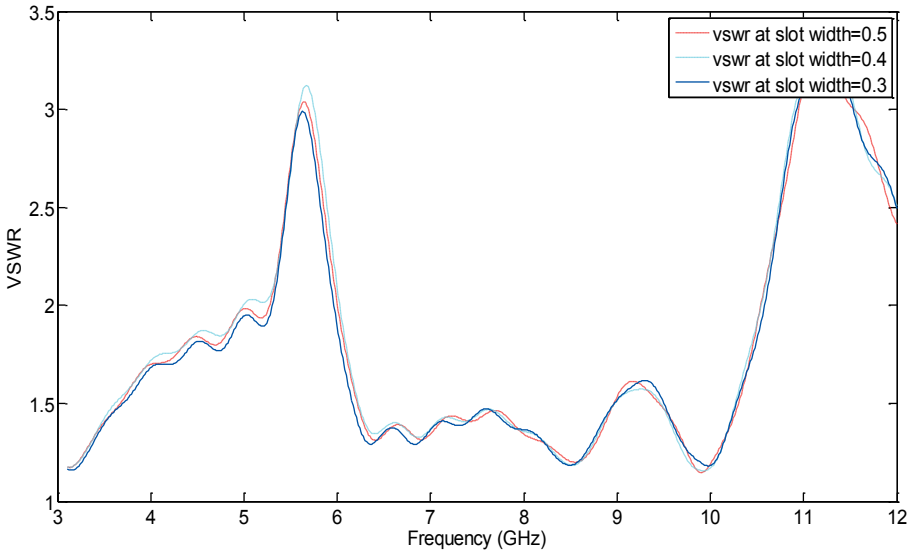

Fig. 8. Effect on VSWR due to slot width.

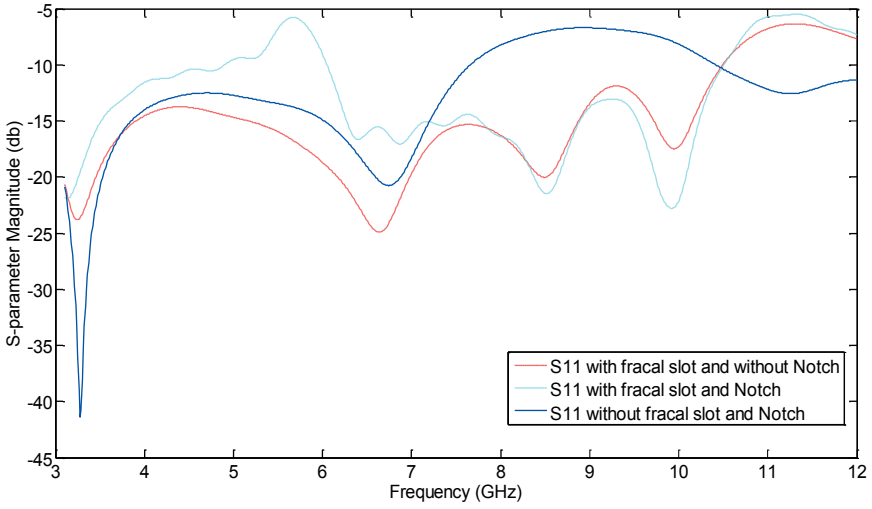

Fig. 9. S_{11} for proposed UWB antenna using Ansoft HFSS.

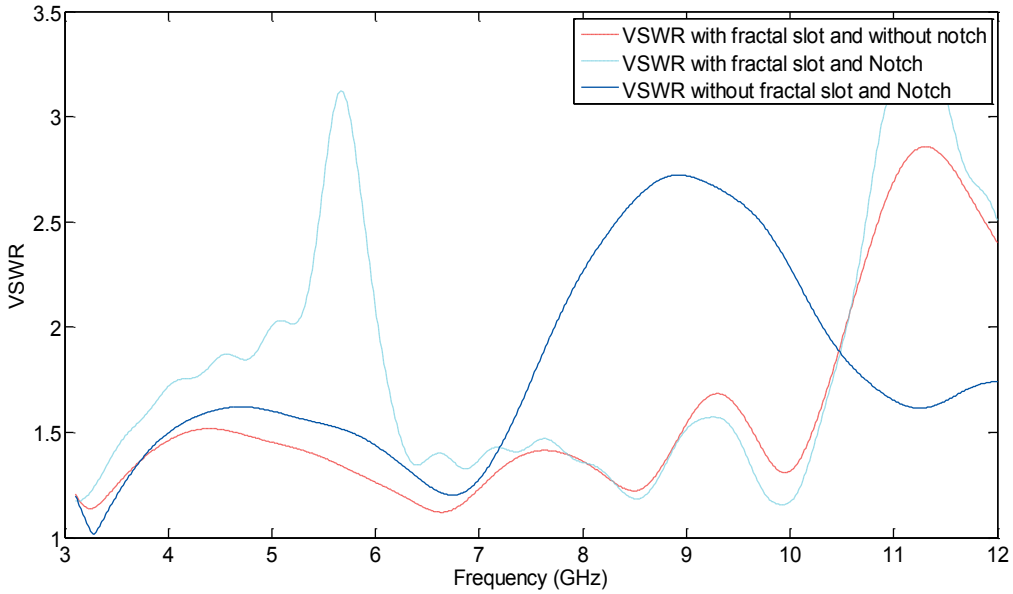

Fig. 10. VSWR for the simulated UWB antenna using Ansoft HFSS.

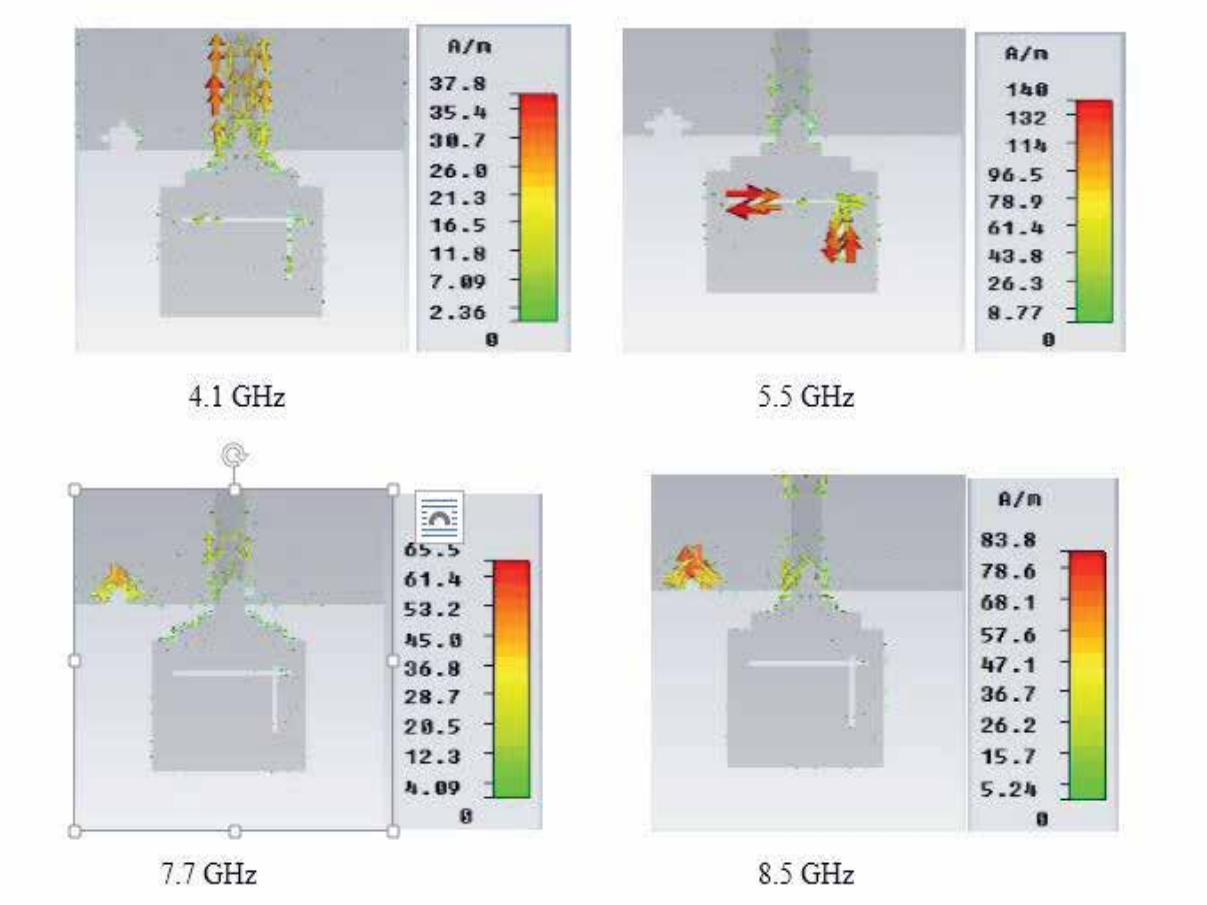

Fig. 11. Simulated current distributions on the surface of the proposed antenna at different frequencies.

$$f_{notch} = \frac{C}{2 \times L \sqrt{\frac{\varepsilon r + 1}{2}}}$$

Where, L is the length of the slot, εr is the relative permittivity and C is the speed of light. The length of the slot resonator is calculated from (1) while its position is analyzed from surface current distribution as shown in Fig. 11. The width of the slot is selected by simulating at different slot widths as shown in Fig. 8. The resonator will introduce high reflection at resonance which will lead to band notching effect. The length (Lxx and Lyy) of the slot is the important parameter in notching the desired band. It is cleared from the VSWR curves that the antenna performance is slightly changed by changing the slot width, so we can choose any value of the above. We selected the width of the slot to be 0.5 mm.

So, by introducing the slot of length and width discussed above in the radiating patch, the VSWR in the 5.8 GHz WLAN band is greater than 2 as shown in Fig. 8, which shows that the antenna performance is not good in this band.

The antenna results have also been verified using Ansoft HFSS. The verified results have been plotted in Fig. 9 and Fig. 10. The S_{11} and VSWR plot has been analyzed first with and without fractal slots in the partial ground plane and then analyzed with and without notch in the radiating patch. These results show that there is one notch band at 5.8 GHz WLAN band. A very small difference is observed between the results simulated through CST and HFSS.

3.3 Surface Current Distribution

Fig. 11 shows the simulated current distributions on the surface of the proposed antenna at 4.1, 5.5, 7.7, and 8.5 GHz. At 4.1, 7.7, and 8.5 GHz, the current mostly flows along the microstrip feed line, while very small current is around the slot. On the other hand, the surface current distribution on the antenna at 5.5 GHz is concentrated around the slot.

3.4 Radiation Patterns

The simulated far field radiation pattern of the proposed antenna at different frequencies is shown in Fig.11.The radiation pattern is stable throughout the operating band and shows that no ripples are present at higher frequencies.

3.5 Far Field 3D Radiation Pattern

The simulated far field of the proposed antenna at two different frequencies is shown in Fig. 12. The antenna has a maximum gain of 5.31 dB at 8.5 GHz and an average gain of almost 4.2 dB.

3.6 Transmission Response of UWB Antenna

This section considers the communication between the two designed UWB band-notched antennas. The two antennas are designed and the distance between the transmitting and the receiving antennas is kept 60cm, which is almost 6 wavelengths of the considered band of operation at the lowest frequency. Also we consider that the antennas are at the far field of each other. Now by exciting the transmitting antenna with different input pulses such as modulated Gaussian pulse, first order Rayleigh pulse, fifth derivative of Gaussian pulse and fourth order Rayleigh pulse.

We also consider that the antennas operate in two orientations: (a) face to face and (b) side by side as shown in Fig. 13. Fig. 14 shows the transfer function, S_{21} versus the frequency in two different orientations. By analyzing the figures it is clear that the transfer function of face to face orientations is better than that of side by side.

4. CONCLUSIONS

A compact UWB rectangular radiating patch antenna along with fractal slots in the partial ground plane has been proposed. Wide band matching is achieved by introducing fractal slots in the partial ground plane and non-uniform stair cased impedance steps at the radiating patch. The potential interference between the UWB system and WLAN band has been minimized by introducing slot in the radiating patch, which rejects the WLAN band. The antenna results have been analyzed showing high Gain and good radiation pattern. The antenna exhibits low VSWR in the frequency band from 3 to 10.6 GHz with a band-notching effect at the frequency band at 5.8 GHz. The antenna has a compact size which makes it a potential candidate for UWB portable devices.

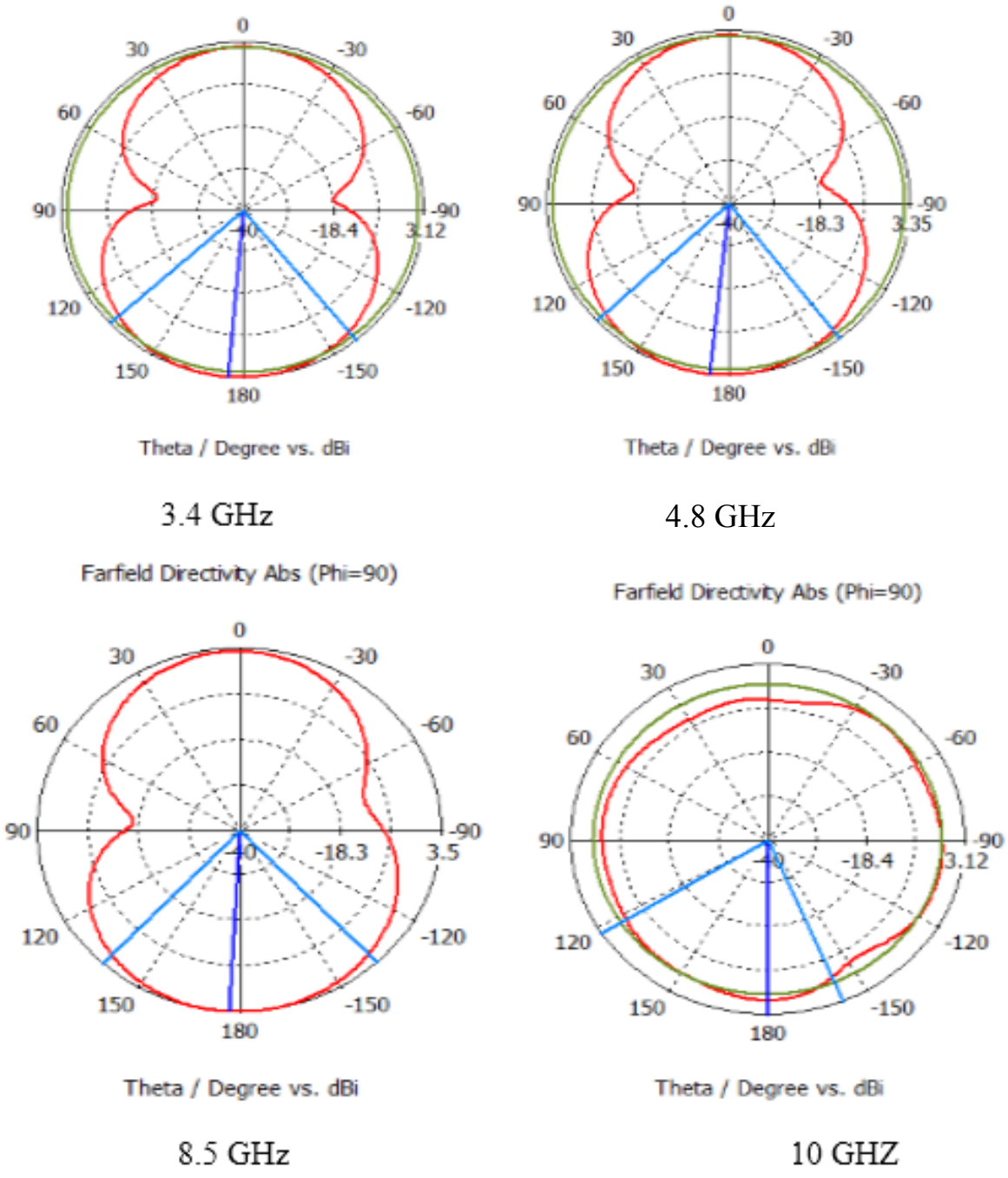

Fig. 12. Simulated far field radiation pattern of the proposed antenna at different frequencies.

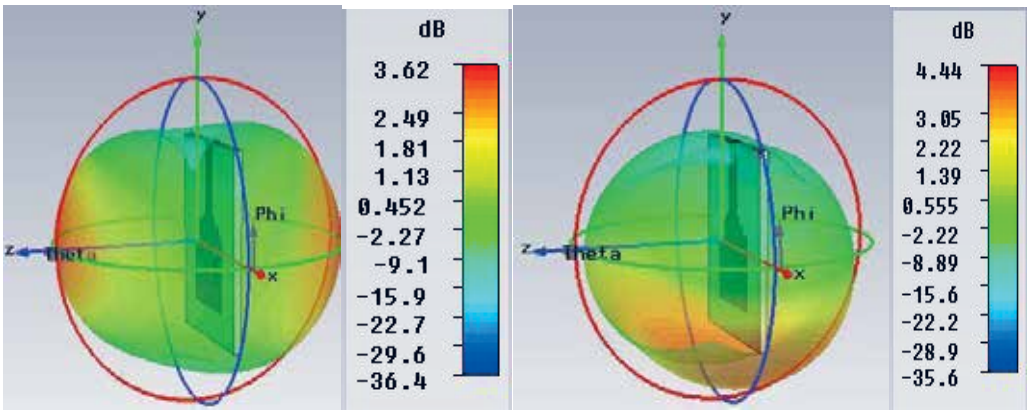

Fig. 13. Simulated 3D far field of the proposed antenna at 5.45GHz and 7GHz.

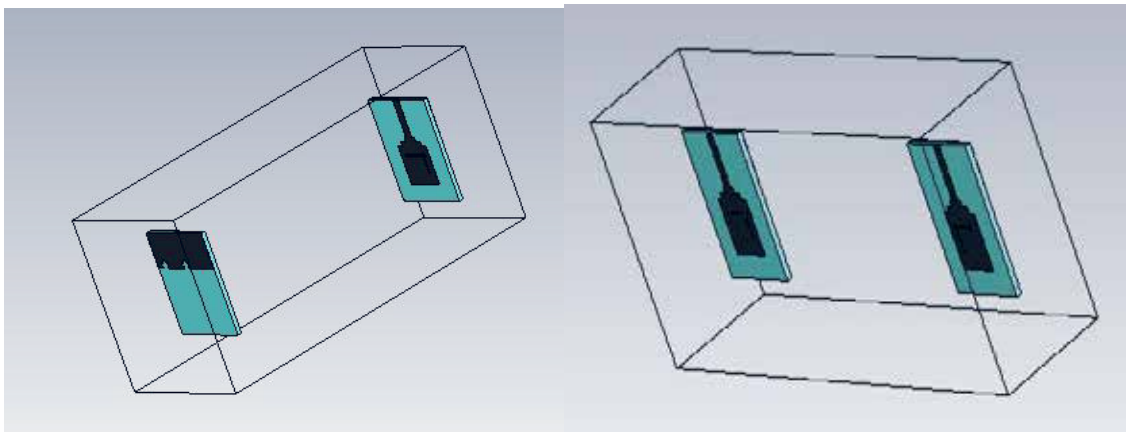

Fig. 14. Transmitting and Receiving antennas in two different orientations: (a) Face to face; (b) Side by side.

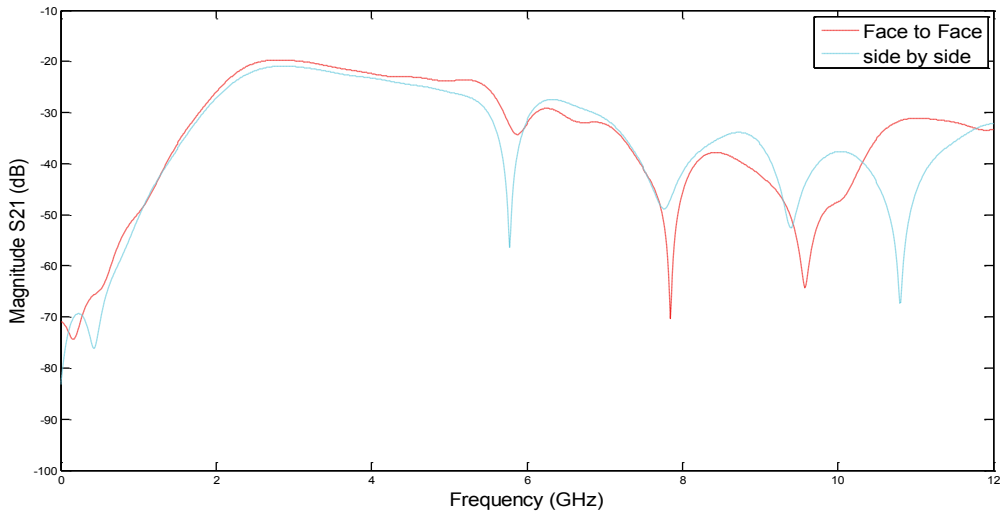

Fig. 15. Transmission Coefficient S_{21} between two UWB antennas.

5. REFERENCES

1. Naghshvarian, M.J. Compact UWB bandnotch antenna with transmission-line-fed. *Progress In Electromagnetics Research B* 3: 283-293 (2008).
2. Low, Z.N., J.H. Cheong & C.L. Law. Low-cost PCB antenna for UWB applications. *Antennas and Wireless Propagation Letters, IEEE* 4: 237-239 (2005).
3. Xu, J., D-Y. Shen, G-T. Wang, X-H. Zhang, X-P. Zhang & K. Wu. A small UWB antenna with dual band-notched characteristics. *International Journal of Antennas and Propagation* Volume 2012: 1-7 (2012).
4. Singh, A., & R.K. Raj. Dual band notched small monopole antenna with a novel T-shaped slot for UWB applications. In: *International Conference on Advances in Engineering and Technology Research (ICAETR), IEEE 2014*, p. 1-4 (2014).
5. Lu, B., T. Li, X. Wang & L. Li. A new miniaturized planar UWB antenna with WLAN Band notch characteristics. In: *2014 3rd Asia-Pacific Conference on Antennas and Propagation (APCAP)*, *IEEE,* p. 426-429 (2014).
6. Islam, M.R. & S. Khan. Designing an UWB patch antenna with band notched by using L-shaped slot and unsymmetrical feedline. In: *Canadian Conference on Electrical and Computer Engineering, 2008.* CCECE, IEEE, 2008, p. 000101-000104 (2008).
7. Liu, J., S. Gong, Y. Xu, X. Zhang, C. Feng & N. Qi. Compact printed ultra-wideband monopole antenna with dual band-notched characteristics. *Electronics Letters* 44(12): 710-711 (2008).
8. Chu, Q-X. & Y-Y. Yang. A compact ultrawideband antenna with 3.4/5.5 GHz dual band-notched characteristics. *IEEE Transactions on Antennas and Propagation* 56.12 (2008): 3637-3644.
9. Wong, K-L, Y-W. Chi, C-M. Su & F-S. Chang. Band-notched ultra-wideband circular-disk monopole antenna with an arc-shaped slot. *Microwave and Optical Technology Letters* 45(3): 188-191 (2005).
10. Cho, Y.J., K.H. Kim, D.H. Choi, S.S. Lee & S-O. Park. A miniature UWB planar monopole antenna with 5-GHz band-rejection filter and the time-domain characteristics. In: *IEEE Transactions on Antennas and Propagation* 54(5): 1453-1460 (2006).
11. Kim, K-H., Y-J. Cho, S-H. Hwang & S-O. Park. Band-notched UWB planar monopole antenna with two parasitic patches. *Electronics Letters* 41(14): 783-785 (2005).
12. Islam, M. T., R. Azim & A.T. Mobashsher. Triple band-notched planar UWB antenna using parasitic strips. *Progress in Electromagnetics Research* 129: 161-179 (2012).
13. Garg, R. *Microstrip Antenna Design Handbook*. Artech House (2001).
14. Falconer, K. *Fractal Geometry: Mathematical Foundations and Applications*. John Wiley & Sons (2004).
15. Hong, T., S-X.i Gong, Y. Liu & W. Jiang. Monopole antenna with quasi-fractal slotted ground plane for dual-band applications. *Antennas and Wireless Propagation Letters, IEEE* 9 (2010): 595-598 (2010).
16. Karmakar, A., U. Banerjee, R. Ghatak & D. R. Poddar. Design and analysis of fractal based UWB monopole antenna. In: *2013 National Conference on Communications (NCC),* IEEE, p. 1-5 (2013).

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Research Article

Systematic Literature Review Protocol for Green Software Multi-sourcing with Preliminary Results

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Abstract: Green Software Engineering is a research area gaining reputation rapidly in order to create green software. Software development multi-sourcing is a modern Global Software Engineering (GSE) pattern for creating high quality software at minimum cost and time at low wages countries by contracting out the software development work to multiple vendors located at multiple geographical locations across the globe. Green Software multi-sourcing is a modern research area which lies at the intersection of both the aforementioned research areas (Green Software Engineering and Software Multi-Sourcing). The goal of this paper is to present the systematic literature review (SLR) protocol which has been developed, validated and in the process of implementation. The implementation of the SLR protocol will yield in the identification of success/risk factors, and real-world green practices to be considered by multi-sourcing vendor organizations that can contribute towards the development of green and sustainable software. Systematic Literature Review procedure has been used as research practice. SLR is a new research method in software engineering which is dissimilar from conventional literature analysis. SLR is more comprehensive and is grounded on a pre-defined protocol. We have developed a Systematic Literature Review (SLR) protocol and are currently in the process of its implementation. The anticipated conclusions of the SLR execution will be the identification of success/risk factors and practices that can contribute towards the development of green software in multi-sourced projects.

Keywords: Green software development, green software multi-sourcing, systematic literature review protocol, green software multi-sourcing models

1. INTRODUCTION

This study is inspired by two interests: the need for sustainable and green software development and the development of a model for measuring vendors' green capabilities in software multi-sourced projects. Green computing is an emerging research field and several solutions have been proposed for this purpose [1].

Green computing can be demarcated as the employment of maximizing the proficient use of computing assets to minimize its deleterious influence on environment [2]. The environmental effects and concerns due to information and communication technologies (ICTs) have flagrantly raised these days [3-6]. Business organizations and ICT firms consider green computing as an integral part of their overall business strategies [7]. From both business and consumers perspective energy efficiency is on the highest priority. The reasons reported include, increasing energy outlays and ecological change, originated by mounting global warming and greenhouse gas [8].

Literature reveals that in recent years a number of struggles have been made while creating green and sustainable software. Several researchers has focused on hardware oriented solutions for environmental sustainability whereas others have worked on software oriented solutions [9]. Imtiaz and Mahmoud [10] have proposed green model having two levels, for sustainable software engineering, the first level presents guidelines for green software development and the

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second level is consist of methods occupied by the software during its execution to support green computing. Erdelyi [11] has presented green rules for developing long-living software in the industrial automation domain. Appasami and Suresh [12] have presented some general guidelines and tips for green computing. GREENSOFT model for sustainable software has been presented by [13], the model supports software engineers/administrators/software users in building, sustaining, and using software in a more ecological way. Shenoy and Earatta [14] have presented a model for green software development in which they suggest some guidelines and practices that support sustainability. Some green software quality factors are presented by [15] which contribute in green software development. Mahaux and Canon [16] have argued that requirements engineering is critical for sustainable software development in the whole software life cycle. Other studies [17, 18] also argue that requirements engineering play important role in sustainable software development. According to Imtiaz and Mahmoud [10] software manages and operates the underlying hardware and therefore has indirect effect on the environment. Similarly software oriented practices can utilize hardware resources efficiently and limit the need for new hardware due to updates. Sedef et al., [19] came up with a guide for green software developers, in the context of environmental sustainability and software quality criteria.

The latest literature in the domain reveals that there is scarcity of software engineering models and tools that can well define how software can be designed and maintained in an eco-friendly manner [14]. Software engineering researchers have developed keen interest in green software development due to the demand of environmental sustainability [20]. Academia and software houses consider green software engineering as a strength due to its vital role in environmental sustainability and energy consumption [1]. The ICT power utilization is increasing day by day. Consequently, a number of hardware oriented solutions exist, but the involvement of green software is still not well matured [21]. A diversity of research on green computing has generally focused on ecological sustainability in terms of computer hardware [10]. Green Software has been defined in the literature as, that Software, whose direct and indirect negative influences on economy/humanity/ human beings/environment due to its creation, deployment, and usage minimal and/or which has a positive consequence on sustainable development [21].

In voluminous software development tasks, developers treat sustainability as postscripts because developers/designers are focused by time-to-market force and are often less conversant about green software practices and techniques [22]. Creating energy efficient and green software is becoming popular subject rapidly [23]. Various aspects of green computing and green ICT have been explored by researchers and scholars in general [11, 24-31]. However, very petite attention has been compensated to the development of green software in multisourcing environment. Multi-sourcing is a contemporary paradigm in software outsourcing domain which offers the benefits of using multiple vendors for the development of software in a shorter time span. The outsourced software is developed in modules/components by various software vendors to expedite the development process [32].

Consequently, Green Software Multi-sourcing has developed a new area of research interest in Global Software Engineering domain in order to develop green and sustainable software in multi-sourced software development projects. Research in this area will assist multi-sourcing industry to deliver green software in general and to gauge vendors' readiness for the development of greener software in particular. For this purpose we have proposed Green Software Multi-sourcing Readiness Model. The proposed structure of the model has been published [33]. In current paper we have presented/exposed our systematic literature Review (SLR) protocol. The preliminary/pilot results of the SLR protocol are also presented.

2. OBJECTIVES

Our main objective is to develop Green Software Multi-souring Readiness Model (GSMRM), to assist multi-sourcing vendor organizations in assessing their readiness for developing green software in multi-sourced projects. The preliminary structure of the model has been published [33].

The proposed model will be developed in three phases i.e. first phase is protocol development and its implementation, second phase

is empirical validation of protocol findings and the third phase is model development and evaluation phase as shown in Fig.1.

In this paper we have presented the first segment/phase of our ongoing research i.e. systematic literature review protocol. We have produced the SLR protocol for identification of the success factors, risk factors and real-world green practices to be considered by multi-sourcing vendor organizations that can contribute towards the development of green and sustainable software in multi-sourced software development projects. While designing our protocol, we studied a number of SLR protocols for guidance [34-36].

3. RESEARCH METHODOLOGY

In order to fulfil the aim of our research/study project, the research work will be carried out in three phases/stages. In the first stage success factors, risk factors and their practices, regarding the development of green and sustainable software, will be investigated via Systematic Literature Review process. A systematic literature review (SLR) process is a new research method in the software engineering domain which is used for the identification, assessment and clarification of existing literature on a specific research topic/research question [37]. According to [38, 39] SLR has become an imperative research methodology in software engineering. SLR is difficult to conduct as compared to ordinary literature review. However the results generated by SLR are comparatively unbiased and to some extent repeatable because it utilizes a pre-defined and validated protocol [40].

In second phase we will conduct empirical studies in software multi-sourcing industry in order to authenticate the SLR outcomes and to find any new factors/features apart from the findings of the SLR. An analogous method has been used by other software engineering experts and researchers [41, 42].

The third phase is the development of the proposed model. The outcomes of the SLR and empirical study will provide inputs to the development of the GSMRM. A similar approach has been used by other software engineering researchers[41]. The proposed GSMRM model will be validated through conduction of five case studies in software development multi-sourcing industry. A similar approach has been used by other researcher [43]. The current study discusses the first phase/stage of our research study i.e. Systematic Literature Review Protocol Development. In current paper we have only presented first phase of our ongoing research i.e. SLR protocol with its preliminary results.

4. SYSTEMATIC LITERATURE REVIEW PROTOCOL

The SLR protocol is a set of activities that have to be completed in order to response the research questions listed in the *Section 4.1*. Our review protocol is made of seven components as shown in Fig. 2. Each component is explained in the following sub-sections:

4.1 Research Questions

We have formulated the following Research Questions (RQs).

RQ 1: What are the success factors, as identified in the literature, to be addressed by multi-sourcing vendor organizations which can contribute towards the development of green and sustainable software?

RQ 2: What are the risk factors, as identified in the literature, to be avoided by multi-sourcing vendor organizations which can contribute towards the development of green and sustainable software?

RQ 3: What are the real-world practices, as identified in the literature, to be adopted by multi-sourcing vendor organizations which can contribute towards the development of green and sustainable software?

4.2 Building Search Term

We have used population, intervention and outcomes of relevance for designing search term from our research questions.

Population: Software Multi-sourcing Vendor Organization

Intervention: Success Factors, Risk Factors and Real-world practices

Outcomes of relevance: Green Software Multi-sourcing Readiness Model (GSMRM)

Experimental Design: SLR, Empirical studies, case studies, expert's opinions and theoretical studies.

4.3 Search strategy/policy

The following search strategy is used for the construction of search terms.

a) We will identify major search terms from population, intervention and outcome of the formulated research questions.

b) We will find the alternate (synonyms) for these major search terms.

c) Appropriate keywords will be considered from relevant papers.

d) Use 'AND' for the concatenation of major terms and 'OR' for concatenation of alternative spellings and synonyms.

Results for a) Green software, practices, success factors, risks factors, multi-sourcing.

Results for b)

Green software :("green computing" OR "green IT" OR "green software" OR "sustainable software")

Practices: ("practices" or "methods")

Success Factors: ("factors" OR "elements" OR "parameters")

Risks Factors: ("risks" OR "barriers" OR "problems")

Multi-sourcing: ("multi-sourcing" OR "multisourcing" OR "multi vendors")

Results for c)

("Green software", "sustainable software", "green computing", "green ICT", "green IT")

Results for d)

Track 1: ("Green software development" OR "green software" OR "sustainable software" OR "green IT" OR "green computing") AND ("practices" OR "methods") AND ("factors" OR "elements" OR "parameters") AND ("risks" OR "barriers" OR "problems") AND ("multi-sourcing" OR "multi vendors outsourcing")

Track 2: ("Green computing" OR "green IT" OR "green software" OR "sustainable software") AND ("practices" OR "methods") AND ("factors" OR "elements" OR "parameters") AND ("risks" OR "barriers" OR "problems")

Where Track1 represents search string designed specifically to retrieve literature regarding green software in software development multi-sourcing context, whereas Track 2 represents search string designed with the intent to retrieve literature regarding green software in general context. The results of search string mentioned in Track 1 were very poor and almost negligible as shown in Table 1. Therefore we decided, after thorough discussions with fellows of software engineering research group (SERG-UOM) at the university, to move towards the implementation of Track 2 as shown in Table 2.

The factors, to be identified through the SLR, will be validated through empirical studies in multi-sourcing software industry in order to know whether these findings are applicable specifically, or can be adopted, in software multi-sourcing environment. A similar approach has already been used by other researchers [44]. Further very limited numbers of empirical research studies have been conducted GSD context in general and software multi-sourcing in particular [45].

Table 1. Track 1 results.

Digital Libraries	Total publication Found
Science Direct	00
ACM	00
IEEE Xplore	00
Springer Link	02
Google Scholar	14

Table 2. Track 2 results.

Digital Libraries	Total publication Found
Science Direct	1494
ACM	696
IEEE Xplore	33
Springer Link	1869
Google Scholar	3390

4.4 Resources Searched

We have searched the following digital libraries using 4.3(d) as a search strings.

- Science Direct (http://www.sciencedirect.com/)
- ACM (http://dl.acm.org/)
- IEEE Xplore (http://ieeexplore.ieee.org/)
- Springer Link (http://link.springer.com/)
- Google Scholar (https://scholar.google.com.pk/)

4.5 Publication Selection

The publication selection criteria are used to select relevant literature from the execution of search string. We have defined the following inclusion/exclusion criteria in order to extract relevant research papers from available literature.

4.5.1 Inclusion Criteria

- Research papers that are relevant to our research questions.
- Research work that describe green software in multi sourcing environment/General context.
- The research papers/ articles /books/review paper is in English Language.
- The article/paper is available in full text.
- Research papers that describes success factors, risk factors and practices of green software development in multi-sourcing/General context.

4.5.2 Exclusion Criteria

- Article/papers/books etc. that do not fulfil inclusion criteria as mentioned above will be excluded

4.5.3 Primary Selection of Relevant Literature

Primary selection of relevant literature has been performed by reviewing the abstract, title and keywords of the papers. The purpose of primary selection is to eliminate those outcomes which have no relevance to our research questions. The primary selected research papers have been checked against the aforementioned publication selection criteria by reading full text of the selected papers.

4.6 Publication Quality Assessment

The main drive of quality valuation is to check and assess the quality of finally selected papers. The quality checklist contains the following questions:

a. Is the objective of the research is clearly defined?
b. Is the outcomes of the research is connected to the objective of the research?
c. Whether the term green software in multi sourced projects/General software development context is discussed clearly?
d. Is it clear how the factor/practice was identified?
e. Each of the above interrogations will be marked as 'YES', 'NO', or 'N.A'.

Apart from the above quality criteria we will also use the following quality criterion which is based on our own experience. We have used the same criteria in our previous research [46].

Criteria for A-quality papers: In this category we list those papers which fulfill the following criteria's:

- Paper published in impact factor Journal
- Having clear methodology
- Having sample size of the following condition
  a. Case Study ≥ 3
  b. Interviews ≥ 12
  c. Survey ≥ 50
  d. Literature Review ≥ 50

Criteria for B-quality papers: In this category we list those papers which fulfill the following criteria's:

- Paper published in well reputed conference
- Having clear methodology
- Having sample size of the following condition
  a. Case Study = 2
  b. Interviews: $\geq 5$and ≤ 11
  c. Survey: ≥ 30 and ≤ 49
  d. Literature Review: ≥ 30 and ≤ 49

Criteria for C-quality papers: In this category we list those papers which fulfill the following criteria's:

- Experienced reports/ articles, published in less reputed venues (Journal, Conference)
- Having clear methodology
- Having sample size of the following condition:

Table 3. SLR Protocol preliminary results.

S. No	Digital library	Search String Used	Date Constraint	Total Publications Found	Primary/ Initial Selection	Final Selection
1	Science Direct	Track 2	All Years	1494	22	09
2	ACM	Track 2	All Years	696	25	11
3	IEEE Xplore	Track 2	All Years	33	05	02
4	Springer Link	Track 2	All Years	1869	17	06
5	Google Scholar	Track 2	All Years	3390	22	13
6		**Total**		**7,482**	**91**	41
7		**Publication through Snowballing: 33** **Total Publications through SLR: 41** **Total Finally Selected Publications: (N=74)**				

Table 4. Data extraction form.

Data to be Extracted	Description/Values
Extraction Form S.No	
Paper ID	
Date Of Review	
Title Of The Paper	
Reference	
Year (Data Collection / Publication Year)	
Database/Digital Library/Online Resource	
Publication Quality (A/B/C/Other)	
Population (Sample/Target)	
Study Strategy (OLR, SLR, Survey, Interview, Case Study, Experiment, Experience Report etc)	
Company Type (National, Multi-National (MNC),NA)	
Company Size (Small, Medium, Large, NA)	
Company SPI Status (CMMI,ISO etc, NA)	
Country (Data collection / Author)	
Multi-Sourcing Context /General Context	
Success Factors	
Risks Factors	
Practices	

a. Case Study: 1
b. Interviews ≤ 5
c. Survey: ≥ 1 and ≤ 29
d. Literature Review: ≥ 1 and ≤ 29

4.7 Data Extraction and Synthesis Process

The practice of mining data from finally selected research papers/articles is called extraction of data. The fundamental goal of our SLR protocol is to mine data (success factors, risk factors and green software practices) which fulfills our framed research questions that is; RQ1, RQ2 and RQ3. We have successfully extracted data from 74 publications. The review was mainly undertaken by a single researcher (principal author), however secondary researcher (Research Supervisor) was

Table 5. List of factors for green and sustainable software development.

S. No.	Success Factor	N= 74	References
1	Green software design and efficient coding	57	[α 1][α 3][α 4][α 5][α 6][α 9][α 10][α 11][α 12][α 16][α 17][α 18][α 19][α 21][α 22][α 23][α 24][α 25][α 36][α 37][α 38][α 40][α 41][α 43][α 45][α 48][α 52][α 53][α 74][α 58][α 60][α 62][α 63][α 65][α 66][α 67][α 68][α 71][α 26][α 29][α 30][α 32][α 33][α 33][α 2][α 7][α 8][α 20][α 34][α 39][α 55][α 57][α 27][α 28][α 29][α 32]
2	Power-saving software strategies	55	[α 2][α 5][α 1][α 6][α 7][α 8][α 10][α 11][α 12][α 13][α 14][α 15][α 17][α 18][α 21][α 22][α 23][α 24][α 25][α 35][α 36][α 37][α 39][α 40][α 41][α 43][α 44][α 46][α 47][α 48][α 49][α 50][α 51][α 52][α 53][α 54][α 55][α 56][α 73][α 58][α 60][α 61][α 63][α 65][α 66][α 68][α 72][α 26][α 29][α 30][α 32][α 33]
3	Low carbon emission throughout the software development process	45	[α 2][α 5][α 1][α 6][α 8][α 9][α 10][α 11][α 13][α 15][α 17][α 18][α 22][α 25][α 36][α 37][α 40][α 43][α 44][α 46][α 48][α 52][α 54][α 55][α 57][α 58][α 59][α 60][α 63][α 64][α 65][α 66][α 67][α 68][α 70][α 71][α 72][α 26][α 28][α 29][α 30][α 32][α 33]
4	Efficient Resource Utilization	44	[α 2][α 3][α 4][α 6][α 8][α 9][α 10][α 11][α 12][α 14][α 21][α 22][α 23][α 24][α 25][α 34][α 35][α 37][α 38][α 40][α 41][α 42][α 43][α 44][α 47][α 50][α 51][α 52][α 53][α 54][α 55][α 73][α 58][α 60][α 61][α 62][α 63][α 64][α 65][α 68][α 29][α 30][α 32]
5	Paperless communication	41	[α 2][α 5][α 1][α 6][α 8][α 11][α 12][α 23][α 25][α 34][α 36][α 40][α 43][α 45][α 48][α 50][α 51][α 52][α 53][α 54][α 55][α 56][α 57][α 74][α 58][α 59][α 60][α 62][α 64][α 66][α 65][α 67][α 71][α 72][α 26][α 28][α 31][α 32][α 33]
6	Filtration of requirements through green evaluator	40	[α 1][α 3][α 4][α 5][α 6][α 8][α 9][α 10][α 11][α 12][α 16][α 19][α 20][α 21][α 22][α 24][α 34][α 36][α 38][α 41][α 42][α 43][α 46][α 53][α 58][α 59][α 61][α 63][α 64][α 65][α 67][α 71][α 26][α 28][α 29][α 30][α 32]
7	E-Waste Management	40	[α 1][α 5][α 6][α 20][α 22][α 25][α 43][α 48][α 52][α 58][α 60][α 62][α 64][α 65][α 67][α 71][α 29][α 32][α 21][α 23][α 25][α 35][α 37][α 40][α 43][α 44][α 46][α 47][α 48][α 49][α 50][α 52][α 57][α 58][α 63][α 65][α 67][α 30][α 33][α 1][α 2][α 3[α 5]
8	Green utilization of the software	20	[α 5][α 11][α 13][α 17][α 18][α 20][α 21][α 22][α 24][α 25][α 43][α 46][α 54][α 58][α 65][α 70][α

Contd.....

Table 5 (Contd.)

S. No.	Success Factor	N= 74	References
			71][α 26][α 29][α 32]
9	Sustainable maintenance of the software	19	[α 5][α 11][α 13][α 17][α 18][α 20][α 21][α 22][α 24][α 25][α 43][α 46][α 54][α 58][α 65][α 70][α 71][α 26][α 29][α 32]
10	Sustainable testing	17	[α 1][α 4][α 5][α 6][α 12][α 13][α 16][α 19][α 21][α 22][α 34][α 43][α 46][α 53][α 29][α 30][α 32]
11	Use of cloud for software distribution	16	[α 1][α 2][α 3][α 5][α 12][α 16][α 20][α 22][α 43][α 53][α 58][α 61][α 65][α 71][α 32]
12	Use of agile strategies	15	[α 3][α 6][α 16][α 21][α 24][α 34][α 43][α 53][α 59][α 27][α 28][α 38][α 48][α 13]
13	Software reusability	14	[α 5][α 1][α 6][α 7][α 16][α 18][α 34][α 38][α 42][α 53][α 61][α 63][α 68][α 33]
14	Green graphical user interface	14	[α 4][α 8][α 12][α 14][α 16][α 18][α 21][α 36][α 53][α 61][α 26][α 29][α 33]
15	Software flexibility and legacy systems support	14	[α 4][α 5][α 6][α 8][α 9][α 10][α 22][α 23][α 38][α 39][α 58][α 63][α 71][α 29]
16	Green project management	11	[α 23][α 34][α 35][α 43][α 53][α 56][α 57][α 73][α 64][α 71][α 33]
17	Provision of green infrastructure	10	[α 1][α 22][α 25][α 37][α 48][α 59][α 62][α 63][α 65][α 29]
18	Green application development environment	09	[α 6][α 19][α 21][α 25][α 36][α 39][α 61][α 68][α 32]
19	Efficient estimation strategies	03	[α 38][α 45][α 13]
20	Green economy	01	[α 69]

Table 6. List of finally selected research papers in the SLR.

Final ID	Paper Title	Tracing Number	Year	Database
α 1	Green software development model: an approach towards sustainable software development	Snowballing 1	2011	IEEE
α 2	Exploration of green computing	Snowballing 2	2013	CiteSeer
α 3	Green software engineering process: moving towards sustainable software product design.	Snowballing 3	2013	Google scholar
α 4	Towards software sustainability guidelines for Long-living Industrial Systems	Snowballing 4	2011	CiteSeer
α 5	Green RM: reference model for sustainable software development	Snowballing 5	2013	Google scholar
α 6	A green model for sustainable software engineering	Snowballing 6	2013	Google

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Tracing Number	Year	Database
				Scholar
α 7	Green software: greening what and how much?	Snowballing 7	2014	IEEE
α 8	Evaluation of software product quality attributes and environmental attributes using ANP decision framework	Snowballing 8	2014	Google scholar
α 9	Deploying and provisioning green software	Snowballing 9	2014	IEEE
α 10	Green software development and design for environmental sustainability	Snowballing 10	2013	Google scholar
α 11	Exploring initial challenges for green software engineering: summary of the first GREENS workshop.	Snowballing 11	2013	ACM
α 12	A review on green software development in a cloud environment regarding software development life cycle: (SDLC) perspective.	Snowballing 12	2013	Google scholar
α 13	Toward sustainable software engineering (nier track)	Snowballing 13	2011	Google Scholar
α 14	Towards a software product sustainability model	Snowballing 14	2013	Google scholar
α 15	A systematic literature review on green software metrics	Snowballing 15	2013	Google Scholar
α 16	Developing sustainable software solutions for Bioinformatics by the “Butterfly ” paradigm	Snowballing 16	2014	Google Scholar
α 17	Green web engineering-measurements and findings	Snowballing 17	2012	Google Scholar
α 18	Analysis and principles of green UI design for web portals	Snowballing 18	2014	Google scholar
α 19	Approach to improve energy efficiency of information systems	Snowballing 19	2014	Google Scholar
α 20	Enhancing software engineering processes towards sustainable software product design	Snowballing 20	2010	Google scholar
α 21	Special factors of development of green software Supporting eco sustainability	Snowballing 21	2013	Google scholar
α 22	Sustainable development, sustainable software, and sustainable software engineering	Snowballing 22	2011	IEEE
α 23	Measuring the sustainability performance of software projects	Snowballing 23	2010	IEEE
α 24	A systematic literature review on sustainability studies in software engineering	Snowballing 24	2014	Google scholar
α 25	Optimization of operating systems towards green computing	Snowballing 25	2011	Google scholar
α 26	Principles and holistic design of green web portal	Snowballing 26	2013	Google scholar
α 27	Green agile maturity model for global software development vendors	Snowballing 27	2014	Google scholar
α 28	Green and sustainable technologies in Software Engineering	Snowballing 28	2015	Google

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Tracing Number	Year	Database
				scholar
α 29	Greening an existing software system using the GPU	Snowballing 29	2013	Google scholar
α 30	Green computing and software defects in open source software: an empirical study.	Snowballing 30	2014	Google scholar
α 31	Understanding green software development: a conceptual framework	Snowballing 31	2015	IEEE
α 32	Lifecycle energy assessment of mobile applications.	Snowballing 32	2013	Google scholar
α 33	Sustainability issues in human computer interaction design	Snowballing 33	2014	Google scholar
α 34	Balancing software product investments	ACM1_page1	2009	ACM
α 35	Integrating sustainability into undergraduate computing education	ACM4_page2	2010	ACM
α 36	The software perspective for energy-efficient mobile applications development	ACM9_page4	2012	ACM
α 37	Fostering green it	ACM10_page5	2013	ACM
α 38	Sustainability evaluation of software architectures: a systematic review	ACM11_page6	2011	ACM
α 39	Want to green application software? – mind the target hardware	ACM17_page10	2013	ACM
α 40	An agenda for 'Green' information technology and Systems research	ACM21_page14	2011	ACM
α 41	Green software services: from requirements to business models	ACM25_page22	2013	ACM
α 42	Sustainable software development	ACM28_page25	2004	ACM
α 43	Green software engineering with agile methods	ACM29_page28	2013	ACM
α 44	The evolution of green ICT practice: UK higher education institutions case study	ACM31_page35	2011	ACM
α 45	Green IT Maturity: developing a framework based on practices and actions	GS8_page1	2013	Google scholar
α 46	Green wall: a methodology for sustainable development using green computing	GS10_page1	2014	Google scholar
α 47	Green IT adoption and sustainable value creation	GS14_page2	2014	Google scholar
α 48	Green computing technologies towards the development of ICT: a critical study	GS15_page2	2013	Google scholar
α 49	Green computing: practice of efficient and eco-friendly computing resources	GS17_page2	2009	Google scholar
α 50	Impact of green computing in it industry to make ecofriendly	GS19_page3	2014	Google

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Tracing Number	Year	Database
	environment			scholar
α 51	A research agenda on managerial intention to green it adoption: from norm activation perspective	GS20_page3	2014	Google scholar
α 52	Green Computing: an essential trend for secure future	GS22_page3	2013	Google scholar
α 53	Summary of the First Workshop on Sustainable Software for Science: Practice and Experiences	GS23_page3	2014	Google scholar
α 54	Green software and green software engineering-definitions, measurements, and quality aspects	GS27_page5	2013	Google scholar
α 55	Approaches to green computing to reduce global warming	GS28_page5	2013	Google scholar
α 56	A green IS taxonomy	GS30_page8	2014	Google scholar
α 57	A maturity model for green ICT: The case of the SURF green ICT maturity model,	GS31_page20	2014	Google scholar
α 58	The GREENSOFT model: a reference model for green and sustainable software and its engineering	SD1_page1	2011	Science Direct
α 59	A categorization of green practices used by Dutch data centers	SD3_page1	2013	Science Direct
α 60	Exploring the role of IT for environmental sustainability in China: An empirical analysis	SD7_page1	2013	Science Direct
α 61	Is software ‘‘green’’? Application development environments and energy efficiency in open source applications	SD8_page1	2012	Science Direct
α 62	Risk identification in green IT practice	SD9_page1	2013	Science Direct
α 63	Energy efficiency and low carbon enabler green IT framework for data centers considering green metrics	SD10_page1	2012	Science Direct
α 64	Designing IT systems according to environmental settings: A strategic analysis framework	SD12_page1	2011	Science Direct
α 65	Promoting green ICT in China: A framework based on innovation system approaches	SD17_page1	2012	Science Direct
α 66	Green WSUS	SD19_page2	2012	Science Direct
α 67	Green computing practices as a part of the way to the sustainable development.	SL1_page1	2013	Springer Link
α 68	Software level green computing for large scale systems	SL7_page1	2012	Springer Link
α 69	Green challenges to system software in data centers	SL17_page5	2011	Springer Link
α 70	Green software and green IT: an end users perspective	SL2_page1	2011	Springer Link
α 71	A Model and selected instances of green and sustainable	SL3_Page1	2010	Springer

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Tracing Number	Year	Database
	software			Link
α 72	Unpacking green IS: a review of the existing literature and directions for the future	SL10_page1	2012	Springer Link
α 73	Estimating the energy consumption of executing software processes	IEEE1_page1	2013	IEEE
α 74	Green IT-outsourcing assurance model	IEEE2_page1	2013	IEEE

approached for assistance in case of an issue about the data mining.

We have successfully performed inter-rater reliability test after data extraction. We have used analogous methodology in former research [47]. We have extracted data from finally selected 74 publications using data extraction form as shown in Table 4. In order to properly synthesize the extracted data, we will use data synthesis process by presenting the required data (success factors, risk factors and practices) in tabular format according to the formulated research questions i.e. RQ1, RQ2, RQ3.

5. PRELIMINARY RESULTS OF THE PROTOCOL

The protocol preliminary results are presented in Table 3. The protocol has retrieved 7,482 research papers/articles. We have selected 111 research papers as a primary/initial selection by reading title of the paper, and examining abstract. After removing duplicate papers we got 91 papers. By applying our pre-defined quality criteria, our publication sample size was squeezed to 74 research papers (Table 3). These include 41 papers from SLR protocol search strategy and 33 papers through Snowballing. Snowballing approach is normally used to complement the SLR search results by searching through the reference/bibliography of a specific research paper or the citations to the paper in order to identify additional/new research papers. We have used the guidelines [48] for conducting snowballing methodology. We have preliminary extracted a list of 20 factors; these factors are shown in Table 5, where Table 6 presents the details of finally selected papers. The identified factors support software development organizations to develop green and sustainable software. We have studied a number of studies for the design and development of the presented SLR protocol [49-56].

6. CONCLUSIONS

Software development multi-sourcing is an emerging global software engineering (GSE) paradigm for producing high quality software at minimum cost and time, by hiring more than one software vendors. Consequently, green software multi-sourcing has raised a novel research area in the context of GSE domain which aims to produce greener software in multi-sourced software development projects. Research in this area will assist multi-sourcing industry to deliver green software in general and to gauge multisourcing vendors' readiness for the development of greener software in particular. In this paper we have presented our SLR protocol to identify the success factors, risk factors and green practices from the available literature on green software development. During implementation of the protocol, while using our search string defined in Track 2 in various search libraries, we have identified a publication sample of (N=7482) research papers out of them (N=91) have been selected as the primary sample by reviewing through title and abstract of the papers. After reviewing through full text of the primary sample of publications, finally we selected 74 papers as our final sample. We have shown these search results and publications selection in Table 3.

We have completed the data extraction phase of the protocol, by extracting data from the finally selected papers on our pre-defined data extraction form as shown in the Table 4.

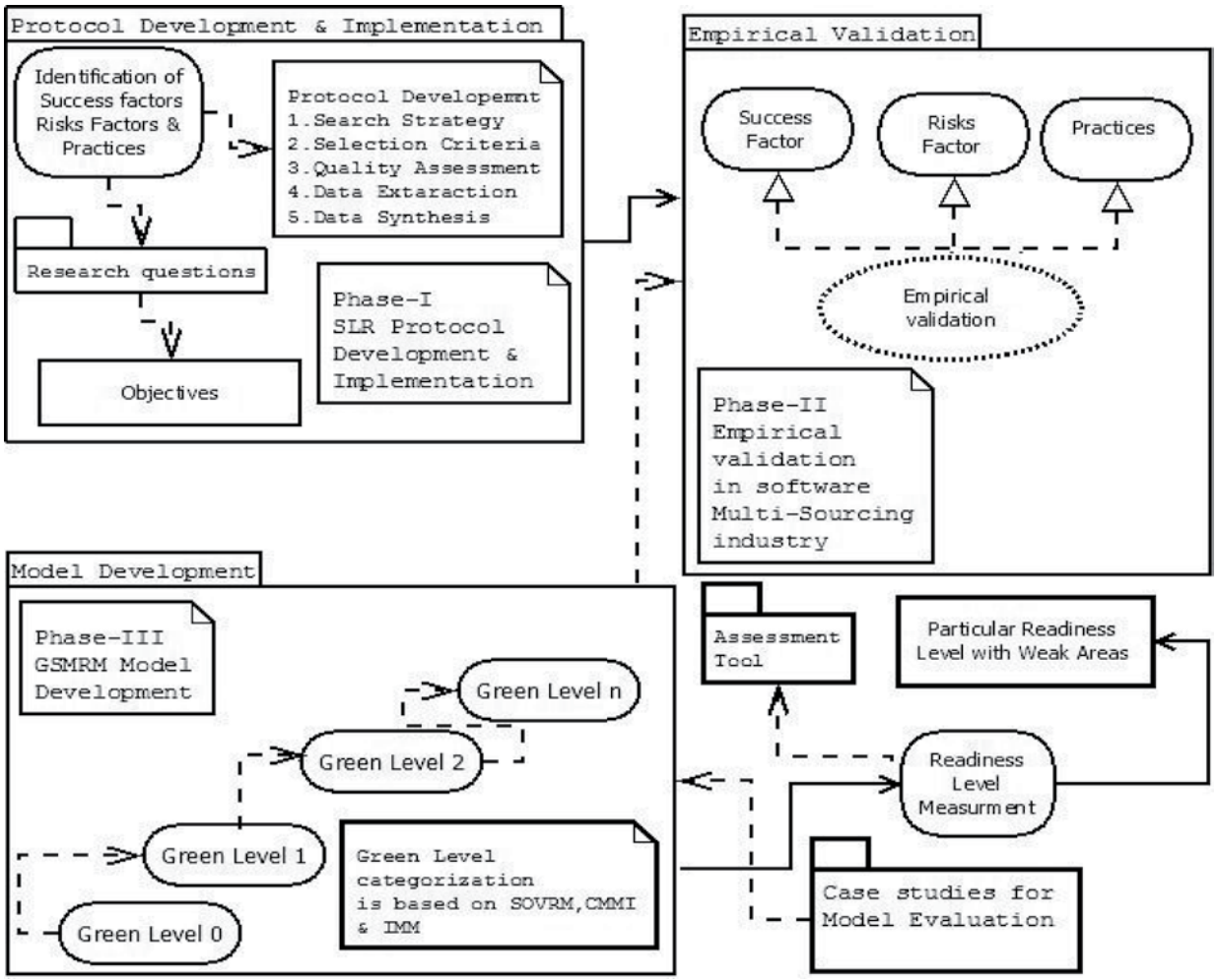

Fig. 1. Phases in our Proposed GSMRM model.

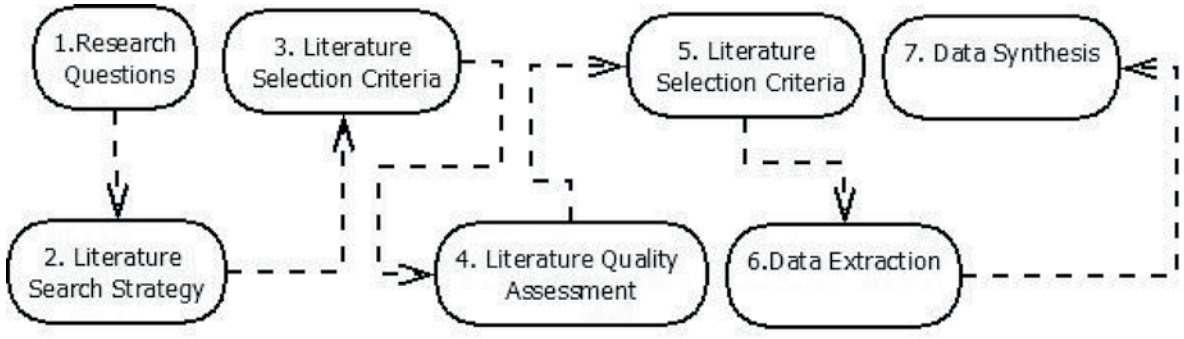

Fig. 2. Components of SLR protocol.

Currently we are in the process of SLR data synthesis in order to synthesize the extracted data. We have preliminary extracted a list of 20 factors (Table 5); Table 6 presents the details of finally selected papers. The identified factors support software development organizations to develop green and sustainable software. Further our ultimate goal is the development of Green Software Multi-Sourcing Readiness Model (GSMRM) as shown in Fig.1.

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8. REFERENCES

1. Cai, S., X. Chen & I. Bose. Exploring the role of IT for environmental sustainability in China: An empirical analysis. *International Journal of Production Economics* 146: 491-500 (2013).
2. Harmon R.R., & N. Auseklis. Sustainable IT services: Assessing the impact of green computing practices. In: *International Conference on Management of Engineering & Technology*, PICMET, Portland, p. 1707 - 1717 (2009).
3. Gu, Q., P. Lago., H. Muccin., & S. Potenza. A categorization of green practices used by Dutch data centers. *Procedia Computer Science* 19:770-776 (2013).
4. Harmon, R., H. Demirkan., N. Auseklis & M. Reinoso. From green computing to sustainable IT: Developing a sustainable service orientation. In: *System Sciences (HICSS), 43rd Hawaii International Conference*, p. 1-10 (2010).
5. Davidson, E., E. Vaast & P. Wang. The greening of IT: how discourse informs IT sustainability innovation. In; *IEEE 13th Conference on Commerce and Enterprise Computing (CEC*), p. 421-427 (2011).
6. Harris, J. *Green Computing and It Best Practices on Regulations and Industry Initiatives, Virtualization, Power Management, Materials Recycling and Telecommuting*. Emereo Pty Ltd, London, UK (2008).
7. Harbla, A., P. Dimri., D. Negi & Y. S. Chauhan. Green computing research challenges: a review. *International Journal of Advanced Research in Computer Science and Software Engineering* 3(10): 1-13 (2013).
8. Moshnyaga, V.G. An assessment of software lifecycle energy. In: *Power and Timing Modeling, Optimization and Simulation (PATMOS), 23rd International Workshop*, Karlsruhe, p. 112-119 (2013).
9. Sierszecki, S., K. Sierszecki., T. Mikkonen., M. Steffens., T. Fogdal., & J. Savolainen. Green software: greening what and how much?. *Software, IEEE* 31: 64-68 (2014).
10. Mahmoud, S.S. & I. Ahmad. A green model for sustainable software engineering. *International Journal of Software Engineering and Its Applications* 7(4): 55-74 (2013).
11. Erdelyi, K. Special factors of development of green software supporting eco sustainability.In: *Intelligent Systems and Informatics (SISY), IEEE 11th International Symposium,* Subotica, p. 337-340 (2013).
12. Appasami, G. & S. Joseph. Optimization of operating systems towards green computing. *International Journal of Combinatorial Optimization Problems and Informatics* 2(3):39-51 (2011).
13. Naumann, S., M. Dick., E. Kern & T. Johann. The GREENSOFT Model: a reference model for green and sustainable software and its engineering. *Sustainable Computing: Informatics and Systems* 1(4): 294-304 (2011).
14. Shenoy, S. S., & R. Eeratta. Green software development model: an approach towards sustainable software development. *India Conference (INDICON), India Annual IEEE,* p. 1-6 (2011).
15. Taina, J. Good, bad, and beautiful software-In search of green software quality factors. *CEPIS UPGRADE* 12(4): 22-27 (2011).
16. Mahaux, M. & C. Canon. Integrating the complexity of sustainability in requirements engineering. In: *First International Workshop on Requirements for Sustainable Systems.* Belgium, p. 31-33 (2012).
17. Mahaux, M., P. Heymans & G. Saval. Discovering sustainability requirements: an experience report. In: *Requirements Engineering: Foundation for Software Quality SE-3, Lecture Notes in Computer Science*, 6606 ed. Springer Berlin Heidelberg, p.19-33 (2011).
18. Winschiers, H. & B. Paterson. Sustainable software development. In: 2*004 Annual Research Conference of the South African Institute Of Computer Scientists and Information Technologists on IT Research in Developing Countries*. South Africa, p. 274-278 (2004).
19. Koçak, S.A., G.I. Alptek & A.B. Bener. Evaluation of software product quality attributes and environmental attributes using ANP decision framework. In: *Third International Workshop on Requirement Engineering for Sustainable Systems,* p. 37-44 (2014).
20. Bozzelli, P., Q. Gu & P. Lago. *A Systematic Literature Review on Green Software Metrics*. VU University, Amsterdam (2013).
21. Dick, M., J. Drangmeister, E. Kern & S. Naumann. Green software engineering with agile methods. In: *Green and Sustainable Software (GREENS), 2nd International* Workshop, Netherlands, p. 78-85 (2013).
22. Durdik, Z., B. Klatt., H. Koziolek., K. Krogmann., J. Stammel & R. Weiss. Sustainability guidelines

for long-living software systems. In: *28th IEEE International Conference Software Maintenance (ICSM)*, Trento, p. 517-526 (2012).
23. Sabharwal, M.R. Software power optimization analysis and optimization for energy-efficient software. In: *Proceedings of the 17th IEEE/ACM international symposium on Low-power electronics and design*, Fukuoka, p. 63-64 (2011).
24. Khan, R.U. & S. U. Khan. Green IT-outsourcing assurance model. In: *Global Software Engineering Workshops (ICGSEW), IEEE 8th International Conference*, Bari, p. 84-87 (2013).
25. Chauhan, N.S. & A. Saxena. A green software development life cycle for cloud computing. *IT Professional* 15(1): 28-34 (2013).
26. Johann, T., M. Dick., E. Kern & S. Naumann. Sustainable development, sustainable software, and sustainable software engineering: an integrated approach. In: *Humanities, Science & Engineering Research (SHUSER), International Symposium*, Kuala Lumpur, p. 34-39 (2011).
27. Dustdar, S., F. Li., H. Truong, S. Sehic, S. Nastic, S. Qanbar, M. Vögler & M. Claeßens. Green software services: from requirements to business models. In: *Green and Sustainable Software (GREENS), 2nd International Workshop* USA, p. 1-7 (2013).
28. Liangli, M., C. Yanshen., S. Yufei & W. Qingyi. Virtualization maturity reference model for green Software. In: *International Conference on Control Engineering and Communication Technology (ICCECT),* Liaoning, p. 573-576(2012).
29. Sahin, C., F. Cayci., J. Clause., F. Kiamilev., L. Pollock & K. Winbladh. Towards power reduction through improved software design. In: *Energytech IEEE*, Cleveland, p. 1-6 (2012).
30. Noureddine, A., A. bourdon., R. Rouvoy., & L. Seinturier. Runtime monitoring of software energy hotspots. In: *27th IEEE/ACM International Conference on Automated Software Engineering. Essen,* Germany, Essen, p. 160-169 (2012)
31. Khan, M.A., C. Hankendi., A. K. Coskun & M. C. Herbordt. Software optimization for performance, energy, and thermal distribution: Initial case studies. In: *Green Computing Conference and Workshops (IGCC),* Orlando, p. 1-6 (2011).
32. Kehal, H. Outsourcing and offshoring. In: *The 21st Century: A Socio-Economic Perspective*. Lgi Global (2006).
33. Salam, M. & S.U. Khan. Green software multi-sourcing readiness model (GSMRM) from vendor's perspective. *Science international (Lahore)* 26:1421-1424 (2014).
34. Turner, M., & S. *Charters. Protocol for a Systematic Literature Review of the Technology Acceptance Model and Its Predictive Capabilities.* Keele University, UK (2006).
35. Sarah, B., N. Baddoo., T. Hall., H. Robinson & H. Sharp. *Protocol for a Systematic Literature Review of Motivation in Software Engineering.* University of Hertfordshire UK and the Open University, UK, (2006).
36. Khan, S.U., M. Niazi & N. Ikram. *Systematic Literature Review Protocol for Software Outsourcing Relationships Trust (SORT)*. School of Computing & Maths, Keele University, UK (2009).
37. Kitchenham, B. *Procedures for Performing Systematic Reviews*. Keele University, UK (2004).
38. Zhang, H., M.A. Babar & P. Tell. Identifying relevant studies in software engineering. *Information and Software Technology* 53(6): 625-637 (2011).
39. Kitchenham, B. & S. Charters. *Guidelines for performing Systematic Literature Reviews in Software Engineering.* Keele University and Durham University Joint Report EBSE 2007-001 (2007).
40. Ilyas, M. & S.U. Khan. Software integration model for global software development. In: *5th International Multitopic Conference (INMIC),* Islamabad, p. 452-457 (2012).
41. Khan, S.U. & M.K. Niazi. *Systematic Literature Review Protocol for Software Outsourcing Vendors Readiness Model (SOVRM).* School of Computing and Maths, Keele University TR/08-0,1 UK (2008).
42. Ahmad, A. & S.J. Kolla. *Effective Distribution of Roles and Responsibilities in Global Software Development Teams*. School of Computing Blekinge Institute of Technology SE-371 79 Karlskrona Sweden, MSE-20XX-XX (2011).
43. Arslan, M, & M.A. Riaz. *A Roadmap for Usability and User Experience Measurement during early phases of Web Applications Development.* School of Computing, Blekinge Institute of Technology Box 520 SE - 372 25 Ronneby Sweden (2010).
44. Korkala, M. & F. Maurer. Waste identification as the means for improving communication in globally distributed agile software development. *Journal of Systems and Software* 95: 122-140 (2014).
45. Sriram, R. & S.K. Mathew. Global software development using agile methodologies: a review of literature. In: *Innovation and Technology (ICMIT), IEEE International Conference,* Sanur Bali, Indonesia, p. 389-393 (2012).
46. Ali, I. & S.U. Khan. Usability of agile methods in software development outsourcing: a systematic literature review protocol. *Science International (Lahore)* 26(5): 2023-2027 (2014).
47. Khan, S.U. & M.K. Niazi., & N. Ikram. Software development outsourcing relationships trust: a systematic literature review protocol. *Evaluation*

and Assessment in Software Engineering, EASE 2010, Keele University, UK 1-5 (2010).

48. Wohlin, C. Guidelines for snowballing in systematic literature studies and a replication in software engineering. In: *Proceedings of the 18th International Conference on Evaluation and Assessment in Software Engineering. London, England*, p. 1-10 (2014).
49. Niazi, M., S. Mahmood, M. Alshayeb & A. Hroub. Empirical investigation of the challenges of the existing tools used in global software development projects. *IET Software* 9(5): 135-143 (2015).
50. Verner, J.M., O.P. Brereton, B.A. Kitchenham., M. Turner & M. Niazi. Risks and risk mitigation in global software development: A tertiary study. *Information and Software Technology* 56(1): 54-78 (2014).
51. Verner, J.M., O.P. Brereton, B.A. Kitchenham, M. Turner & M. Niazi. Systematic literature reviews in global software development: a tertiary study. In: *16th International Conference on Evaluation & Assessment in Software Engineering (EASE)* Spain, p. 2-11 (2012).
52. Niazi, M. Software Process Improvement: A road to success. In: *Product-Focused Software Process Improvement: Springer,* p. 395-401 (2006).
53. Niazi, M. An instrument for measuring the maturity of requirements engineering process. *The 6th International Conference on Product Focused Software P*rocess Improv*ement, LNCS, Oulu, Finland*, p. 13-16 (2005).
54. Khan, S.U., & M. Niazi & R. Ahmad. Empirical investigation of success factors for offshore software development outsourcing vendors. *Journal of IET Software* 6:1-15 (2012).
55. Niazi,M. & D. Wilson., D. Zowghi & B. Wong. A model for the implementation of software process improvement: An empirical study. In: *Product Focused Software Process Improvement:* Springer, p. 1-16 (2004).
56. Niazi, M., D. Wilson & D. Zowghi. A model for the implementation of software process improvement: a pilot study. In: *Third International Conference on Software Quality (QSIC03), IEEE Computer Society,* Dallas, USA, p. 196-203 (2003).

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Research Article

Green Agility for Global Software Development Vendors: A Systematic Literature Review Protocol

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Abstract: Global software development (GSD) is now-a-days pervasive in software industry aiming to develop global standard software through geographically distributed skilled teams in minimum time and cost. In order to meet the demand for green software production and frequent changes in requirements of the clients, GSD developers have revamped traditional methods and trying to incorporate green principles with agile methods for rapid and energy efficient software development. This paper presents our contribution to building a systematic literature review (SLR) protocol for green agile maturity (GAM) for GSD vendor organizations. The protocol aims to systematically review the available literature for the identification of success/risk factors that may have a direct or indirect effect on green and sustainable software development using agile methods. The desired outcome of SLR protocol will be a group of success/risk factors and their concerned practices that will be helpful for vendors to produce green and environmentally sustainable software by incorporating agile principles in global software development.

Keywords: Green software, sustainable software, green agile, global software development, SLR Protocol.

1. INTRODUCTION

Global software development (GSD) is growing rapidly due to increase in globalization of software business industry [1]. In GSD, software engineers and developers from various countries with different cultures and time zones participate in the development process. Distributed experts at diverse locations coordinate through the latest knowledge sharing and communication tools [2]. GSD offers tremendous benefits that include access to skilled pool of software developers, production of high standard software, business advantage of proximity to markets, quick access to software development updates and the possibility to use "follow-the-sun" and "round–the-clock" development. Hence software development is now considered as a globally distributed endeavour [3-4].

However, GSD unlocks new doors for software business yet it also yields a number of challenges that comprises hidden agreement costs, dearth of client involvement, splitting and allocation of work at different sites, lack of trust among the outsourcing companies and scarcity of software development outsourcing practices [5-6].

Agile software development is invigorating approach towards quick and interactive software development. It provides a conceptual structure for undertaking any software project that is co-located or globally distributed. Unlike traditional methods of software development, agile methods attempt to reduce risks and maximize software productivity by developing software in short iterations [7]. Agile approaches rely on individual developers' skills rather than formalized processes and cumbersome amount of documentation [8]. Thus, agile methods pursue to avoid suggesting overwhelming processes, having little contribution to software product [9].

Using agile methods in distributed software development offers several benefits like constant communications and scheduled delivery of software, continuous integration of software code, improved project's quality and efficiency, nominal

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documentation and early expert customer feedback [10-11]. Green or sustainable software is the design and production of software, having direct or indirect negative effect on country's economy, people, society and environment that result from software pre-development, development and post-development phases are negligible and/or which have a positive impact on sustainable software production [12]. Green software engineering is an emerging paradigm and is growing rapidly that aims to develop software with green features to reduce negative impact on environment.

This research work presents a systematic literature review protocol for the development of green agile maturity model for GSD vendors as well as the preliminary results as shown in Table 5. The findings will contribute to the development of first phase of our proposed model that is aimed to assist GSD vendors to measure their green-agile maturity for the design and production of green and sustainable software [13]. The detailed structure of the proposed model is shown in Fig. 1.

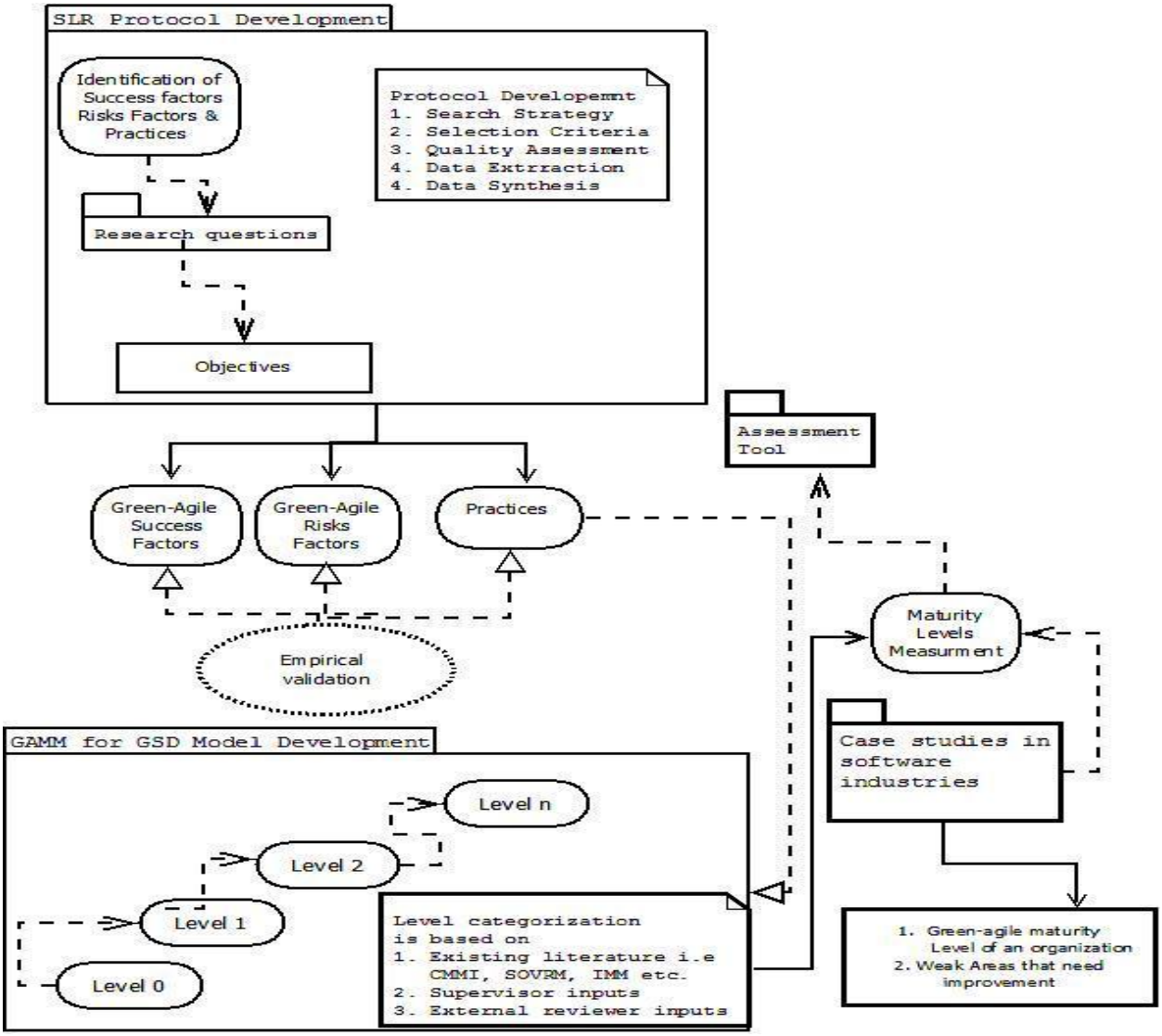

Fig. 1. Overview of proposed model (GAMM for GSD vendors).

The following research questions stimulated the research work presented in this paper:

RQ 1. What are the success factors, as mentioned in the relevant literature, for adapting agile techniques that can assist software engineers in GSD organizations for the design and production of green and sustainable software?

RQ 2. What are the risks involved, as mentioned in the relevant literature, to be avoided by software engineers in GSD organizations for the design and production of green and sustainable software using agile techniques?

RQ 3: What are the relevant practices in GSD organizations, as mentioned in the literature, to be applied by software engineers that can be valuable in the design and production of green and sustainable software using agile techniques?

Table 1. Track 1 search results.

Source	Total publications found
Google Scholar	7
Springerlink	26
ACM	4
IEEEXplore	0
Science Direct	39

Table 2. Track 2 search results.

Source	Total publications found
Google Scholar	350
Springerlink	887
ACM	42
IEEEXplore	70
Science Direct	949

2. BACKGROUND

Research in the area of green-agile is growing. A number of researchers have worked on agile methods with green aspects to promote quick and interactive development of green software. Several researchers have proposed enhanced model of agile methods specifically SCRUM and extreme programming (XP) that integrates green aspects of software engineering for co-located software development teams [14-16].

Agile methods are emerging techniques in software engineering that have reshaped software development life cycle and assure the delivery of sustainable software through iterative and quick development. Agile processes are based on a set of some major principles such as strong team work, close association between practitioners and business organizations, face-to-face meetings with customers, early and frequent delivery of workable product and accepting flexibility towards dynamic requirements from customers [17-20].

Sara et al [21] have proposed a two-level green software model that comprises sustainable software life cycle as well as software tools that claim to produce greener and environment friendly software using agile approaches. The model integrates some of the agile principles that lead to green software development. It consists of different stages that are based on hybrid processes of sequential and agile methods for production of sustainable software. Green guidelines and green processes have also been proposed for each stage of software development. Among the green agile principles flexibility in changing requirements, interactive software development that involves customer, incremental and iterative software development and early testing techniques for defects prevention have been mentioned that aid in promoting green and environmental sustainable software product.

Tate [22] coined the importance of green and sustainable software development through iterative agile development. He suggests that culture of environment friendly software development should incorporate the principles of agile approaches in software development life cycle. According to his work presented, continuous refinement of software development by accepting changes in the requirements any time, scheduled delivery of working code, emphasis on simple design with simple contents, early defect prevention through regular testing are the core principles of agile techniques that can surely add in the eco-friendly software development.

Koontz et a. [23] have worked on re-architecting of software products and have identified the integral principles of agile methods that proved to be helpful in sustainable software development. Out of the identified principles, incremental and quick development reduced working cycle times, iterative development, mini builds with simple designs and continuous

integrations are the ones that can help to produce greener and sustainable software products.

The literature described above listed out some intrinsic principles of agile methods and their importance for the development of green and sustainable software product. However, none of the identified factors and principles has been dug out through systematic literature review. The mentioned work lacks to explore the applicability of agile principles for greener and sustainable software development in global software projects. Our proposed work, systematic literature review protocol for the identification of green agile factors for GSD vendors thus has a significant value for contribution to software engineering domain and is evident that no such effort has been spent before regarding such work.

3. RESEARCH METHODOLOGY

To achieve the ultimate objectives, the research work has been planned in three inter-dependent phases. In first portion of our research work, the success factors, risks and relevant practices, regarding the production of green software through the use of agile approaches will be investigated by means of SLR. SLR is an efficient mechanism for identifying, evaluating and interpreting all currently available research, relevant to particular research questions or area of interest [24]. SLR is now an interesting research methodology used by empiricists for conducting empirical research [25-26]. Though, it is a time consuming method and a bit hard to conduct over other review techniques but the results retrieved are unbiased as it follows a pre-defined and validated protocol [27].

Empirical study has been planned in the second phase of the research to validate the results of systematic literature review, as shown in Table 5, and to explore some new factors, if any, in GSD organizations. Same methodology has been followed by other software engineers and researchers [28-29].

In third phase of our protocol, proposed model (GAMM for GSD vendors) will be developed on the basis of the SLR outcomes and empirical study to be conducted in GSD organizations. We intend to validate the proposed model through five case studies in GSD organizations as well. Same approach has been adopted by other researchers [30].

Table 3. Data extraction form.

S. No.	Data to be extracted
1	Form No.
2	Paper review date
3	Paper tracing No.
4	Title of the paper
5	Author name(s)
6	Reference
7	Paper venue
8	Quality of publication (A/B/C/Other)
9	Country
10	Year (Publication year)
11	Strategy of study (Ordinary review, SLR, empirical study etc.)
12	Agile methodology discussed (XP, Scrum, FDD, Crystal etc)
13	Population (Sample and Target).
14	Company size (Small, Medium, Large)
15	Company type (Software industry, Academic/Research Institute etc)
16	Company scope (Local, Global)
17	Company SPI status (CMMI, ISO etc)
18	Success factors/motivators in the adoption of agile methods for the development of green software specific to GSD or in general context.
19	Challenges/risks in adoption of agile methods for the development of green software specific to GSD or in general context.
20	Practices for green agility specific to GSD or in general context.

4. SYSTEMATIC LITERATURE REVIEW PROTOCOL

Performing a systematic review involves several discrete activities that can be clustered into three major phases: planning, conducting and reporting the review. Systematic reviews deal with the problem of accumulating empirical evidences, obtained using various techniques and in broadly different contexts, used mainly in software engineering domain [31-33].

This section covers the first phase of a systematic review process (review planning).

Table 4. SLR protocol preliminary results.

S. No.	Digital library	Search string	Date constraint	Total publications found	Primary selection	Final selection
1	Science Direct	Track 2	All Years	949	28	09
2	ACM	Track 2	All Years	42	12	04
3	IEEE Xplore	Track 2	All Years	70	14	05
4	Springer Link	Track 2	All Years	887	21	16
5	Google Scholar	Track 2	All Years	350	50	15
			Total	**2298**	**125**	**49**

Publications found through snowballing: 31
Total publications found through SLR: 49
Total publications selected (final): *N*=80

Table 5. Preliminary results of the systematic literature review protocol.

S. No	Success factor	Frequency	Percentage N=80
1	Efficient utilization of time and computing resources	21	26
2	Minimal documentation	25	31
3	Minimal rework/reengineering	18	23
4	Reduced cost	17	21
5	Improved quality	25	31
6	Improved management of product life cycle	09	11
7	E-waste minimization	09	11
8	Simple design	38	48
9	Refactoring	15	19
10	Continuous integration	14	18
11	Standard coding	27	34
12	Early defect prevention	14	18
13	Optimization of processes	05	6
14	Changes requirements anytime/late in development	01	1
15	Agile planning and agile requirements	04	5
16	Fast delivery	23	29
17	Iterative development	31	39
18	Early development	08	10
19	Early testing/continuous validation	16	20
20	Review and preview	08	10
21	Development of small size software components	02	3
22	Efficient collaborations between developers and customers	21	26
23	Energy efficient software development	01	1
24	Efficient coordination among the agile team members.	19	24

*N = Total number of finally selected publications

Identifying Search Terms

Search terms are designed according to the following rules.

a. Find major terms in research questions such as population, intervention and expected outcomes.
b. Find out synonyms for the identified major terms.
c. Verify the major terms along with synonyms in relevant literature.
d. Concatenate the major terms using 'AND' and 'OR' operators to get the final search strings.

Results for (a)

RQ 1 and RQ 2:

Green software, agile methods and global software development, success factors, risks.

Results for (b)

RQ 1 and RQ 2:

Green software: ("green software" OR "greener software" OR "sustainable software" OR "green computing" OR "green IT" OR "green software engineering")

Agile methods: (agile OR "agile methods" OR "green agile" OR "extreme programming" OR scrum)

Global software development: ("global software development" OR "distributed software development")

Results for (c)

RQ 1 and RQ 2:

green software, greener software, sustainable software, green computing, green IT, green software engineering, agile, agile methods, green agile, extreme programming, scrum, global software development, distributed software development.

Results for (d)

RQ 1 and RQ 2:

Track 1: ("green software" OR "greener software" OR "sustainable software" OR "green computing" OR "green IT" OR "green software engineering") AND (agile OR "agile methods" OR "green agile" OR "extreme programming" OR scrum) AND ("global software development" OR "distributed software development")

Track 2: ("green software" OR "greener software" OR "sustainable software" OR "green computing" OR "green IT" OR "green software engineering") AND (agile OR "agile methods" OR "green agile" OR "extreme programming" OR scrum)

Here Track1 signifies the search string being designed to explicitly retrieve available literature relevant to green agility in GSD, where Track 2 will retrieve relevant literature specific to green software development using agile methods. The results retrieved through Track 1 are almost insignificant as shown in Table 1. The tracks described earlier were discussed with experts of software engineering research group at university of Malakand and it was decided to follow Track 2 for the development of protocol, as shown in Table 2.

The identified factors as shown in Table 5 will be validated in GSD organizations through empirical studies. This will provide guidance to GSD vendor organizations to know better how these factors are applicable and adopted by software practitioners in global software development for the development of green and sustainable software using agile methods. Same method has been followed by other researchers in software engineering community [34-38].

5. PUBLICATION SELECTION

5.1. Inclusion Criteria

The following criteria are used to determine which piece of literature found by the search strings will be considered for the data extraction:

a. Studies that describe green software development using agile methods specific to GSD or in general context
b. Studies that describe applicability of agile methods in GSD that can assist towards green and sustainable software development.
c. Studies that describe green software engineering principles that are supported

by agile methods (such as XP, Scrum etc.) specific to GSD or in general context.

d. Studies that describe the agile maturity of GSD vendors that can assist software engineers for the design and development of eco-friendly software.

e. Studies that describe success or risk factors or relevant practices that can add value in the development of green software specific to GSD or in general context.

f. Studies written in English language will be considered only.

5.2 Exclusion Criteria

The following exclusion criteria describe which piece of literature found by the search strings will be excluded:

a. Studies that is not relevant to the research questions.

b. Studies that do not describe green agility.

c. Studies that are not related to green software development using agile techniques.

5.3 Selecting Primary Sources

Primary selection of literature is performed on the basis of title and abstract only. The main purpose is to exclude the irrelevant results against the research questions and problem domain. Selected literature is then carefully reviewed according to the inclusion/exclusion criteria as mentioned above. In case of any confusion regarding inclusion or exclusion principles, secondary reviewer will be contacted for guidance and expert decision.

6. PUBLICATION QUALITY ASSESSMENT

After final selection of the papers, quality and standard of the publications is measured according to the following criteria.

a. Is it clear how green agility is measured in global software development?

b. Is it clear how agile methods are adapted for green software development in GSD or in general context?

c. Is it clear how the adoption of agile methodologies is difficult to be integrated with green software development in GSD?

d. Is it clear how the factors for enhancement and improving green software development with agile methods were identified in outsourcing projects?

Each of the above list items will be marked as 'YES', 'NO', 'Partially' or N.A'.

Publication's quality of selected papers is further analyzed on the basis of below mentioned criteria and only qualifying papers are selected as our final sample size. A similar method has been followed in our earlier research [39].

6.1. Criteria for Category-A Papers

In category-A, we included those papers only which satisfy the following criteria:
C=Case study
I=Interview
S=Survey

1. L=Literature review journal publications (impact factor)
2. Clear methodology
3. Must have sample size as follow;
   a. $C \geq 3$
   b. $I \geq 12$
   c. $S \geq 50$
   d. $L \geq 50$

6.2. Criteria for Category-B Papers

In this category we included only those papers which satisfy the following criteria:

1. Conference publications
2. Clear methodology
3. Must have sample size as follow:
   a. $C = 2$
   b. $5 \leq I \leq 11$
   c. $30 \leq S \leq 49$
   d. $30 \leq L \leq 49$

6.3. Criteria for Category-C Papers

In category-C we included only those papers which satisfy the following criteria:

1. Literature published in less reputed venues (Journal, Conference)
2. Clear methodology
3. Having sample size as follow;
   a. C = 1
   b. I ≤ 5
   c. 1 ≤ S ≤ 29
   d. 1 ≤ L ≤ 29

7. DATA EXTRACTION STRATEGY

In this phase the required data is extracted that answers the above mentioned research questions. Secondary reviewer is approached for necessary guidance and to resolve the ambiguities in data extraction. The secondary reviewer randomly selected research publications and compared the results produced by the primary reviewer in order to ensure the quality of publications. Extraction form as shown in Table 3 is used for data extraction process.

8. DATA SYNTHESIS

Data synthesis is the process of grouping the identified factors from finally selected publications. For research question1, the data will be synthesized in a table that will illustrate the identified factors with frequency. Complete details of each factor will be maintained separately having details (S. No, Factor group name, factor's subgroups and paper tracing number). The same process of data synthesis will be adapted for the Research Question 2.

9. RESULTS AND DISCUSSION

Table 4 depicts primary search results of the protocol. Total number of publications found through Track 2 is 2,298. Out of which 125 papers were initially selected by reading its title and abstract. After removal of the duplicate publications found, we got 118 papers. Finally a total of 80 research publications were selected, as shown in Table 6, out of which 49 have been selected by following SLR protocol and 32 by snowballing respectively. Complete list of the selected papers is shown in Table 6. Snowballing is a distinct search mechanism that uses author name, reference list of a selected paper or its citations to find more papers that may have missed by the SLR search string during its search phase. Guidelines mentioned in [40] have been used for conducting systematic reviews supported by snowballing approach. Furthermore, we have studied a number of papers [41-49] for the design and development of the presented SLR protocol.

Table 6. List of finally selected papers in the systematic literature review.

Final ID	Paper Title	Database
P1	Software evolution for industrial automation systems: Literature overview	Google scholar
P2	Collaboration in mature XP teams	Google scholar
P3	Sustainability guidelines for long-living software systems	Google scholar
P4	Appropriate information system development	Google scholar
P5	A systematic mapping study on sustainable software engineering: A research preview	Google scholar
P6	Toward an XP evaluation framework	Google scholar
P7	MI Copa: Micro credit operation automation	Google scholar
P8	Sustainable software development: An agile perspective	Google scholar
P9	Chaos issues on communication in agile global software development	Google scholar
P10	Green software engineering process : Moving towards sustainable software product design	Google scholar
P11	Enhancing software engineering processes towards sustainable software product design	Google scholar
P12	A green model for sustainable software engineering	Google scholar
P13	Bio mimicry as a super systems metaphor for software engineering?	Google scholar
P14	An IT perspective on integrated environmental modeling: The SIAT case	Google scholar
P15	Modeling to support communication and engineering of service-oriented software	Google scholar

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Database
P16	Smart green infrastructure for innovation and transformation hosting environments	IEEE
P17	Impact of web 2.0 and cloud computing platform on software engineering	IEEE
P18	GETA for information technology: Go green, eat green, think green, and act green information technology	Google scholar
P19	The agile manifesto	Google scholar
P20	Towards better understanding of agile values in global software development	Google scholar
P21	Review of agile methodologies in software development	Google scholar
P22	Analysis and design of a novel agile methodology	Google scholar
P23	Sustainability in software engineering	Google scholar
P24	Cloud software Finland	IEEE
P25	Processes and practices for quality scientific software projects	
P26	Adopting key lessons from agile manufacturing to agile software product development: A comparative study	Google scholar
P27	Success factors of agile software development	Science direct
P28	Review on traditional and agile cost estimation success factor in software development project	Google scholar
P29	Limitations of agile software processes	IEEE
P30	A survey study of critical success factors in agile software projects	Science direct
P31	Performance evaluation of software development models	Google scholar
P32	The agile software development series	IEEE
P33	Usage and perceptions of agile software development in an industrial context: An exploratory study	IEEE
P34	Agile software development practices: Evolution, principles and criticisms	Google scholar
P35	Agile software development: novel approaches for software engineering	IEEE
P36	What is agile software development?	Springer link
P37	Learning from agile software development	IEEE
P38	Agile methodologies and process discipline	Google scholar
P39	Odyssey and other code science success stories	Google scholar
P40	Agile process for integrated service delivery	IEEE
P41	Role of agile methodology in software development	Google scholar
P42	Human resource planning in agile projects	Google scholar
P43	Extreme programming – agile method used in project management	Google scholar
P44	Using factor analysis to generate clusters of agile practices	IEEE
P45	Scaled agile framework: A blight	Google scholar
P46	Software development methodologies for reducing project risks	Google scholar
P47	Green software engineering with agile methods	IEEE
P48	Common agile practices in software processes	IEEE
P49	Factors influencing the agile methods in practice literature survey & review	IEEE
P50	Introducing agile development practices from the middle	IEEE
P51	Green as the new lean: How to use lean practices as a catalyst to greening your supply chain	IEEE
P52	Supporting distributed extreme programming	Springer link
P53	Outsourcing and offshoring with agility: A case study	Springer link
P54	Agile processes in software engineering and extreme programming	Springer link
P55	Sustainable software: A study of software product sustainable development	Springer link
P56	Agile software construction	Springer link

Contd.....

Table 6 (Contd.)

Final ID	Paper Title	Database
P57	REM in agile projects	Springer link
P58	Assessing software agility: An exploratory case study	Springer link
P59	Comparative analysis of agile maturity model	Springer link
P60	A Failure to Learn in a Software Development Team: The unsuccessful introduction of an agile method	Springer link
P61	Derivation of green metrics for software	Springer link
P62	Aggregated survey of sustainable business models for agile mobile service delivery platforms	Springer link
P63	Agility meets system engineering: A catalog of success factors from industry practice	Springer link
P64	Factors affecting effectiveness of agile usage: Insights from the BBC worldwide case study	Springer link
P65	Obstacles to agile software development	Springer link
P66	Creating environmental awareness in service oriented software engineering	Springer link
P67	Introduction to mechanism design for sustainability	Springer link
P68	Empirical studies of agile software development: A systematic review	Science direct
P69	Progressive outcomes: A framework for maturing in agile software development	Science direct
P70	Risks in distributed agile development: A review	Science direct
P71	Agile software architecture: Aligning agile processes and software architectures	Science direct
P72	Drivers of agile software development use: Dialectic interplay between benefits and hindrances	Science direct
P73	Essential communication practices for extreme programming in a global software development team	Science direct
P74	Review of life cycle assessment towards sustainable product	Science direct
P75	Model of efficient and sustainable improvements in a lean production system through processes of environmental innovation	Science direct
P76	Processes versus people: How should agile software development maturity is defined?	Science direct
P77	A literature review of agile practices and their effect in scientific software development	ACM
P78	Necessary and neglected? An empirical study of internal documentation in agile software development teams	ACM
P79	A systematic literature review of agile software processes and user centered design integration	ACM
P80	Effort estimation in agile software development: A survey on the state of practice	ACM

A list of identified factors through SLR has been shown in Table 5. There are a total of 24 factors, extracted from finally selected papers (N=80). All these factors are considered as green factors of agile methods that can help software development organizations to produce green and sustainable software in GSD. Among the identified factors from selected publications, simple design, iterative development, minimal documentation, efficient utilization of time and computing resources, standard coding and fast delivery of software have shown high frequency. Our results reveal that the identified factors, as shown in Table 5, should be taken into account by vendor organizations for the development of green and sustainable software using agile techniques.

10. CONCLUSIONS

This paper presents a distinct approach towards the development of software by describing a systematic literature review protocol. We have a

particular focus on agile methods in, that support the development of sustainable software. Agile methods promise quick and scheduled delivery of software in short increments. It has reduced the complexity of software development through customer involvement and continuous interaction. To avail green agility, there is a need to explore green principles of agile methods that can contribute in software development life cycle that yields green and sustainable software. Keeping in view the importance and potential benefits of global software development, we intend to develop green agility maturity model that will help to measure the agile maturity of software organization and will also help to identify the weak areas that need to be addressed.

This research work presents the development of SLR protocol and its subsequent results as shown in Table 4 and Table 5. While implementing the protocol using the designed search string as mentioned in Track 2, we got publication sample of (N=2,298) , out of them (N=125) have been primary selected by reading the title and abstract of the papers. After a thorough review of the full text of the primary selected publications, we got a final sample of N=80.

We have almost finalized the data extraction of final selected publications by elicitation of the required data using an extraction form, as shown in Table 3. We have initially synthesized the extracted data in the form of factors as our preliminary results. These factors will be beneficial to software vendors for the production of green and sustainable software products using agile methods in global projects. Furthermore, these findings will ultimately help us to develop Green Agile Maturity Model for GSD Vendors (GAMM for GSD Vendors), in order to measure their green agile maturity for GSD projects. The detailed view is shown in Fig. 1.

11. REFERENCES

1. Holmström, B. Fitzgerald & J. Agerfalk. Agile practices reduce distance in global software development. *Information Systems Management* 23(3): 7-18 (2006).
2. Sahay, S. Global software alliances: The challenge of' standardization. *Scandinavian Journal of Information Systems* 15(1): 11-17 (2003).
3. Smite, D., C. Wohlin & T. Gorschek. Empirical evidence in global software engineering: A systematic review. *Journal of Empirical Software Engineering* 15(1): 91-118 (2010).
4. Khan, S.U., M. Niazi & R. Ahmad. Barriers in the selection of offshore software development outsourcing vendors: An exploratory study using a systematic literature review. *Journal of Information and Software Technology* 53(1): 693–706 (2011).
5. Khan, H.H., N. Mahrin & B. Chuprat, Factors generating risks during requirement engineering process in global software development environment. *International Journal of Digital Information and Wireless Communications (IJDIWC) 4*(1): 63-78 (2014).
6. Zafar, A., S. Ali & R. K. Shahzad. Investigating integration challenges and solutions in global software development. In: *IEEE Conference, Frontiers of Information Technology (FIT),* Islamabad, Pakistan, p. 291- 297 (2011).
7. Vithana, V.N., Fernando & M. Kapurubandara. Success factors for agile software development: A case study from Sri lanka. *International Journal of Computer Applications* 113(17): 119-125 (2015).
8. Alistair., A. *Agile Software Development: The Cooperative Game*. Pearson Education (2006).
9. Highsmith, J. & A. Cockburn. Agile software development. *The Business of Innovation Computer* 34(9): 120-127 (2001).
10. Shrivastava, S. V. & H. Date. Distributed agile software development: A review. *Journal of Computer Science* 7(3): 10–16 (2010).
11. Singh, A., K. Singh & N. Sharma. Agile in global software engineering: an exploratory experience. *International Journal of Agile Systems and Management* 8(1): 23-38 (2015).
12. Kern, E., M. Dick & M. Naumann. Impacts of software and its engineering on the carbon footprint of ICT. *Environmental Impact Assessment Review* 52: 53-61 (2015).
13. Rashid., N. & S.U Khan. Green agile maturity model for global software development vendors. *Science International Journal* (Lahore, Pakistan) 5(26): 2041-2043 (2014).
14. Mohan, K., B. Ramesh & L. Cao. Managing disruptive and sustaining innovations in Green IT. *IT Professional* 6: 22-29 (2012).
15. Layman, L., Williams & D. Damian. Essential communication practices for extreme programming in a global software development team. *Journal of Information and Software Technology* 48(9): 781-794 (2006).
16. Ray, S. Green software engineering process: moving towards sustainable software product design. *Journal of Global Research in Computer Science* 4(1): 25-29 (2013).

17. Misra, S., V. Kumar & K. Fantazy. Agile software development practices: evolution, principles, and criticisms. *International Journal of Quality & Reliability Management* 29(9): 972-980 (2012).
18. Brhel, M., H. Meth & A. Maedcher, Exploring principles of user-centered agile software development: A. *Information and Software Technology* 61: 163-181 (2015).
19. Dybå, T. & T. Dingsøyr. Empirical studies of agile software development: a systematic review. *Information and Software Technology* 50(9): 833-859 (2008).
20. Dingsyr, T., S. Nerur & V. Balijepally. A decade of agile methodologies: towards explaining Agile software development. *Journal of Systems and Software* 85(6): 1213-1221 (2012).
21. Mahmoud, S.S. & I. Ahmad. A green model for sustainable software engineering. International Journal of Software Engineering and Its Applications 7(4): 55-74 (2013).
22. Tate, K. *Sustainable Software Development: An Agile Perspective*. Addison-Wesley Professional (2005).
23. Koontz, R.J. & R.I. Nord. Architecting for sustainable software delivery. Cross Talk (May/June 2012), In: *Rapid and Agile Stability*, p. 14–19 (2012).
24. Khan, S.U., M.K. Niazi & R. Ahmad. Barriers in the selection of offshore software development outsourcing vendors: an exploratory study using a systematic literature review. *Information and Software Technology* 53(7): 693-706 (2011).
25. Zhang, H., M.A. Babar & P. Tell. Identifying relevant studies in software engineering. *Information and Software Technology* 53(6): 625-637 (2011).
26. Kitchenham, B., S.L. Pfleeger & L.M. Pickard. Preliminary guidelines for empirical research in software engineering. *Software Engineering, IEEE Transactions* 28(8): 721-734 (2002).
27. Khan, S. U., M. Niazi & R. Ahmad. Factors influencing clients in the selection of offshore software outsourcing vendors: an exploratory study using a systematic literature review. *Journal of Systems and Software* 84(4): 686-699 (2011).
28. Khan, A.W. & S.U. Khan. Critical success factors for offshore software outsourcing contract management from vendors' perspective: an exploratory study using a systematic literature review. *Journal of IET Software* 7(6): 327-338 (2013).
29. Khan, S.U. & M.I. Azeem. Intercultural challenges in offshore software development outsourcing relationships: an exploratory study using a systematic literature review. *Journal of IET Software* 8(4): 161-173 (2014).
30. Firdaus, A., I. Ghani & N.M.I. Yasin. Developing secure websites using feature driven development (FDD): a case study. *Journal of Clean Energy Technologies* 1(4): 322-326 (2013).
31. Brereton P., B.A. Kitchenham & D. Budgen. Lessons from applying the systematic literature review process within the software engineering domain. Journal of Systems and Software 80(4): 571-583 (2007).
32. Kitchenham, B., O.P. Brereton & D. Budgen. Systematic literature reviews in software engineering–a systematic literature review. *Information and Software Technology* 51(1): 7-15 (2009).
33. Niazi, M., N. Ikram & M. Bano. Establishing trust in offshore software outsourcing relationships: an exploratory study using a systematic literature review. *Journal of IET Software* 7(5): 283-293 (2013).
34. Korkala, M. & F. Maurer, Waste identification as the means for improving communication in globally distributed agile software development. Journal of Systems and Software 95(1): 122-140 (2014).
35. Shrivastava, S.V. & U. Rathod. Categorization of risk factors for distributed agile projects. *Information and Software Technology* 58*:* 373-387 (2015).
36. Ramesh, B., L. Cao & R. Baskerville. Agile requirements engineering practices and challenges: An empirical study. *Information Systems Journal* 20(5): 449-480 (2010).
37. Niazi, M., D. Wilson & D. Zowghi. A maturity model for the implementation of software process improvement: An empirical study. *Journal of Systems and Software 74*(2): 155-172 (2005).
38. McIntosh, S., Y. Kamei & B. Adams. An empirical study of the impact of modern code review practices on software quality. *Empirical Software Engineering* 4(1)*:* 1-44 (2015).
39. Ali, I. & S.U. Khan. Usability of agile methods in software development outsourcing: A systematic literature review protocol. *Science International Journal* (Lahore, Pakistan) 5(26): 2023-202 (2014).
40. Petersen, K., S. Vakkalanka & L. Kuzniarz. Guidelines for conducting systematic mapping studies in software engineering: an update. *Information and Software Technology* 64: 1-18 (2015).
41. Niazi, M., S. Mahmood & M. Alshayeb. Empirical investigation of the challenges of the existing tools used in global software development projects. *Journal of IET Software,* 9(5): 135-143 (2015).
42. Verner, J.M., O.P. Brereton, B.A. Kitchenham, M.K. Niazi. Risks and risk mitigation in global software development: A tertiary study. *Information and Software Technology* 56: 54-78 (2014).

43. Verner, J.M., O.P. Brereton, B. Kitchenham, M. Turner & M.K. Niazi. Systematic literature reviews in global software development: a tertiary study. In: *16th International Conference on Evaluation & Assessment in Software Engineering (EASE),* Spain, p. 2-11 (2012).
44. Niazi, M. Software process improvement: a road to success. *Product-Focused Software Process Improvement:* Springer, p. 395-401 (2006).
45. Niazi, M.K. An instrument for measuring the maturity of requirements engineering process. In: *6th International Conference on Product Focused Software Process Improvement*. *LNCS*, Oulu, Finland , p. 13-16 (2005).
46. Khan, S.U., M. Niazi & R. Ahmad. Empirical investigation of success factors for offshore software development outsourcing vendors. *Journal of IET Software* 6: 1-15 (2012).
47. Niazi, M., D. Wilson & D. Zowghi. A model for the implementation of software process improvement: an empirical study. In: *Product Focused Software Process Improvement.* Springer, p. 1-16 (2004).
48. Niazi, M., D. Wilson & D. Zowghi. A model for the implementation of software process improvement: a pilot study. In: *Third IEEE International Conference on Quality Software,* (*QSIC*03). IEEE Computer Society, Dallas, USA, p. 196-203 (2003).
49. Niazi, M. Software process improvement implementation: avoiding critical barriers. *The Journal of Defense Software Engineering* 22(1): 24-27 (2009).

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Research Article

Empirical Investigation of Success Factors for Establishing Software Outsourcing Partnership from Vendor's Perspective

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Abstract: Software outsourcing partnership (SOP) is a trustful relationship between client and vendor organisations for shared goals. A SOP is different than ordinary software development outsourcing relationship. Usually a successful outsourcing relationship may lead to SOP. Software companies currently use a wide variety of mechanisms to outsource software development work. Besides all, SOP is an emerging strategy. The overarching target of this research paper is to find and analyse those factors that are considered significant for vendors in conversion of their existing outsourcing contractual relationship to partnership. In this paper an empirical study has been conducted and the results are distributed and analysed based on 'expert job location' and 'company size'. We have executed a questionnaire based survey with 35 experts from different software companies. The participants were questioned to rank prominence of critical success factors (CSFs) on a seven point likert scale. We have find out 26 success factors including the CSFs like, 'organisational proximity' 'mutual trust', 'effective and timely communication', 'flexible service level agreements (FSLA)', 'bidirectional transfer of knowledge (BTK)', 'mutual interdependence and shared values', 'quality production' and '3C (coordination, cooperation and collaboration)' that generally impact in conversion of the existing contract based outsourcing relationship to a partnership based relationship. Vendors should address all of the identified success factors, specifically the CSFs in order to attain partner position with their clients in software development outsourcing business.

Keywords: Software outsourcing partnership, empirical study, success factors, vendors

1. INTRODUCTION

Software outsourcing partnership (SOP) is a global software development outsourcing (SDO) paradigm for developing high quality software product at reduced cost. SOP is different to software outsourcing. This is because software outsourcing is a contract-based relationship between client-vendor organisations whereas SOP is a collaborative relationship beyond organisational boundaries. Client vendor relation in this fashion often crossing the traditional contractual limits agreed at the initial level of the collaboration. Here risks and benefits, investments and work load of joint labours are equally divided among the collaborative members. Companies achieve competitive advantages through inter and intra-organisational collaboration. In SOP, long term working relationships are developed based on bidirectional trust, mutual interdependence and win-win mind-set between partners. Companies usually develop collaboration to decrease the costs of obtaining appropriate information/ understanding and capabilities or competencies needed for well-organized professional processes. Collaborative relationships usually are in the form of joint ventures, alliances, association or partnerships [1].

However, developing a fruitful long term cooperative relationship based on collaboration between two diverse businesses are more challenging and complex than commonly estimated. In view of Kelly et al [2] disappointment proportion for collaborative relationships (like associations, alliances, joint ventures or partnerships) varied from 50% to 60%. Bamford et al [1] reported in his research article that success rate was only 53%. When collaboration like partnership is in developing stage, the focus might very often be in financial

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and legal aspects. Beside all the complexities, collaboration still happens amongst organisations. Focusing on social aspects beside legal and financial, might lead to better-quality and long-lasting results when developing partnership [3].

Building a successful inter-organisational partnership is a multi-dimensional and iterative process in which legal, psychosocial, economical sub processes are concurrently taking place [4]. Shared goals and ownership, mutual interdependence, mutual trust, long term commitment, effective and timely communication, quality production and partner's proximity are constituent parts of a successful partnership [3].

Software companies currently use a wide variety of mechanisms to source software development; they outsource development work, develop insource, expand insource capability through acquisitions, build partnerships and joint ventures with counterpart organisations [7]. Four of the strategies are highlighted by Moe et al [7] including insource, outsource, separate profit centre and strategic partnership. Similarly Roy et al [8] present four internal outsourcing strategies including internal governance, recuperation, outsourcing and partnership.

According to Kishore [9] outsourcing relationship can be categorised into four categories. These are support (provision type), alignment (arrangement types), reliance (dependence type) and alliance (coalition type). Alliance is a relation with high trust and low control. SOP is a type of an coalition relationship [10].

1.1 Outsourcing Partnership – What it is?

Outsourcing partnership is a widely used terminology in the literature but still no precise definition exists for it. It is a relationship composed of two words outsourcing and partnership and therefore thoughtful understanding of individual terms is desirable for its definition. Outsourcing is the contracting of various system's sub-functions, programming, data entry, facilities management, maintenance operation, system integration, disaster recovery, data centre management, and telecommunication by client firm to external vendor [11].

Oxford English dictionary [12] define outsourcing as "the procurement of services, components or goods from an outdoor or overseas provider, particularly in place of an inside one". In view of Kinnula et al [10] "outsourcing is the transferring of duty for a particular corporate task from a servant group to a non- servant group".

The main reasons for outsourcing are cost savings, increased flexibility in bidirectional decision making, access to specialist expertise, improved quality of service, free management time when there is lack of resources, improved financial control [13]. According to Brinkerhoff and Jennifer [14] the reasons for outsourcing, includes marked pressure on organisation to reduce costs, increase core competencies, and to provide specialized expertise more effectively.

In the management literature the partnership type correlation between companies has been studied extensively [4]. For example inter-firm cooperation has examined in the marketing discipline, partnering between manufacturers and distributors, manufacturers and sales agents, buyers and sellers as well as auditors and clients [4]. While in computer literature empirical literature survey on the partnership relationship between outsourcer and outsourcee started to grow after 2000 in the Europe, US and Asia. La Londe et al [15] define it as "an association amongst two organisations that involves the sharing of workloads, benefits and risks over some pre-defined vanishing point". Lambert et al [16] have the view "it is a business associations between two or more organisations founded upon, openness, mutual trust, shared rewards and risks that produce a competitive benefits, resulting from performing in this association more than that might be attained by the either organisation individualistically".

In nut shell 'an outsourcing partnership' is a commonly used word with no clear-cut definition. It is used quite charitably by the academics, without proper definition. However, obliquely it is conceivable to develop a universally acceptable understanding of the combine word. It is a relationship based on partnership, brings about from the contract out process, unlike to other categories of relationship that can be engendered from the outsourcing process. It is a long term SDO relationship.

In this research paper, we consider software outsourcing partnership as "a strategic partnering relationship for software development between client and vendor organisation(s) with mutual adjustment and renegotiations of tasks and

commitment that exceeds mere contractual obligations stated in the initial phase of the collaboration. SOP is a mutually beneficial, continuous and long term relationship, in which future plans, visions and confidential information is shared with partner organisations proactively and willingly, with the aim to help each other, in concentrating their skills and resources towards the right track.

1.2 Difference between Ordinary Outsourcing and Partnership Outsourcing

Ordinary software development outsourcing (SDO) relationship is different to SDO partnership. This is because, in ordinary outsourcing relation a client tendered software development effort to a an external vendor(s), who provides development facilities for payment while outsourcing partnership is the superior form of ordinary outsourcing relationship [17-19]. SOP is a relation for long time based on the renegotiations of mutually adjusted task and commitment that supersede the initially agreed contractual terms and conditions that are mentioned at the start of the association [19]. It is flexible, long term relationship established based on sharing of benefits, risks, future goals and visions. In practice only a fruitful outsourcing relationship is eligible to promote to outsourcing partnership [9]. It cannot be instantly developed, but rather, it shapes with the passage of time [6]. A key difference is in the level of depth; SOP is deeper relationship in which many traditional border line between companies are wrecked [10]. A relationship is said to be SOP, where the parties share confidential information about future plans, work together, combine resources, share ownership, risks and benefits [5] and take joint decisions to undertake mutually beneficial business [20]. Outsourcing partnership is a good tool to overcome technological uncertainty, because outsourcing partnership is the unique type of outsourcing relationship where partners share information of unexpected events [10]. Here both the parties share tacit information, human resources, and work load, to achieve mutual goals [20]. The main difference between partnership and contractual relationship is that, in partnership relationship the stress is given on trust and achievement of general business goals while in contractual relationship the stress is given on the obligation of formally written contract and on achieving specific business goals. In summary partnerships are about relationships, not contracts [4, 10]. To understand SOP from the perspectives of the vendor, the following research questions are verbalized.

RQ 1.What are the critical success factors, as identified in the real practice, to be developed by software outsourcing vendor organisations which assist in promoting the exiting outsourcing relationship into partnership with client organisation

RQ 2. How are these factors related to the respondent job location?

RQ 3. Do the identified factors vary based on company size?

The reaming paper is structured as background and associated work is presented in section 2. Section 3 is the methodology. Section 4 describes study results. Section 5 is limitations while Section 6 demonstrates the conclusion and future work of this research project.

2. BACKGROUND AND ASSOCIATED WORK

A number of approaches exist for collaboration, such as sub-contracting, partnership, alliance, reliance and joint venture, etc. Kinnula [10] has presented a summary of the research areas of partnership in the context of SDO. These include (1) motivation towards partnership (2) performance evaluation of partnership (3) scope of partnership (4) success of partnership and (5) decision making frameworks for partnership.

Numeral researchers have enraged to statement certain matters of the SOP, e.g (Ellram and Edis [21], Bowersox et al [20], Sehic et al [22], Kinnula [6], Dominguez [23], Lane et al [4], Mohr et al [24], Sehic et al [22], Oza et al [25], Ellram and Edis [21] Virolainen and Veli-Matti [26] etc). Summary of some of these research works are presented below:

Bowersox et al [20] state that, partnership is formed in order to achieve shared benefits greater than the firms would achieve individually. It is a long term process in which partners with mutual goals makes joint decisions, work closely together, share information, ownership, benefits, risks, resources and achieve mutually beneficial results.

A research study was carried out in USA on factors affecting partnership formation [25], the

main results of the investigation were mutual trust and cultural differences. A comparable study was carried out by Kinnula [6] to investigate the formation of outsourcing partnership and has proposed outsourcing partnership life cycle model. Sehic et al [22] proposed a strategic partnership model (SPM) and have identified various external factors (such as social, political, competitive and technology) and internal factors (such as organisational perspective, cost, resource, history and competitiveness).

Ellram and Edis [21] explain how traditional outsourcing relationship is moved towards partnering relationship. Previously alliance has been highlighted with opportunism, doubt and distrust, contracts for single projects, strictly watched over communication between client and vendor, limited objectivity, restraint access of organisational resources, retribution for slip-ups, blame and distance and connection for specific project only. This type of out-dated mind-set is not fruitful to an outsourcing partnership relationship. To bring the relationship on right way, a key change in the approach is required. In partnership type relationships shared aims and objectives, mutual trust, openness and honesty in dealings, effective and in time communication, objective critique, long-term commitment, innovative and supportive work place, organisational access to new technology, complementary skills and market, knowledge and resources sharing, teamwork, complete company engrossment at every levels of contacts and organisational proximity provide foundation for the partnership relationship formation.

Dominguez [23] argues the partnership as a demonstration of trust. The need for partnering relationships arises in case where countless and faster co-operation is demanded. One of the constituent's elements of partner type relationship is the provision of trustful atmosphere between the outsourcing client and vendor(s). Open communication, information sharing and mutual goals are all tools for getting partner position.

Mohr et al [24] have identified various critical factors such as coordination, communication quality, commitment, trust, information sharing, active participation, honesty and openness, and joint problem solving in partnership formation [24]. Similarly other identified factors include bidirectional information sharing, shared goals, trust, early communication with client, distinct value addition by vendor, top management support, mutual commitment and mutual understanding [26].

This paper is one component of our proposed Software Outsourcing Partnership Model (SOPM) [27]. The overarching target of this research paper is to find out and analyses those factors that are considered significant for vendors in conversion of their existing outsourcing contractual relationship to partnership. Initially we have conducted systematic literature review (SLR) for the identification of success factors for SDO partnership and the results have been published [19]. SLR is more comprehensive to conventional literature review [11, 19] . For validation of the SLR findings, we have executed a questionnaire based survey with 35 experts from different software companies. The participants were questioned to rank prominence of critical success factors (CSFs) on a seven point likert scale. This survey validated and confirms the findings of SLR.

3. STUDY DESIGN

We have executed a questionnaire based survey, in the form of online survey by using the online tool Google drive, in software outsourcing industry. Our intent was to confirm results of our previously published SLR through industrial practitioners and to discover any new factors other than the identified ones. Survey research is considered particularly a suitable method of collecting tacit quantitative and qualitative data [28]. In this section, we describe the data collection, the approach taken for the selection of participants, the questionnaire procedures and the data analysis strategy. The details of the research methodology are given in the following sub-sections.

3.1 Empirical Study

A survey is a method of empirical investigation for obtaining a numeric (quantitative) description on the sample. It is the most widely used research methodology used for data collection, in order to obtain tacit information on a particular phenomenon or problem of interest [28]. A similar method has been undertaken by other investigators [11]. We have used Google drive, a free online Google application for the design and distribution of online survey questionnaire. The detailed process of executing the empirical survey is presented in the succeeding sub-sections:

3.2 Designing an Online Questionnaire Survey

Questionnaire survey comprises of two main steps, design and sampling. The process of finding, approaching and selecting the appropriate fields experts to participate in the questionnaire survey is termed as sampling [28]. After sampling design of questionnaire occur. Here a set of questions is presented for participant to be answered. Both are elaborated in the subsequent subcategories.

3.2.1 Sampling

For sampling we have two choices: 1) systematic approach; and 2) non-systematic approach [28]. In the 1st approach, samples are drawn from list of the available entire population, using some statistic, while 2nd approach is used for small survey where the entire population is not available [28]. We have used the 2nd non-systematic approach because our investigation was on minor scale. Further it was also impossible for us to collect contacts of each and every software house and to list and categorise all the employees and selecting professionals from that. Other researchers like Cox et al [29], Khan et al [11] and Kinnula [10] used a similar approach.

3.2.2 Design of Questionnaire

The questionnaire was designed at the department of CS & IT UOM, in order to validate the results of our previously published SLR through industry practitioners and to discover any new factors other than the identified ones. For the design of questionnaire, the SLR findings were used as the key contributions to the questionnaire design. The questionnaire questions are distributed into four diverse sections. Demographic information is the first section. Section-2 presents a list of 26 success factors which are evaluated on a seven argument likert scale and section-3 contains the submission information. We have provided a combination of close and open ended questions in our survey. We have queried the respondent to give their answer on a seven argument likert scale (1-Extremely Agree (EA), 2-Moderately Agree (MA), 3-Slightly Agree (SA), 4-Not Sure (NS), 5-Extremely Disagree (EDA), 6-Moderately Disagree (MDA) and 7-Slightly Disagree (SDA). We have also provided some open ended questions to the participants. "Give some additional factors other than the listed ones" is an example of open ended question. The survey questionnaire was tested through five members of SERG_UOM@ yahoogroups.com at the university.

3.3 Data Gathering

The objective of the research study is to reconnoitre the experiences and opinions of the practitioners, working in the software industry, in the context of SOP. It can be considered primarily as being qualitative in nature. Qualitative research focuses on investigating and understanding social and cultural phenomena in context and is appropriate where the purpose is to explore a topic and to obtain an overview of a complex area [28, 29]. Questionnaire survey is particularly suitable for collecting qualitative data because, it provides the opportunity for discussion or exploration of new topics that arise during data collection. A questionnaire provides a considerable autonomy to researcher in arrangement of questions and in the quantity of time and consideration given to each theme. Questions can be open-ended, allowing for a variety of responses. This approach of data collection helps to reduce the risk of bias relating to the researchers preconceptions and it allows for the use of elaboration probes to encourage the participant to give his/her own opinion about a particular subject [28, 29].

3.3.1 Pilot Questionnaire

The questionnaire procedures were tested with the help of five associates of SERG_UOM@ yahoogroups.com, at the campus, who had undertaken several questionnaire surveys. This experience confirmed our expectation that the questionnaire would take approximately 30 minutes and also led to some changes in the delivery and sequencing of the questions.

3.3.2 Selection of Participants

Before the distribution of the questionnaire we wrote a letter of invitation having some briefing of the research study and were mailed to the below mentioned websites.

- Yahoo (https://groups.yahoo.com/neo)
- LinkedIn (www.linkedin.com)
- Facebook (www.facebook.com) and
- Software Companies at Pakistan.

We also requested to the authors of the industrial papers through email, for participation in the survey. These industrial papers were selected through our previously published SLR. In

comeback to these emails, 101 industry professional's experts consented for contribution. Upon receiving their willingness the questionnaire web link was sent to these experts. In the predefined time limit, we received 42 filled questionnaires. Applying the quality criteria 07 questionnaires were dropped/ excluded. A final of 35 responses remains for further analysis. Among the 35 participants 8 experts are foreigners while the remaining 27 experts are Pakistani nationals.

3.3.3 Questionnaire Procedures

Questionnaires were carried out between April and May 2015. Prior to questionnaire, each participant was sent questionnaire invitation letter. This letter outlined the main themes to be covered during the questionnaire, the expected duration, and measures which would be taken to ensure privacy and confidentiality. All questionnaires were conducted online, using the Google drive free online survey tool.

3.3.4 Executing Online Surveys

A total of 101 professionals were invited for participation in the questionnaire survey through emails. Fig. 1 shows the number of received responses /day. Fig. 1 indicates that we had received rapid response to our survey request from some participants. To increase the response rate, we also sent an email reminder to the participants. This reminder helped significantly.

3.3.5 Data Analysis Strategy

Upon receiving willingness of the experts, the questionnaire web link was sent to these experts. In the predefined time limit, we received 42 filled questionnaires. Applying the quality criteria 07 questionnaires were dropped/ excluded. A final of 35 responses remains for further analysis. Thus, we caught 34.65% response rate for our survey. These 35 filled questionnaires were further analysed based on different variables.

3.3.6 Nonresponse Bias Measurement

It was clear from the previous heading that numerous participants had not answered to our questionnaire survey. We have conducted t-tests for checking non-response biasness cause and effects. We did not find any important dissimilarity ($p<0.05$) amongst non-respondents and respondents [30].

4. RESULTS

In results section the findings related to research questions are discussed. In the sub sections 4.1, 4.2, and 4.3 we have answered RQ1, RQ2, and RQ3 respectively. For the analysis purpose we have find significant difference of success factors using different variables. For significant differences chi-square (linear by linear association) test is used. For analysis of the significant difference amongst nominal and ordinal variables the linear by linear chi-square test is consider more powerful as compared to Pearson chi-square test [30].

In the questionnaire different participants have selected different options on the 7-point likert scale suggestion for each of the listed CSF. For analysis, we have categorised their responses into three categories as shown in Table 2. First category is Positive (Extremely Agree + Moderately Agree + Slightly Agree). Second category is Negative (Extremely Disagree + Moderately Disagree + Slightly Disagree). Third category is Neutral neither (Neither Positive nor Negative).

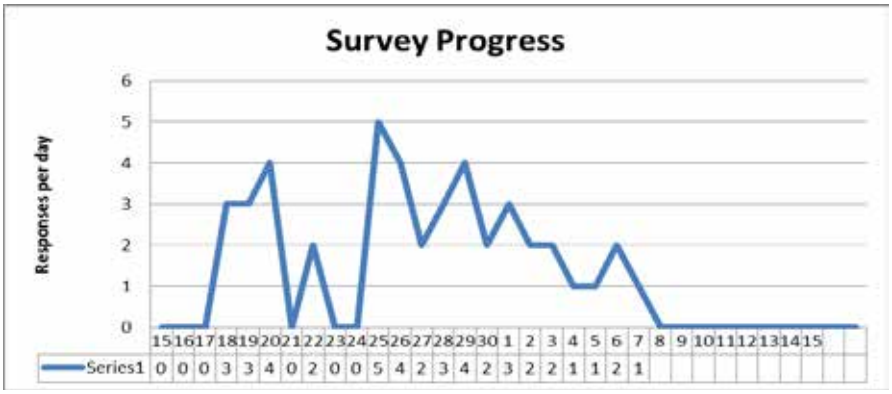

Fig. 1. Number of daily responses.

Table 1. Summary of CSFs identified through empirical study.

S. No.	Success Factors	Total Expert Responses = 35									
		Positive				Negative				Neutral	
		Extremely Agree frequency	Moderately Agree frequency	Slightly Agree frequency	Agree %	Extremely Disagree frequency	Moderately Disagree frequency	Slightly Disagree Frequency	Disagree %	Not sure	%
1	Mutual interdependence and shared values	17	13	3	94%	0	0	0	0%	2	6%
2	Mutual trust	27	4	2	94%	0	0	0	0%	2	6%
3	Effective and timely communication	24	7	3	97%	0	0	0	0%	1	3%
4	Quality production	23	9	2	97%	0	0	0	0%	1	3%
5	Organisational proximity	9	11	5	71%	0	0	1	3%	9	26%
6	3C (coordination, cooperation and collaboration)	22	9	2	94%	0	0	0	0%	2	6%
7	Flexible Service Level Agreements (SLA)	10	14	8	91%	0	0	0	0%	3	9%
8	Bidirectional transfer of knowledge (BTK)	16	12	4	91%	0	0	1	3%	2	6%
9	Long-term commitments	16	12	4	91%	0	0	1	3%	2	6%
10	Joint management infrastructure	15	6	5	74%	0	2	2	11%	5	14%
11	Cross Cultural understanding and sensitivity	15	10	5	86%	1	0	0	3%	4	11%
12	Success stories of previous projects	22	9	3	97%	0	0	0	0%	1	3%
13	Access to new markets, technologies and complementary skills	17	11	6	97%	0	0	0	0%	1	3%
14	Governance and control	9	11	5	71%	1	1	3	14%	5	14%
15	Financial stability and relation specific investment	13	14	5	91%	0	0	1	3%	2	6%
16	Organisational transparency and receptivity	13	14	6	94%	0	0	0	0%	2	6%
17	Flexibility and reliability	15	13	4	91%	0	0	0	0%	3	9%
18	Spurring innovation	11	10	4	71%	0	0	1	3%	9	26%
19	Win-Win strategy	12	11	6	83%	0	0	0	0%	6	17%
20	Effective relationship management	21	7	6	97%	0	0	0	0%	1	3%
21	Constructive conflicts resolution mechanism	10	16	3	83%	0	0	0	0%	6	17%
22	Top management engagement	22	8	4	97%	0	0	0	0%	1	3%
23	Social networking	7	11	4	63%	0	4	1	14%	8	23%
24	New business opportunity	10	8	4	63%	1	4	4	26%	4	11%
25	Honesty and openness	16	10	3	83%	1	0	1	6%	4	11%
26	Human Resource Management	15	4	3	83%	0	1	0	6%	4	11%

4.1 Summary of Factors, Identified through Empirical Study

Table 1 illustrates that all the identified factors have been ranked positively/agreed by more than 60% of the participants. Similarly others most high ranked success factors, in the survey, are ‘effective and timely communication’ ‘quality production’, ‘success stories of previous projects’ ‘access to new markets, technologies and complementary skills’, ‘top management engagement’ and ‘effective relationship management’. All these

factors are positively agreed by 97% of the experts. This confirms the findings of the literature as reported below:

- According to Webb and Laborde [31] 'effective and efficient communication' between client and vendor organisations gives them an opportunity for the formation of strong relationship for quality production. Partnership is the most suitable way to get access to new markets, new technologies and complementary skills that are not available in house/insource. In view of Berger et al [32] effective communication between outsourcing partners is assumed to be of crucial importance for the successful relationship like partnership.
- Effective communication provide strong opportunity to enter into partnership [31].
- Today's outsourcing relationship is not formed just for cost saving, but for the best quality that the counterpart offer [14].
- Partnership offers the opportunity for both parties involved to get access to new markets, new technologies and complementary skills that are not available in house/insource, for undertaking complimentary activities to achieve mutual benefits [33].
- Effective relationship management is the key to outsourcing partnership formation [10].
- Success stories of previous projects is an important factor, generally a mature and successful outsourcing arrangement may convert to outsourcing partnership [17, 18].

Similarly the second most highly agreed success factors, in the survey, are 'mutual trust', 'mutual interdependence and shared values', 'organisational transparency and receptivity' and '3C (coordination, cooperation and collaboration)'. This in turn supports the findings of the literature which is reported as:

- According to Bowersox et al [20] in order to attain mutual benefits, greater than the organisations would be able to achieve individually, a long-term partnering relationship is formed. Where organisations with common goals make joint decisions, work together, share resources and information, benefits and risks, and achieve mutually valuable outcomes.
- Literature reveals that the current inter-organisational trend is changing from competition to coordination, cooperation and collaboration [34].
- In view of Alexandrova [34], special attention should be put on the way in which organisations "learn" from their partners, as this appears to be one of the means for the development of key competences.

Our results also indicate that that the following success factors got the third rank (91%) in the identified list for vendors. These include 'flexible service level agreements (SLA)', 'bidirectional transfer of knowledge (BTK)', 'long-term commitments', 'financial stability and relation specific investment', 'and 'flexibility and reliability'. Literature reveals the importance of these factors in the following manner:

- Most conventional organisational relationship uses a formally written contract as a control instrument, because it helps to put mutually enforceable bounds on the power and activities of each partner [35]. While partnership relationship uses flexible service level agreement (FSLA) as a controller instrument in the SDO arrangements, that would implement control approaches through mutual trust [35].
- BTK emerges when optimal (in terms of quantity and quality) information necessary for the realization of the service is provided through the channels of effective communication between the partners. The knowledge could have two forms: implicit i.e. informal, tacit, and explicit i.e. formal [11].
- Commitment guarantee parties that the association will be continued till long time and has been categorised as "a lasting wish to continue a value added relationship" [36].
- According to Klepper and Jones [37] financial stability of the partner is necessary factors of the SOP because partnership requires asset investment by both client and vendor organisation.
- Partnership type relationship is flexible, i.e it allows changes in the SLA/contract and project requirement throughout the agreed time spans. This relationship is a long lasting

and may result in contracts renewal several times [37].

We also found that 'cross cultural understanding and sensitivity' i.e. 86% in the fourth ranked significant factor to SOP vendor's organisation. The literature reveals the importance of these factors as:

- Many cross cultural software development relationship failures have been endorsed to a cultural differences and lack of capability to boost 'cross cultural understanding and sensitivity' [38].

'Win-Win strategy', 'constructive conflicts resolution mechanism', 'honesty and openness' and 'human resource management' are ranked as fifth (83% positively agreed) in our survey findings.

- Conventional outsourcing is based on win-loss mentality, while partnership outsourcing is based on win-win mentality where both parties win [19].
- Conflicts represent the level of disagreement in the functioning partnership. Resolve disagreement as they arise [10].
- In partnership both parties share information of the project status openly, which vanishes chances of conflicts [6].
- According to Khan and Niazi [11, 42] HRM is an important factor to be addressed by vendors efficiently in outsourcing relationship.

Seventy % of the respondents are positive about 'organisational proximity', 'governance and control', and 'spurring innovation' is an important factor of outsourcing partnership. Literature reports the importance of these factors as follow:

- 'Governance and control' is concerned with taking corrective action in problems situation, for resolution, among distributed partners. It also involves the settlement of conflicts of concern between numerous outsourcing partners [39].
- Organisational proximity is the factor, which controls organisational differences as it is "belonging to the same space of references". It is also the development of shared norms, representations, work standards and practices [40].
- According to Lee and Kim [33] outsourcing vendor must be capable of innovation in order to become partner.

'Social networking' and 'new business opportunity' are the two factors having percentages less than 70%, i.e., 63% in the survey findings. Literature also reveals that:

- Social networking is an essential element of partnership [19, 43].
- Partnership provides new business opportunity' [19].

4.2 Distribution of the Factors, Identified through Empirical Study, across Respondents Job Locations

A total of 35 industrial practitioners/experts participated in the survey. We have divided all these professionals into two groups (vicinity and foreign) grounded on their company's location. By local experts we mean those experts who are involved in Pakistani software outsourcing industry while 'foreign' experts are those working in overseas organisations i.e other than Pakistan.

Table 2 demonstrates distribution the identified success factors across these two kinds of SDO experts. Fig. 2 graphically illustrates responses of professional experts.

In our empirical results eight experts are 'foreigners' while the remaining twenty seven experts are 'locals' as shown in Table 2. Table 2 reveals that most of the success factors are positively agreed by both foreign and local experts.

We have found only one significant difference based on expert's job location i.e bidirectional transfer of knowledge (BTK), for which 'p' significance is less than the standard significance level 0.05 as shown in Table 2. BTK having percentage 96% in local group while 25% in overseas group. This shows that it is more known factor in local group as compared to overseas.

4.3 Distribution of the Factors, Identified through Empirical Study, across different Company Size

We have categorised the survey participants into three categories based on their company size. According to the standard definition of organisation size, reported by Australian bureau of statistics [41], these three categories are: large (more than 200 employees), medium (20 to 199 employees), small (0 to19 employees).

Table 2. Distribution of factors based on job location.

S. No.	Success factors	Expert Responses = 35						Chi Square Test (Linear-by-Linear association ∝ =0.05)	
		Vicinity (N = 27)			Overseas (N = 8)				
		A	DA	NS	A	DA	NS	X^2	P
1	Mutual interdependence and shared values	93%	0%	7%	100%	0%	0%	0.611	0.435
2	Mutual trust	93%	0%	7%	100%	0%	0%	0.611	0.435
3	Effective and timely communication	96%	0%	4%	100%	0%	0%	0.296	0.586
4	Quality production	96%	0%	4%	100%	0%	0%	0.296	0.586
5	Organisational proximity	70%	0%	30%	75%	13%	13%	2.254	0.133
6	3C (coordination, cooperation and collaboration)	93%	0%	7%	100%	0%	0%	0.611	0.435
7	Flexible Service Level Agreements (SLA)	93%	0%	7%	88%	0%	13%	0.198	0.656
8	Bidirectional transfer of knowledge	96%	0%	4%	75%	25%	0%	5.818	0.016
9	Long-term commitments	89%	4%	7%	100%	0%	0%	0.097	0.756
10	Joint management infrastructure	70%	11%	19%	88%	13%	0%	0.927	0.336
11	Cross cultural understanding and sensitivity	81%	4%	15%	100%	0%	0%	0.546	0.460
12	Success stories of previous projects	96%	0%	4%	100%	0%	0%	0.296	0.586
13	Access to new technologies, markets, and complementary skills	96%	0%	4%	100%	0%	0%	0.296	0.586
14	Governance and control	70%	15%	15%	75%	13%	13%	0.000	1.000
15	Financial stability and relation specific investment	89%	4%	7%	100%	0%	0%	0.097	0.756
16	Organisational transparency and receptivity	93%	0%	7%	100%	0%	0%	0.611	0.435
17	Flexibility and reliability	93%	0%	7%	88%	0%	13%	0.198	0.656
18	Spurring innovation	67%	4%	30%	88%	0%	13%	0.463	0.496
19	Win-Win strategy	81%	0%	19%	88%	0%	13%	0.153	0.696
20	Effective relationship management	96%	0%	4%	100%	0%	0%	0.296	0.586
21	Constructive conflicts resolution mechanism	93%	0%	7%	100%	0%	0%	0.611	0.435
22	Top management engagement	96%	0%	4%	100%	0%	0%	0.296	0.586
23	Social Networking	59%	15%	26%	75%	13%	13%	0.203	0.652
24	New business opportunity	59%	26%	15%	75%	25%	0%	0.329	0.566
25	Honesty and openness	81%	15%	4%	88%	0%	13%	1.987	0.159
26	Human Resource Management	81%	15%	4%	88%	0%	13%	1.987	0.159

Fig. 2. Respondent Job location.

Table 3. Distribution of factors base on company size.

S. No.	Success Factors	Expert Responses = 35									Chi Square Test(Linear-by-Linear association ∝ 0.05)	
		Small (N = 11)			Medium (N = 14)			Large (N = 10)				
		A	DA	NS	A	DA	NS	A	DA	NS	X^2	P
1	Mutual interdependence and shared values	82%	0%	18%	100%	0%	0%	100%	0%	0%	3.245	0.72
2	Mutual trust	82%	0%	18%	100%	0%	0%	100%	0%	0%	3.245	0.72
3	Effective and timely communication	91%	0%	9%	100%	0%	0%	100%	0%	0%	1.575	0.209
4	Quality production	91%	0%	9%	100%	0%	0%	100%	0%	0%	1.575	0.209
5	Organisational proximity	82%	0%	18%	64%	0%	36%	70%	20%	10%	0.118	0.731
6	3C (coordination, cooperation and collaboration)	82%	0%	18%	100%	0%	0%	100%	0%	0%	3.245	0.72
7	Flexible Service Level Agreements	91%	0%	9%	86%	0%	14%	100%	0%	0%	0.494	0.482
8	Bidirectional transfer of knowledge	91%	0%	9%	93%	7%	0%	100%	0%	0%	0.811	0.368
9	Long-term commitments	82%	0%	18%	100%	0%	0%	90%	10%	0%	4.817	0.028
10	Joint management infrastructure	73%	0%	27%	71%	21%	7%	80%	10%	10%	1.596	0.207
11	Cross Cultural understanding and sensitivity	73%	0%	27%	100%	0%	0%	80%	10%	10%	2.903	0.088
12	Success stories of previous projects	91%	0%	9%	100%	0%	0%	100%	0%	0%	1.575	0.209
13	Access to new technologies, markets, and complementary skills	91%	0%	9%	100%	0%	0%	100%	0%	0%	1.575	0.209
14	Governance and control	73%	9%	18%	86%	0%	14%	50%	20%	30%	0.000	1.000
15	Financial stability and relation specific investment	82%	0%	18%	93%	7%	0%	100%	0%	0%	2.121	0.145
16	Organisational transparency and receptivity	91%	0%	9%	100%	0%	0%	90%	0%	10%	0.003	0.958
17	Flexibility and reliability	91%	0%	9%	93%	0%	7%	90%	0%	10%	0.004	0.947
18	Spurring innovation	55%	0%	45%	79%	7%	14%	80%	0%	20%	1.524	0.217
19	Win-Win strategy	73%	0%	27%	86%	0%	14%	90%	0%	10%	1.090	0.296
20	Effective relationship management	91%	0%	9%	100%	0%	0	100%	0%	0%	1.575	0.209
21	Constructive conflicts resolution mechanism	73%	0%	27%	100%	0%	0	80%	0%	20%	0.278	0.598
22	Top management engagement	91%	0%	9%	100%	0%	0	100%	0%	0%	1.575	0.209
23	Social Networking	64%	18%	18%	57%	7%	36	70%	20%	10%	0.106	0.744
24	New business opportunity	73%	18%	9%	71%	21%	7	40%	40%	20%	0.172	0.678
25	Honesty and openness	82%	9%	9%	100%	0%	0	60%	30%	10%	1.166	0.280
26	Human Resource Management	82%	9%	9%	100%	0%	0	60%	30%	10%	1.166	0.280

Table 3 characterises the results of empirical survey from these three types of outsourcing professionals. In our empirical results 11 participants are from small, 14 from medium and 10 participants from large sized organisation as shown Fig. 3. The results indicate that out of 26 CSF, 18 CSFs are such that more than 80% of small organisation experts are agree that these CSFs can play positive role in conversion of SDO vendor to outsourcing partner. It is clear from Table 3 that 21 out of 26 CSFs are such that more than 80% of the medium size organisation experts are agree that these CSF can positively impact SDO client(s) in promotion of SDO vendor organisation. Table 3 illustrate that 20 out of 26 CSFs are such that more than 80% of large companies experts are agree that these CSFs can help SDO vendor organisation in order to enter

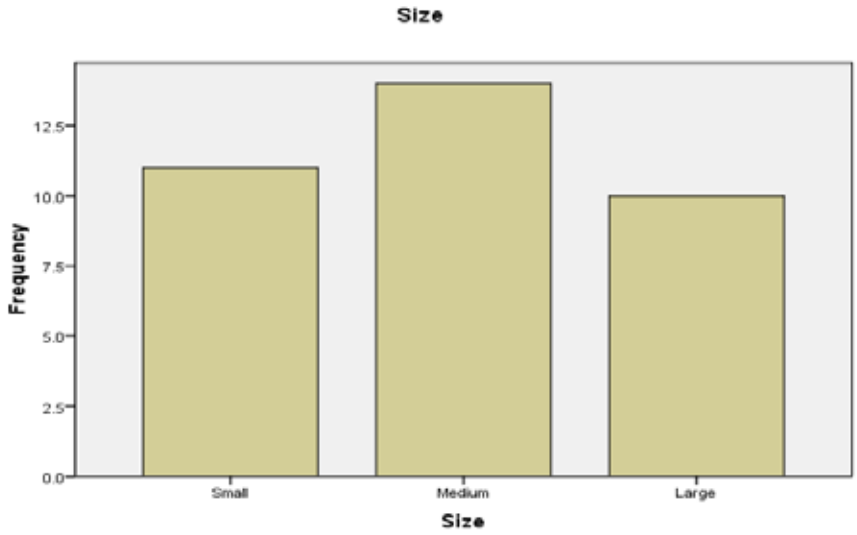

Fig. 3. Company.

into outsourcing partnership with client organisation.

We have found only one significant difference, based on expert's company size i.e. Long-term commitments, for which 'p' value is less than significance level 0.05 as shown in Table 3. 'Long-term commitments having percentage 82% in small, 100% in medium while 90% in large organisation. It shows that 'long-term commitments' is greater in medium and large size organisations, as compared to small type organisations. The reason might be that, it is difficult for large and established organisation to change outsourcing partner.

5. STUDY LIMITATIONS

This section discusses the validity of our findings of empirical study. One possible threat to internal validity is the small size of the foreigner participants. Out of 35 just 8 participants are from abroad. We should have to include more foreigners SDO experts, in order to obtain better results, but due to scarcity of resources and time it was not possible for the current study. We have tried our best in present resources to reach international experts. We have posted email requests to experts of various LinkedIn software development outsourcing and outsourcing partnership groups for participation in our survey. Because of the volunteer participation only limited numbers of respondents from abroad participate in our survey. Therefore reader should be careful while generalising the results. One possible threat to internal validity is that for any specific response, the respondent does not provide the reasons to report success factors. We are not able to independently control this treat. In this study the participants were not questioned to provide the underlying causes of SOP success factors in outsourcing partnership.

For the current study we have executed online questionnaires. In questionnaire based survey respondents are provided with a list of possible CSF and asked to rank the CSF that plays a vital role in outsourcing partnership formation, which is main disadvantage of questionnaire survey. We have address this issue up to some extent by asking the participant to also write in SOP other than those already mentioned on the questionnaire.

Though, these findings confirms the findings of our previous SLR study [19]. There is no major difference between the finding of the SLR and the empirical study. That's why we have full confidence in our results. This may fill the gap amongst industry and academia in the context of SOP.

6. CONCLUSIONS

We have identified 26 success factors for SOP in total, through empirical study, faced by vendors in SOP formation. Out of these twenty success factors have occurrences of greater than or equal to 80% as shown in Table 1. These twenty most

reported factors are 'mutual trust', 'quality production', 'mutual-interdependence and shared values', 'effective and timely communication', '3C (coordination, cooperation and collaboration)', 'flexible service level agreements (SLA)', 'bidirectional transfer of knowledge (BTK)', 'long-term commitments', 'cross cultural understanding and sensitivity', 'success stories of previous projects', 'access to new markets, technologies and complementary skills', 'financial stability and relation specific investment', 'organisational transparency and receptivity', 'flexibility and reliability', 'win-win strategy', 'effective relationship management', 'constructive conflicts resolution mechanism', 'top management engagement', 'honesty and openness', 'human resource management.

Beside all the stated limitations, we have confidence in that; our study is contributing one to both academia and industrialist. The results of this study are expected to:

1. Provide guiding information to SDO vendors that can support them in designing and implementing successful SOP initiatives. The findings of this study recommend that SDO vendors must adopt all of the reported success factors especially the critical one in order to gain partner position.
2. Increase partnership cohesiveness, as it will guide both sides to understand each other's requirements and goals, in order to sustain long term commitment.
3. Provide guidance to SDO client, in taking factual decision regarding continuing, renewing or terminating their agreements with their current vendor.
4. Will serve as guidepost for future planning for software outsourcing relationship especially SOP.
5. Provide assistance in well understanding of CSFs for SOP, to ensure successful partnership.

Overall these results complement the results of our SLR [19]. There is no major dissimilarity between the SLR and the empirical study findings. This may fill the gap between industry and academia in the context of SOP.

Our research aimed at providing SDO vendors with a guiding knowledge that can assist them to implement and design successful outsourcing partnership initiatives. This paper recommends that SDO vendors should focus on all of the reported CSFs as mentioned in Table 1. SDO vendors should also emphasis on the given percentage of SOP factors in Table 2 and Table 3.We have noted the following points as a future plan from the findings of this study:

- The solutions/practices will be identified and analysed in SOP relationships from vendor's perspectives.
- To analyses the critical risk in the conversion to, or formation process of SOP from vendor perspective.
- To find the underlying reasons of why some factors are not important for specific group of SDO organisations.

Our ultimate future work is focussed on the development of a Software Outsourcing Partnership Model (SOPM). This paper gives input to the development of the 1st phase of the SOPM, such as the identification of various CSFs through empirical study. The SOPM will assist SDO vendors in promoting their existing contractual SDO relationship into SOP with client organisation. The SOPM will provide guidance and boost the work undertaken till date on models of key success factors development for outsourcing partnership.

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8. REFERENCES

1. Bamford, J., D. Ernst & D.G. Fubini. Launching a World-Class Joint Venture. *Harvard Business Review* 82: 90-100 (2004).
2. Kelly, M.J., J.L. Schaan & H. Joncas. Managing Alliance Relationships: Key Challenges in the Early Stages of Collaboration. *R&D Management* 32: 11-22 (2002).
3. Ylitalo, J, E. Mäki & K. Ziegler. *Building Mutuality and Trust in Strategic Partnership. Meaning of Early Stages in Relationship Formation: A Case Study*. Working Paper, Helsinki University of Technology BIT Research Centre, Espoo, Finland (2004).

4. Lane, M.S. & W.H. Lum. Examining client perceptions of partnership quality and the relationships between its dimensions in an IT outsourcing relationship. *Australasian Journal of Information Systems* 17: 5-34 (2011).
5. Kinnula, M. & V. SeppAnen. *Information technology requirements in an outsourcing partnership*. In: *Frontiers of e-Business Research Conference*, Tampere, Finland, p. 493-502 (2003).
6. Kinnula, M., V. Seppanen, J. Warsta & S. Vilminko. The formation and management of a software outsourcing partnership process. In: *40th Hawaii International Conference on System Sciences.* Washington, USA, p. 1530-1605 (2007).
7. Moe, N., B, Mite, S. Darja & G.K, Hanssen. From offshore outsourcing to in sourcing and partnerships: Four failed outsourcing attempts. *Empirical Software Engineering* 19:1225-1258 (2014).
8. Roy, V. & B.A. Aubert. A resource-based analysis of IT sourcing. *ACM Special Interest Group on Management Information Systems* 33(2): 29-40 (2002).
9. Kishore, R., H.R. Rao., K. Nam, S. Rajagopalan & A. Chaudhury. A relationship perspective on IT outsourcing. *Communications of the ACM - Mobile Computing Opportunities and Challenges* 46(12): 86-92 (2003).
10. Kinnula, M. *The Formation and Management of a Software Outsourcing Partnership.* Dissertation, Department of information processing science*:* University of Oulu, Finland, 187 pp. (2006).
11. Khan, S.U, M. Niazi & R. Ahmad. Empirical investigation of success factors for offshore software development outsourcing vendors. *Institution of Engineering and Technology Software* 6(1): 1-15(2010).
12. Dictionary, O. E. *Oxford English dictionary online*: Oxford University Press, Oxford, UK, http://www.oed.com/ (2008).
13. Bocij, P., D. Chaffey. A. Greasley & S. Hickie. *Business Information Systems: Technology, Development and Management for the E-Business 4th ed*. FT Press, 736 pp. (2009).
14. Brinkerhoff, J.M. Government non-profit partnership: A defining framework. *Public Administration and Development* 22: 19-30 (2002).
15. La Londe, B.J. & M.C. Cooper. *Partnerships in Providing Customer Service: A Third-Party Perspective.* Council of Logistics Management, Oak Brook, Illinois, USA, 139 pp. (1989).
16. Lambert, D.M., M.A. Emmelhainz & J.T. Gardner. Building Successful Logistics Partnerships. *Journal of Business Logistics* 20: 165-181(1999).
17. Ali, S. & S.U. Khan. Systematic literature review protocol for software outsourcing partnership (SOP). *IOSR Journal of Computer Engineering (IOSRJCE)* 2(1): 08-18 (2012).
18. Ali, S. & S.U. Khan. Software outsourcing partnership (SOP): a systematic literature review protocol with preliminary results. *International Journal of Hybrid Information Technology* 7: 377-392 (2014).
19. Ali, S. & S.U. Khan. Critical Success Factors for Software Outsourcing Partnership (SOP): A Systematic Literature Review, In: *International Conference on Global Software Engineering*, Shanghai China, p. 153-162 (2014).
20. Bowersox, D, D. Closs & T. Stank. How to master cross-enterprise collaboration. *Supply Chain Management Review* 7: 18–27 (2003).
21. Ellram, L.M. & O.R. Edis. A case study of successful partnering implementation. *Journal of Supply Chain Management* 32: 20-28 (1996).
22. Sehic, I. & H. Gilani. *It Outsourcing: A Strategic Partnership between Buyer and Seller Organization*. PhD Dissertation, International Business School, Jönköping University, Jönköping, Sweden. 105 pp. (2010).
23. Dominguez, L.R. *The Manager's Step-by-step Guide to Outsourcing*: McGraw Hill Professional, 226 pp. (2006).
24. Mohr, J. & R. Spekman. Characteristics of partnership success: partnership attributes, communication behavior, and conflict resolution techniques. *Strategic Management Journal* 15:135-152 (1994).
25. Oza, N.V. *An Empirical Evaluation of Client - Vendor Relationships in Indian Software Outsourcing Companies*. PhD Dissertation, School of Computer Science: University of Hertfordshire, UK, 231 pp. (2006).
26. Virolainen, V.M. *Motives, Circumstances and Success Factors in Partnership Sourcing*. PhD Dissertation, Lappeenranta University of Technology Lappeenranta, Finland, 232 pp. (1998).
27. Ali, S. & S.U. Khan. Software Outsourcing Partnership Model. *Science International (Lahore)* 26: 1437-144 (2014).
28. Creswell, J.W. *Research Design: Qualitative, Quantitative and Mixed Methods Approaches*: Sage Publications, USA, 26 pp. (2013).
29. Cox, K, M. Niazi & J. Verner, Empirical study of *Sommerville and Sawyer's* requirements engineering practices. *Institution of Engineering and Technology Software* 3: 339-355 (2009).
30. Martin, B. *An Introduction to Medical Statistics*, 3rd edition: Oxford medical publications, UK 391(2000).
31. Webb, L. & J. Laborde. Crafting a successful outsourcing vendor/client relationship. *Business Process Management Journal* 11: 437-443 (2005).
32. Berger, H. & C. Lewis. Stakeholder analysis is key to client–supplier relationships of global outsourcing project success. *International Journal of Information Management* 31: 480-485(2011).

33. Lee, S. & G. Lim. The impact of partnership attributes on EDI implementation success. *Information & Management* 42:5 03-516 (2005).
34. Alexandrova, M. IT outsourcing partnerships: Empirical research on key success factors in Bulgarian organizations. *Management: Journal of Contemporary Management Issues* 17: 31-50(2012).
35. Goo, J. Structure of service level agreements (SLA) in IT outsourcing: The construct and its measurement. *Information Systems Frontiers* 12: 185-205(2010).
36. Simpson, J.T. & D.T. May. Relationship management: A call for fewer influence attempts? *Journal of Business Research* 39: 209-218 (1997).
37. Klepper, R. & W O. Jones. *Outsourcing Information Technology, Systems and Services.* Prentice-Hall, Upper Saddle River, NJ, US, 392 pp. (1998).
38. Johnson, J.P., T. Lenartowicz & S. Apud. Cross-cultural competence in international business: Toward a definition and a model. *Journal of International Business Studies* 37: 525-543 (2006).
39. Becht, M, P. Bolton & A. Rajell. Corporate governance and control. *Handbook of the Economics of Finance,* 109 pp. (2003).
40. Torre, A. & A. Rallet. Proximity and Localization. *Regional Studies* 39: 47-59(2005).
41. Trewin, D. *Australian Small Business Key Statistics and Analysis*. Department of Industry, Innovation, Science, Research and Tertiary Education, Australia, 109 pp. (2014).
42. Niazi, M., D. Wilson, D. Zowghi, & B. Wong. A model for the implementation of software process improvement: An empirical study. In: *Product Focused Software Process Improvement* (Profes 2004), Japan (2004).
43. Niazi, M., N. Ikram, M. Bano, S. Imtiaz & S.U. Khan, Establishing trust in offshore software outsourcing relationships: An exploratory study using systematic literature review. *IET Software* 7(5): 283 - 293 (2013).

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Research Article

Solutions for Critical Challenges in Offshore Software Outsourcing Contract

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Abstract: Software outsourcing is a contract based relationship between client and vendor organisations, where the client organisation makes a contract for all or part of software-development activities with the vendor organisation(s) that provide agreed services in return for remuneration. Efficient outsourcing contract can yield in successful outcomes of the outsourced projects". In our previous work we have identified nine critical challenges (e.g., 'lack of project management', 'poor monitoring system', 'lack of outsourcing relationship management', 'culture and linguistic issues', 'non-competitive price', 'IPR and regulatory issues', 'failure to manage end-users expectation', 'lack of negotiation strategies' and 'unforeseen contingencies') faced by vendors in managing and execution of offshore software development outsourcing (OSDO) contract. In this paper, our research methodology is composed of both empirical study in OSDO industry and systematic literature review. Based on the aforementioned two well-known methodologies we have identified 91 practices/solutions in total to assist vendor organizations in addressing these nine critical challenges. Implementation of the identified practices will assist in avoidance/mitigation of the critical challenges faced by vendors in management and execution of OSDO contract.

Keywords: Offshore software outsourcing contract, vendor, practices/solutions, empirical study, systematic literature review (SLR)

1. INTRODUCTION

Offshore software development outsourcing (OSDO) is a Global Software Engineering paradigm for developing high quality software at low cost in low-wages countries. Furthermore, software outsourcing is a contract based relationship between client and vendor organisations, where client organisation contracts out with vendor organisation to get high quality software [1]. Outsourcing contract is an agreement between outsourcing vendor and client organisations for the purpose of developing high quality software. Goo et al [2] stated that outsourcing contract management is considered as an important factor in the sensation of any outsourcing project. Efficient management and execution of the outsourcing contract can lead to the successful outcomes of OSDO relationship. However contract management in OSDO relationship bears a number of challenges. We have already identified a number of critical risks confronted by vendors in the management and execution of outsourcing contract [3]. These critical challenges are presented in Table 1. The goal of this work, described in this paper, is to investigate practices/solutions for the identified critical challenges in OSDO contract management from vendor's perspective.

Table 1. List of critical challenges of outsourcing contract management.

CC#	Critical challenge
1	Lack of project management
2	Poor monitoring system
3	Lack of outsourcing relationship management
4	Cultural and linguistic issues
5	Non-competitive price
6	IPR and regulatory issues
7	Failure to manage end-user expectation
8	Lack of negotiation strategies
9	Unforeseen contingencies

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1.1 Software Outsourcing

Software outsourcing is a Global Software Engineering (GSE) paradigm being actively practiced from the last two decades. Software outsourcing is categorized into offshore outsourcing, nearshore outsourcing and onshore outsourcing based on the geographical distance between the stakeholders involved. Offshore software development outsourcing (OSDO) is practiced by many organisations in the developed countries for developing a high-quality software in far flung developing countries at low cost [1]. "Software outsourcing has grown quickly and provides new shape to business process [4]. Majority of US and UK software firms tend towards OSDO due to high-quality software development at low cost [5]. The well-reputed software outsourcing vendors are located at India, Ireland, China and Russia, whereas the client's countries are United States, United Kingdom and Japan [6]. It has been observed from the literature review that most of the CMMI level five companies" belong to India [7].

1.2 Software Outsourcing Contract

Software outsourcing relationship is based on a formal contract that is referred to as outsourcing contract. Khan et al [1] have stated that a good contract management can produce the desired outcomes from the outsourcing projects. A good contract directs toward achieving ultimate goals and mitigates the imminent issues which, lead toward the project failures [8]. Outsourcing contract may be bifurcated into pre-contract and post-contract for the best management of contract implementation [8]. A good contract relationship direct to achieve client trust which, is possible through managerial attention [9].

Software outsourcing contract management plays an important role between client and vendor organisations in software development outsourcing relationship. A poor contract leads toward failure of outsourcing project. To identify the challenges of outsourcing contract and its practices we have formulated the following research questions.

RQ 1. "What are the barriers/risks/challenges to be avoided by vendor organisations at various stages (pre-contract, during-contract and post-contract) in order to design and implement an effective offshore software development outsourcing contract?"

The findings of the RQ1 have already been published in our previous work [3].

RQ 2. What are the practices/solutions, to be adopted by OSDO vendors, for addressing the critical challenges/risks in offshore software development outsourcing contract management?

2. BACKGROUND

"Software outsourcing" is a contract based relationship between the client and vendor organisations where, the client organisations contract out all or part of software-development activities to the vendor organisation(s) and the vendor organisation(s) provides the agreed services in return for remuneration [1]. Software outsourcing has turned into a key supply-chain management practices, where boost in globalization has altered and advanced society to outsource essence and non-essence operational and other activities to nearshore and offshore locations [10]. Outsourcing has become a key practice of supply-chain management, but it has been proven to be hard to manage [11].

Efficient outsourcing contract management can play a vital role for achieving the planned goals and successful relationships between the software outsourcing parties [12]. Contract management has been described as a pivotal factor in the success of outsourcing projects [2, 13]. A well defined contract can ease strategic challenges [14]. The significance of contract is highlighted by contract theory in context of risk behavior of suppliers [13]. Poppo et al [13] stated that "the more complete the contract, the greater the specification of promises, obligations and actions for dispute resolution, which in turn limits the scope for opportunistic behaviour".

According to Popoo et al [13] the award of extra incentives can manage the poor performance of suppliers' products and services regarding contract management. Incomplete contract can be managed through a well-defined and well-documented complete contract and this can mitigate the contract risks such as environment change, poor communication and cultural differences [13]. Complete contract perform two functions such as control and coordination, where the control reduces the single and aberrant behaviour and coordination establishes a linkage among various units [15].

As usual, software outsourcing contract is not a risk-proof activity. Dum and Bradstreet [16] "have conducted a global survey and have found that 50% of the outsourcing projects failed due to poor contract management". We have identified, in our previous work, various critical challenges in the context of OSDO contract management from vendor's perspective [3], such as 'lack of project management', 'lack of outsourcing management', 'cultural and linguistic issues, 'non-competitive price', 'IPR and regulatory issues, 'lack of negotiation strategies, 'unforeseen contingencies', 'poor monitoring system' and 'failure to manage end-user expectations'.

It has been reported in the literature that majority of studies are focused on challenges of global software engineering management rather than identification of solutions/practices or techniques [17]. Hosain et al [18] have identified, through systematic literature review (SLR), the scrum practices in the context of global software development (GSD).

The literature review reveals that most of the work has been done in software outsourcing on challenges' identification but there is no sufficient literature on solutions of these identified challenges. This paper contributes in identification of solutions/practices for addressing the critical challenges faced by vendors in the management and execution of OSDO contract.

3. STUDY DESIGN

Our findings are based on the following two research methodologies:

- Systematic literature review (SLR)
- Questionnaire survey

First, we conducted SLR to identify the solutions/practices for the identified critical challenges. Secondly, we conducted questionnaire survey in OSDO industry to validate findings of the SLR and to find any new practice/solution apart from the identified ones. Details are given in the following sections.

3.1 Systematic Literature Review

"Systematic literature review (SLR) is a mean of investigating, evaluating and interpreting the existing relevant literature for some specific research question(s) in a more systematic way based on a pre-defined protocol that is aimed to reduce biasness. SLR is a secondary study and is more thorough than ordinary literature review as it is composed of three major phases", i.e., planning, conducting and reporting. [19].

We first conducted SLR process [19, 20] for the identification of practices for critical challenges in OSDO contract. We have developed the SLR protocol [21] where we have defined our search string, the relevant resources/on-line libraries to be searched, the inclusion and exclusion criteria, publication selection criteria and data extraction process.

The search terms conceded 114 research articles in primary selection in various digital libraries but after final review based on the inclusion/exclusion criteria, only 84 publications were selected as shown in Table 2. The inclusion and exclusion criteria were used to select the most relevant articles to the research question.

Table 2. List of publication selection.

Digital libraries	No. of publications	Primary selection	Final selection
IEEEXplore	148	49	38
ACM	70	02	02
ScienceDirect	1000	48	34
CiteSeerX	342	00	00
Google Scholar	88	15	10
Total	**1648**	**114**	**84**

We have searched the following digital libraries:

"IEEE Xplore (http://www.ieeexplore.ieee.org/Xplore/guesthome.jsp)

ACM Portal (http://www.dl.acm.org/)

ScienceDirect (http://www.sciencedirect.com)

CiteSeer Digital Library (http://www.citeseer.ist.psu.edu)

Google Scholar (http://www.scholar.google.com)"

Details of the inclusion/exclusion criteria and data extraction form are given in the protocol [21] and can be provided by the authors on request.

3.1.1 Data Synthesis

After completion of the data extraction phase, the data synthesis were performed by both reviewers (the authors) based on the predefined procedure. Initially we identified 124 practices in total. These were further reviewed and some of them were grouped together to form 119 practices in total. Finally, we found 91 practices after external review. These practices were validated through external reviewer. The changes were addressed as suggested by the reviewer. Finally, we have polished these groups and set these groups of practices across the identified critical challenges.

3.2 Questionnaire Survey

After conduction of the SLR, we conducted empirical study in outsourcing industry to validate findings of the SLR and to find any new practice apart from the identified ones. According to Kitchenham and Pfleeger [22] a survey research method is suitable for collecting self-reported qualitative and quantitative data from a huge number of respondents. A survey can be used for collecting data by using different techniques such as interview, questionnaire survey and others [23, 24]. Questionnaire survey is an empirical method use for collecting qualitative data from industry experts about some specific research area. We have preferred to use questionnaire survey for data collection due to various reasons such as collecting data from diverse range of respondents and accessible resources.

A questionnaire survey was designed based on our previous literature findings and followed the format used by other researchers [25-27]. For self-reported data collection a closed format questionnaires has been used in this research work. We have also included open ended questions in questionnaire survey to collect the tacit knowledge from the industry experts apart from the identified practices for critical challenges. This kind of survey assists to mitigate the risks of biasness relating to the researchers preconceptions and it encourages the participant to keep talking about a particular subject [28].

3.2.1 Data Collection

In this study we have tried to explore the experiences and viewpoints of software industry through our developed questionnaire. This study is considered qualitative in nature. Qualitative research method is pertained with studying objects in their natural setting [29]. Creswell [30] stated that "a qualitative researcher attempts to interpret a phenomenon based on the explanations that people bring to them". Ma [31] has stated that "Quantitative research use data that can be represented in the form of numbers or that can be immediately transported into numbers. In qualitative research, data then are represented as words and pictures, rather than numbers". Wohlin et al [29] have argued that "Qualitative research begins with accepting that there is a range of different ways of interpretation. It is concerned with discovering the causes noticed by the subjects in the study, and understanding their view of the problem at hand".

3.2.1.1 Questions: We have formulated various questions as given in our developed questionnaire. We have used seven point likert scale (Extremely satisfied, moderately satisfied, slightly satisfied, neither, slightly dissatisfied, moderately dissatisfied and extremely dissatisfied) for a purpose to take detailed feedback on the identified practices of SLR for the critical challenges.

The questionnaire is divided into four sections: Demographic data are provided in Section 1, challenges of outsourcing contract management are placed in Section 2, in Section 3 practices for each are challenges are reported and instruction of questionnaire submission are given in Section 4.

In order to preserve the privacy of the data and participants, a report of the ethical principles that the research team would obey was delivered to the participants. It was also ensured to the participant that the provided data would not be disclosed to anyone apart from the research team. Furthermore, it was ensured to the participants that the research team would not distribute/share the confidential data with anyone in a way that could disclose any participant's identity or organisation.

3.2.1.2 Questionnaire piloting: The questionnaire was piloted first before its distribution to the target population. It was piloted through 10 members of software engineering research group (SERG-UOM) at University of Malakand. We received some minor suggestions for improvements that were incorporated before its delivery to the target population.

3.2.1.3 Participants' selection: An invitation letter (consent letter) along with the short description of the research was posted on the following links for the distribution of the questionnaire survey.

a. LinkedIn Group (http://www.linkedin.com)

b. Groups on social media (e.g., www.facebook.com)

c. Software companies at Pakistan

d. We also invited for participations the authors of the industry papers selected through the SLR; emails were available in the published papers.

e. We also sent hard copies of the questionnaire to some experts at software industry in Pakistan upon receiving their consents.

3.2.2 Questionnaire Procedure

Questionnaires were carried out between December 2014 and January 2015. Prior to Questionnaire, each participant was sent questionnaire invitation letter. This letter outlined the main themes to be covered during the questionnaire, the expected duration, and measures which would be taken to ensure privacy and confidentiality.

All questionnaires were conducted through online, using the Google Docs free online tool.

3.2.3 Data Analysis Strategy

We received consents for participation in the survey from 103 software outsourcing experts in total, and we sent/shared the questionnaire with all these experts either in soft form or printed form in some cases. We received the responses from 94 experts only. Due to our pre-defined quality criteria, 06 responses were excluded. Thus the final sample of the completed questionnaires received was dropped down to 88 in total. Thus we received the response rate of 85%. Amongst these 65 were responses received from national/local experts in the industry and 29 responses received from foreign experts belonging to 15 different countries.

4. RESULTS AND DISCUSSION

In this section, we have discussed our findings of the SLR for identification of practices for mitigation/avoidance of the critical challenges faced by vendors in OSDO contract. The critical challenges, 09 in total, along with their relevant identifies practices are given in the subsequent sections.

4.1 Lack of Project Management

Project management can play an important role between the stockholders involved in OSDO contract. Buchta et al [32] have "defined various causes of poor project management such as weak relationship of client with outsourcing vendor organisation, cultural and linguistic issues. Remus et al [33] have discussed different standards of project management rating, e.g. high and low level design, risk analysis and testing". We have identified 17 practices as shown in Table 3 for mitigation/avoidance of the critical challenge 'lack of project management'.

4.2 Poor Monitoring System

It is argued that "weak contracting based on inadequate assessment of a vendor bid and backed up by poor monitoring systems, not only results in unanticipated, higher costs; it can create major problems for clients, too" [34]. The drafting of contract is important while monitoring of the work is even more important [33]. We have identified 07 practices as shown in Table 4 for mitigation/avoidance of the critical challenge 'poor monitoring system'.

4.3 Lack of Outsourcing Relationship Management

In outsourcing "software development a good relationship between the outsourcing parties can assist in outsourcing contract management. Herbsleb et al [35] have defined some of the problems in outsourcing relationships such as distance, poor documentation, culture differences and network problems. A poor relationship management between both the outsourcing parties can imply negative impact on the outsourcing contract management" [36]. We have identified 07 practices as shown in Table 5 for mitigation/avoidance of the critical challenge 'lack of outsourcing relationship management'.

Table 3. List practices for addressing lack of project management.

*CC # 1: Lack of project management			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-1.1	"Establish proper definition of project size, milestones and deliverables"	14	48
CCP-1.2	"Prior definition of draft that include project plan and project process".	14	31
CCP-1.3	"Proper coordination between client and vendor from start of the project and effective previous relationship between both the parties enhance the success of project management".	14	41
CCP-1.4	"Ensure technical ability and to train of project management team for outsourcing contract"	4	28
CCP-1.5	"Avoiding of the hidden cost and use of a cost estimation and cost mitigation models such as (COCOMO, COSYSMO, Function Point Analysis, Parametric Estimating, PRICE Systems etc.) to reduce the cost of software development that will lead toward better project management"	4	32
CCP-1.6	"Team leader motivation can reduce issues in project management"	4	33
CCP-1.7	"Establish internal audit, external audit mechanisms and use of proper control mechanism model such as (State-Transition Model of Trust Management, Role-based Access Control, etc.) for an effective project management"	10	25
CCP-1.8	"Negotiate with client over various issues, use of open communication and improvement of ongoing skills to reduce the communication gap".	5	36
CCP-1.9	"Establish detailed written agreement, proper documentation, work product and management of related record between both the parties involved"	5	34
CCP-1.10	"Proper transferring of knowledge/information between onsite and offshore teams involve in offshore outsourcing software development projects".	5	33
CCP-1.11	"Check your project management readiness for the global activity".	6	30
CCP-1.12	"Establish trust building activities between the outsourcing parties reduce chance of lack of project management"	5	27
CCP-1.13	"Implement proper scheduling that guarantee directly toward efficient project management"	2	33
CCP-1.14	"Efficient understanding of the client's language and culture"	4	23
CCP-1.15	"The record of pilot project and good track record direct project toward success"	4	31
CCP-1.16	"Proper infrastructure, definition of rules and regulations of vendor organization"	2	28
CCP-1.17	"Define intellectual property and copyright protection"	2	33

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 4. List of practices for addressing poor monitoring system.

*CC # 2: Poor monitoring system			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-2.1	"An effective control/governance on various project activities such as proper identification , estimation, evaluation and management planning, controlling and monitoring"	8	33
CCP-2.2	"Proper record keeping, efficient documentation, negotiation"	14	35
CCP-2.3	"Define proper monitoring system"	14	31
CCP-2.4	"Performance monitoring can be performed through a peer review process, involvement of experts and third party and use of latest technology"	15	20
CCP-2.5	"Monitoring the performance against objective criteria to achieve the goal"	1	36
CCP-2.6	"Proper schedule obeying reduce monitoring risk"	6	33
CCP-2.7	"Define process for system monitoring"	5	34

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 5. List of practices for addressing lack of outsourcing relationship management.

*CC # 3: Lack of outsourcing Relationship management			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-3.1	"Proper negotiation/ documentation between outsourcing parties"	2	52
CCP-3.2	"Setting/ obeying of terms and conditions by both the outsourcing parties involved"	15	35
CCP-3.3	"Culture understanding by both the outsourcing parties"	1	28
CCP-3.4	"Risk understanding and find immediate solutions"	1	31
CCP-3.5	"Follow tight timing schedule and in-time decision support system by both the outsourcing parties involved"	7	20
CCP-3.6	"Knowledge/ information sharing can improve outsourcing management activities"	4	27
CCP-3.7	"Trust building activities in outsourcing parties can reduce outsourcing risks"	1	30

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 6. List of practices for addressing culture and linguistic issues.

*CC # 4: Cultural and linguistic issues			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-4.1	"The training and interaction of staff on both sides about culture to ensure the quality of what is ultimately supplied and resolve culture issues"	6	39
CCP-4.2	"Knowledge sharing activities avoid a dispute of culture conflict"	6	27
CCP-4.3	"Define awareness about culture to reduce the gap"	17	31
CCP-4.4	"To avoid misunderstanding define negotiation policy"	17	36
CCP-4.5	"Arrangement of face to face meetings between the outsourcing parties involved"	17	30
CCP-4.6	"Define communication channels between stakeholders"	17	24
CCP-4.7	"Define hotline between client and supplier"	18	35
CCP-4.8	"Common methodologies and tools should be used to unite the efforts of different teams working on the project that have different languages and cultures"	2	33

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

4.4 Culture and Linguistic Issues

Culture and linguistic issues can weaken the outsourcing contract between stakeholders involved in OSDO contract. Culture differences can create hurdles in requirement specification and ongoing contract management. Krishna et al [37] have conducted two case studies where it has been found that cultural gap can be covered through culture understanding, training and hiring of skilled employees. This challenge can be handled through cultural knowledge, training and" scheduled/face to face meeting. We have identified 08 practices as shown in Table 6 for mitigation/avoidance of the critical challenge 'cultural and linguistic issues'.

4.5 Non-Competitive Price

The main advantage of offshore software development is low-cost and quality software development. A Non-competitive price can affect outsourcing contract management. It has been found that non-competitive price creates problems for client while the vendor organisation can enjoy considerable advantages from it [38]. Jiang et al [39] stated that vendor organization's main focus is to get the "lowest bidding price, the highest operating cost, and the shortest contract duration". We have identified 13 practices as shown in Table 7 for mitigation/avoidance of the critical challenge 'non-competitive price'.

4.6 IPR and Regulatory Issues

Lack of awareness about intellectual property rights, code of conduct and lack of governance can weak the concept of outsourcing contract management between both the outsourcing parties. Lee and Raisinghani et al [34, 40] have defined various issues of outsourcing contract, e.g., intellectual property matters, information security, staffing, etc. We have identified 09 practices as shown in Table 8 for mitigation/avoidance of the critical challenge 'IPR and regulatory issues'.

Table 7. List of practices for addressing Non-competitive price.

*CC # 5: Non-competitive price			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-5.1	ost reduction through proper negotiation and monitoring between outsourcing parties"	11	34
CCP-5.2	operly define Production costs and transaction costs"	11	31
CCP-5.3	involvement of third party to set the contract for its effective and timely npleteness"	11	23
CCP-5.4	efine clear accountability mechanism"	8	26
CCP-5.5	oper definition for fixed and variable cost setting"	8	24
CCP-5.6	efine suitable pricing policies to enhance outsourcing business"	8	38
CCP-5.7	e of latest technology / tools for proper cost estimation"	6	23
CCP-5.8	nsultancy with domain expert reduce the cost issues"	5	33
CCP-5.9	oper definition of rules and policies to handle non-competitive bidding issues"	2	26
CCP-5.10	oid hidden cost/ extra fees for services beyond the contract"	13	27
CCP-5.11	e of Pareto optimum (win-win) concept to reduce risk and maximize the benefit"	4	30
CCP-5.12	oper caring of schedule timing reduce extra charges and maximize the benefits"	14	34
CCP-5.13	well defined contract management reduce outsourcing cost"	1	38

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 8. List of practices for addressing IPR and regulatory issues.

*CC # 6: IPR and regulatory issues			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-6.1	"Develop indicators to measure Intellectual property (IP) assets"	11	35
CCP-6.2	"Use experienced advisors"	11	39
CCP-6.3	"Decide on a policy for copyright (who is allowed to use protected material and under which circumstances?)"	11	35
CCP-6.4	"Proper role definition for all employees"	11	19
CCP-6.5	"Define code of conduct for both stakeholders"	11	28
CCP-6.6	"Establish and communicate clear security guideline to protect business secret, data protection, IPR and exit management"	5	22
CCP-6.7	"Develop awareness of the role of IP in outsourcing parties"	5	25
CCP-6.8	"Clear process management mechanism can effectively improve IPR, visibility and quality"	21	33
CCP-6.9	"Formulate policies for handling invention/design made by employees"	5	28

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

4.7 Failure to Manage End-User Expectation

This kind of challenge arises due to improper communication, insufficient requirements and lack of employees' training. This may cause to initiate the problem when the end-user is not satisfied from the generated outcome of the project. Vagadia [41] stated that proper contract between the outsourcing parties can fulfill their desired expectations. Nakatsu et al [42] have defined that "expectations must be managed to ensure that the project deliverables will be consistent with the perceptions of the users". We have identified 09 practices as shown in Table 9 for mitigation/avoidance of the critical challenge 'failure to manage end-user expectation'.

4.8 Lack of Negotiation Strategies

"Lack of negotiation issue may arise due to lack of information sharing, improper channels that are using for communication and lack of cooperation between both the outsourcing parties. Tuten and Urban [43] have defined that improper communication between the client, and vendor organisations frail their outsourcing contract and finally leads toward failure of the whole" project. We have identified 09 practices as shown in Table 10 for mitigation/avoidance of the critical challenge 'lack of negotiation strategies'.

Table 9. List of practices for addressing Failure to manage end-user expectation.

*CC # 7: Failure to manage end-user expectation			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-7.1	"Understanding the exact requirements or risk avoidances can satisfy customer"	6	45
CCP-7.2	"Define policy to get all client requirements"	6	32
CCP-7.3	"Proper communication and pilot project record cover the gap of customer satisfaction"	10	31
CCP-7.4	"Arrange frequent visits to carry on contract management"	10	38
CCP-7.5	"Define customer service phone bank, hotline complaint phone to avoid the arising risks"	10	20
CCP-7.6	"Establish control mechanism and the ability to fulfill client requirements"	6	23
CCP-7.7	"Proper definition of quality management procedures for quality software production"	4	33
CCP-7.8	"Dividing the software development into parts to speed up can get user expectation"	11	36
CCP-7.9	"Defining and obeying of rules and regulation for service, time and budget move a project on right track"	6	39

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 10. List of practices for addressing Lack of negotiation strategies.

*CC # 8: Lack of negotiation strategies			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-8.1	"Establish both formal (open interview, survey, and meeting) and informal modes of communication e.g. (rich media like social media, etc.)"	25	31
CCP-8.2	"Use code of conduct for safe and sound communication"	1	34
CCP-8.3	"Provide training to team members about proper negotiation"	5	38
CCP-8.4	"Facilitation of renegotiation satisfy both outsourcing parties"	6	24
CCP-8.5	"Establish regular meeting calendar for outsourcing parties"	6	17
CCP-8.6	"The team members motivation and knowledge sharing can improve negotiation"	5	22
CCP-8.7	"Documentation of the meeting can assist the outsourcing parties"	2	34
CCP-8.8	"Arrangement of audio/ video chat to save the time and cost of project and improve communication between both the parties."	8	35
CCP-8.9	"Project goal specification lead toward positive negotiation"	8	28

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

4.9 Failure to Manage End-User Expectation

This "kind of challenge arises due to improper communication, insufficient requirements and lack of employees' training. This may cause to initiate the problem when the end-user not satisfied from the generated outcome of the project. Vagadia [41] stated that proper contract between the outsourcing parties can fulfill their desired expectations". Nakatsu et al [42] have defined that "expectations must be managed to ensure that the project deliverables will be consistent with the perceptions of the users". We have identified 09 practices as shown in Table 9 for mitigation/avoidance of the critical challenge 'failure to manage end-user expectation'.

4.10 Lack of Negotiation Strategies

"Lack of negotiation issue may arise due to lack of information sharing, improper channels that are using for communication and lack of cooperation between both the outsourcing parties. Tuten and Urban [43] have defined that improper communication between the client, and vendor organisations frail their outsourcing contract and finally leads toward failure of the whole" project. We have identified 09 practices as shown in Table 10 for mitigation/avoidance of the critical challenge 'lack of negotiation strategies'.

Table 11. List of practices for addressing unforeseen contingencies.

***CC # 9: Unforeseen contingencies**			
Practice No.	**Practice**	**% of SLR (n=84)**	**% of Q. Survey (n=88)**
**CCP-9.1	"Establish internal audit and external audit mechanisms"	33	42
CCP-9.2	"Proper arrangement of training for emergency situation"	21	35
CCP-9.3	"Knowledge sharing and educate staff members about project activities/objective and client culture"	12	35
CCP-9.4	"Organisation should develop policies, standards, milestones and review these on periodic basis"	8	32
CCP-9.5	"Recruitment of well qualified and professional staff members reduce future risks"	7	34
CCP-9.6	"Defined staff flexibility, switching of staff between sites and vendor capability reduce chance of unforeseen contingencies."	4	17
CCP-9.7	"Improve the performance and trust building to avoid failure incident which, arise due to requirement specification"	11	33
CCP-9.8	"Proper control on cost consumption and define procedure for audit policy and audit process"	5	20
CCP-9.9	"Proper planning for risk reduction and define their proper solution"	5	35
CCP-9.10	"To avoid hidden cost and information between the outsourcing parties"	2	27
CCP-9.11	"Create team of think tanks to provide solutions for any emerging situation"	6	27
CCP-9.12	"Proper coordination between outsourcing parties reduce the unforeseen risks"	13	41

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

Table 12. List of best practices of critical challenges in OSDO contract.

***CC # 1: Lack of project management**			
Practice No.	**Best Practice**	**%age of SLR (n=84)**	**ES % of Q. Survey (n=88)**
**CCP-1.1	"Establish proper definition of project size, milestones and deliverables"	14	48
CCP-1.2	"Prior definition of draft that include project plan and project process"	14	31
CCP-1.3	"Proper coordination between client and vendor from start of the project and effective previous relationship between both the parties enhance the success of project management".	14	41
CCP-1.4	"Establish internal audit, external audit mechanisms and use of proper control mechanism model such as (State-Transition Model of Trust Management, Role-based Access Control, etc.) for an effective project management"	10	25
CC # 2: Poor monitoring system			
CCP-2.1	"Proper record keeping, efficient documentation, negotiation"	14	35
CCP-2.2	"Define proper monitoring system"	14	31
CCP-2.3	"Performance monitoring can be performed through a peer review process, involvement of experts and third party and use of latest technology".	15	20
CC # 3: Lack of outsourcing relationship management			
CCP-3.1	"Setting/ obeying of terms and conditions by both the outsourcing parties involved"	15	35
CC # 4: Cultural and linguistic issues			
CCP-4.1	"Define communication channels between stakeholders"	17	31
CCP-4.2	"To avoid misunderstanding define negotiation policy"	17	36
CCP-4.3	"Arrangement of face to face meetings between the outsourcing parties involved"	17	30
CCP-4.4	"Define hotline between client and supplier"	17	24
CCP-4.5	"A continuous linkage between client and supplier with a shared, mutual understanding and the goals is likely to be the most successful source to cover culture gap".	18	35

*CC # 1: Lack of project management			
Practice No.	**Best Practice**	**%age of SLR (n=84)**	**ES % of Q. Survey (n=88)**
CC # 5: Non-competitive price			
CCP-5.1	"Cost reduction through proper negotiation and monitoring between outsourcing parties"	11	34
CCP-5.2	"Properly define production costs and transaction costs"	11	31
CCP-5.3	"An involvement of third party to set the contract for its effective and timely completeness"	11	23
CCP-5.4	"Avoid hidden cost/extra fees for services beyond the contract"	13	27
CCP-5.5	"Proper caring of schedule timing reduce extra charges and maximize the benefits"	14	34
CC # 6: IPR and regulatory issues			
CCP-6.1	"Develop indicators to measure intellectual property (IP) assets"	11	35
CCP-6.2	"Use experienced advisors"	11	39
CCP-6.3	"Decide on a policy for copyright (who is allowed to use protected material and under which circumstances?)"	11	35
CCP-6.4	"Proper role definition for all employees"	11	19
CCP-6.5	"Define code of conduct for both stakeholders"	11	28
CCP-6.6	"Clear process management mechanism can effectively improve IPR, visibility and quality"	21	33
CC # 7: Failure to manage end-user expectation			
CCP-7.1	"Proper communication and pilot project record cover the gap of customer satisfaction"	10	31
CCP-7.2	"Arrange frequent visits to carry on contract management"	10	38
CCP-7.3	"Define customer service phone bank, hotline complaint phone to avoid the arising risks".	10	20
CCP-7.4	"Dividing the software development into parts to speed up can get user expectation"	11	36
CC # 8: Lack of negotiation strategies			
CCP-8.1	"Establish both formal (open interview, survey, and meeting) and informal modes of communication e.g. (rich media like social media, etc.)"	25	31
CC # 9: Unforeseen contingencies			
CCP-9.1	"Establish internal audit and external audit mechanisms"	33	42
CCP-9.2	"Proper arrangement of training for emergency situation"	21	35
CCP-9.3	"Knowledge sharing and educate staff members about project activities/objective and client culture"	12	35
CCP-9.4	"Improve the performance and trust building to avoid failure incident which, arise due to requirement specification"	11	33
CCP-9.5	"Proper coordination between outsourcing parties reduce the unforeseen risks"	13	41

** CC stands for Critical Challenge, **CCP stands for Critical Challenges' Practice.*

4.11 Unforeseen Contingencies

"A good and durable outsourcing contract can be managed between the client and vendor organisations through detailed policy and contingency plan definition. Stefan [44] has defined that uncertainty; unforeseen contingencies and contradictory information are major issues faced by the vendor organisations in proper contract" management. We have identified 12 practices as shown in Table 11 for mitigation/avoidance of the critical challenge 'unforeseen contingencies'.

5. LIMITATIONS

We have used two different methods for identification of the practices. First we conducted SLR to review the literature in a systematic way. We have followed all the steps of the SLR process. However due to large number of papers and search

engines retrieval mechanisms, we may have missed some of the relevant papers. However this is not a systematic omission.

Secondly, we conducted questionnaire survey in OSDO industry using online and offline tools in order to validate findings of the SLR and to find out any new practice(s) apart from the identified ones. We followed all the standard guidelines, during the design, piloting, distribution to the target population and analysis of the finally selected completed questionnaires. However due to space limitations we are unable to attach the full questionnaire in the paper.

Our survey received responses from a total of 88 OSDO experts. This sample include 29 foreign and 59 local experts in Pakistan For generalization, it would be better if we should have involved more foreign/international participants instead of the local ones but it was not possible due to limited resources and time at this stage. We have utilized every possible source to approach foreigner experts through different social and professional network groups available on internet; even we have emailed them personally to participate in our questionnaire survey. However their participation was purely on voluntary basis. Due to limited number of respondents from foreign, one should be careful while generalizing the results. To internal validity one possible threat is that for any specific response, that may have not in fact described underlying reasons to report outsourcing contract challenges. This threat may not have been able for us to control independently. In these responses the participants would not be supposed to give the original reasons of OSDO contract.

We have used questionnaires and one weakness of survey method is that participants are given with a list of possible options as we provided the list of practices, identified through SLR earlier, of the critical challenges. This exercise limits the respondents to those reported practices where the respondents only concentrate on the practices given in the list. To overcome this issue we also motivated the respondents to provide other practices for the critical challenges apart from those mentioned in the questionnaire.

Overall there is no major difference between the findings of the SLR and questionnaire survey which gives strengths to the generalization and validity of our findings.

6. CONCLUSIONS

We have found 17 practices for addressing the challenge ‘lack of project management’, 07 practices for addressing ‘poor monitoring system’, 07 practices for ‘lack of outsourcing relationship management’, 08 practices for ‘cultural and linguistic issues ', 13 practices for ‘non-competitive price’, 09 practices for ‘IPR and regulatory issues ', 09 practices for ‘failure to manage end-user expectation’, 09 practices for ‘lack of negotiation strategies’ and 12 practices for ‘unforeseen contingencies’. We have categorized these practices into low-rating and high-rating (best practices) on the basis of its occurrences. We will consider those practices as best practices whose occurrence is greater than or equal to 10% in both SLR and empirical study. We have found 34 best practices using this criterion. Based on this criterion the best practices for each of the critical challenge are given in the Table 12.

We have collected all these practices through the predefined procedure of SLR [45]. In total 91, identified practices can assist vendor organisation to avoid/mitigate the challenges for managing the outsourcing contract.

The identified challenges can influence the performance of outsourcing contract management of both the parties. To avoid or mitigate the effect of these challenges the identified practices can assist the outsourcing vendor organisation in this context. The proper implementation of these practices can improve efficiency of outsourcing vendor through avoidance/mitigation of the critical challenges.

The ultimate goal of this research is to develop software outsourcing contract management model from vendor’s perspective that will assist vendor organisations in managing a good contract with client organisations during the whole period of contract. This paper contributes to the 2nd phase of our proposed model.

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8. REFERENCES

1. Khan, A.W. & S.U. Khan. Critical success factors for offshore software outsourcing contract management from vendors' perspective: An exploratory study using a systematic literature review. *Institution of Engineering & Technology (IET) Software* 7(6): 327-338 (2013).
2. Goo, J. Kishore, H.R. Rao & K. Nam. The role of service level agreements in relational management of information technology outsourcing: an empirical study. *Management Information System (MIS) Quarterly* 33(1): 119-145 (2009).
3. Khan, A.W. & S.U. Khan. *Critical challenges in execution of offshore software outsourcing contract from vendors' perspective: A systematic literature review*. In: *5th International Conference on Information and Communication Systems (ICICS-2014) of IEEE*. Jordan University of Science & Technology (JUST), Irbid, Jordan (2014).
4. United-Nations, World Investment Report. *The Shift Towards Services*, New York and Geneva. (2004).
5. Palvia, S.C.J. Global Outsourcing of IT and IT Enabled Services: Impact on US and Global Economy. *Information Technology Case and Applications* 5(3): 1-8 (2003).
6. Oza, N.V., T. Hall, A. Rainer & S. Grey, Trust in software outsourcing relationships: An empirical investigation of Indian software companies. *Information and Software Technology* 48(5): 345-354 (2006).
7. SEI. *Process Maturity Profile of the Software Community*. Software Engineering Institute, Pittsburgh, PA, USA (2002).
8. Lee, M.K.O. IT outsourcing contract: practical issues for management. *European Management Journal* 96(1): 15-20 (2003).
9. Harris, A., L.C. Giunipero & G.T.M.Hult, Impact of organizational and contract flexibility on outsourcing contracts. *Industrial Marketing Management* 27: 373-384 (1998).
10. Wiengarten, F., M. Pagell & B. Fynes. The importance of contextual factors in the success of outsourcing contracts in the supply chain environment: the role of risk and complementary practices. *Supply Chian Management: An International Journal* 18(6): 630-643 (2013).
11. Handley, S.M. & W.C.J. Benton. The influence of task- and location-specific complexity on the control and coordination costs in global outsourcing relationships. *Journal of Operations Management* 31(3): 109-128 (2013).
12. Khan, A.W. & S.U. Khan. Outsourcing contract management model (OCMM). In: *13th International conference on Product-Focused Software Development and Process Improvement (Profess)*. Madrid, Spain (2012).
13. Poppo, L. & T. Zenger. Do formal contracts and relational governance function as substitutes or complements? *Strategic Management Journal* 23(3): 707-726 (2002).
14. Handley, S.M. & W.C.J. Benton. Unlocking the business outsourcing process model. *Journal Operations Management* 27(5): 344-371 (2009).
15. Mellewigt, T., A. Madhok, & A. Weibel. Trust and formal contracts in interorganizational relationships –substitutes and complements. *Managerial & Decision Economics* 28(8): 833-847 (2007).
16. Dun. and Bradstreet. Dun & Bradstreet's Barometer of Global Outsourcing. *Journal of Information* 3(2): 33-45 (2000).
17. Smite, D., C. Wohlin, T. Gorschek & R. Feldt. Empirical evidence in global software engineering: A systematic review. *Empirical Software Engineering* 15(1): 91-118 (2010).
18. Hossain, E., M. Ali-Babar & P. Hye-young. Using scrum in global software development: A systematic literature review. In: *4th IEEE International Conference on Global Software Engineering, ICGSE09*. Lero, Limerick, Ireland (2009).
19. Kitchenham, B. & C. Charters. *Guidelines for performing Systematic Literature Reviews in Software Engineering*, Keele University and Durham University Joint Report (2007).
20. Niazi, M. Do systematic literature reviews outperform informal literature reviews in the software engineering domain? an initial case study. *Arabian Journal for Science and Engineering* 40(3): 11-29 (2015).
21. Khan, A.W. & S.U. Khan. Systematic literature review protocol for the identification of practices in software outsourcing contract. In: *International Multi-Topic Conference (IMTIC'15)*, Springer: Mehran University, Hyderabad, Pakistan (2015).
22. Kitchenham, B. & S.L. Pfleeger. *Principles of Survey Research*, *Parts 1 to 6*. Software Engineering Notes (2002).
23. Lethbridge, T.C. Studying software engineers: Data collection techniques for software field studies. *Empirical Software Engineering* 10(1): 311-341(2005).
24. Niazi, M, S. Alshayeb & A. Hroub. Empirical investigation of the challenges of the existing tools used in global software development projects. *Institution of Engineering & Technology (IET) Software* 12(10): 112-134 (2015).
25. Niazi, M. A *Framework for Assisting the Design of Effective Software Process Improvement Implementation Strategies*. University of Technology, Sydney (2004).
26. Shivakumar, P. & K.K..Vijapurapu. Tacit knowledge preservation at vendor organizations in offshore outsourcing software development. In:

Blekinge Institute of Technology. School of Computing SE-371 79 Karlskrona Sweden, p. 80 (2014).
27. Khan , S.U., M. Niazi & R. Ahmad. An empirical investigation of success factors for offshore software development outsourcing vendors. *Institution of Engineering & Technology (IET) Software* 6(1): 1-15 (2012).
28. Patton, M.Q, *Qualitative Evaluation and Research Methods*. SAGE Publications, USA (1990).
29. Wohlin, C., M. Höst & K. Henningsson. Empirical research methods in web and software engineering. *Web Engineering* 10: 409-430 (2006).
30. Creswell, J. *Research Design, Qualitative and Quantitative Approaches*. Sage Publications, London, UK (1994).
31. Ma, N. *Building a Narrative Based Requirements Engineering Mediation Model (NREMM)*. PhD thesis, School of Computer Science, Faculty of Engineering and Information Sciences, University of Hertfordshire (2009).
32. Buchta, D., B.H. Lin., H.Roder & R.Ziegler. IT-outsourcing und implikationen fur den standort deutschland. *At Kearney* 12(3): 22-38 (2004).
33. Remus, U. & M. Wiener. Critical success factors of offshore software development projects: The perspective of German-speaking companies, In: *Critical Success Factors of Offshore Software Development Projects*. Deutscher Universi-tätsverlag 22: 326-355 (2006).
34. Lee, M.K.O. IT outsourcing contracts: practical issues for management *Industrial Marketing Management & Data Systems* 96(1): 15-20 (1996).
35. Herbsleb, J. & D. Moitra. Global software development. *IEEE Software* 13:16-20 (2001).
36. Khan, S.U., M. Niazi & R. Ahmad. Barriers in the selection of offshore software development outsourcing vendors: an exploratory study using a systematic literature review. *Information and Software Technology* 53(2): 693-706 (2011).
37. Krishna, S., S. Sahay & G. Walsham. Managing cross-cultural issues in global software outsourcing. *Communications of the ACM* 47(4): 62 - 66 (2004).
38. Shi, L. & A. Susarla. Relational contracts, reputation capital, and formal contracts: evidence from information technology outsourcing, In *Seminars at the University of Washington, Department of Economy* (2010).
39. Jiang, B., T. Yao & B. Feng. Valuate outsourcing contracts from vendors' perspective: a real options approach. *Decision Sciences* 39(3): 31-48 (2008).
40. Raisinghani, M.S., S.B. Hickerson, B.M. Marshelle & H. Michael. Information technology/systems offshore outsourcing: Key risks and success factors. *Journal of Information Technology Research* 1(1): 72-92 (2008).
41. Vagadia, B. Strategic outsourcing: risks, rewards and relationships. *Management for Professionals* 22: 81-91 (2012).
42. Nakatsu, R. & C. Iacovou. A comparative study of important risk factors involved in offshore and domestic outsourcing of software development projects: a two-panel Delphi study. *Information & Management* 46(1): 57-68 (2009).
43. Tuten, T.L. & D.J. Urban. An expanded model of business-to-business partnership formation and success. *Industrial Marketing Management* 30: 149-164 (2001).
44. Stefan, C. Contract roles in outsourcing activities, evidence from operations outsourcing in real estate management. *Decision Support System* 12(3): 66-75 (2011).
45. Barbara, K. & B. Pearl. A systematic review of systematic review process research in software engineering. *Inforamtion and Software Technology* 55(2): 2049-2075 (2013).

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Research Article

Motivators in Green IT-outsourcing from Vendor's Perspective: A Systematic Literature Review

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Abstract: Green IT is becoming an important concept for the organization since over the last decade, however, not every organization is completely ready to implement Green IT initiatives. Therefore, it is necessary to develop an easy-to-use but comprehensive self-assessment to evaluate organizational readiness to implement Green IT. We have reviewed the literature through systematic literature review (SLR) process to identify motivators in Green IT-outsourcing from vendor's perspective. Our final sample of publication selection comprises 82 papers in total. We have identified, through the SLR, a list of 9 motivators. Six of these motivators were identified as significant motivators. These significant motivators are 'energy efficiency', 'development and use of environment-friendly softwares', 'overall business cost reduction', 'promoting reusability and sustainability both in terms of hardwares and softwares', 'improving quality of services' and 'deployment of virtualization strategies with respect to hardwares, softwares and processes'. We have further analyzed the identified motivators on the basis of different variables including, study strategy/methodology and period of years. We have found more similarities than differences in the identified motivators across study strategy/methodology and period of years.

Keywords: Green IT, motivators, systematic literature review, outsourcing vendors

1. INTRODUCTION

Green IT has recently emerged into an active research area in the information systems (IS) discipline. The term 'green IT' is defined as "optimal use of Information and Communication Technology ICT for managing the environment sustainability of enterprise operations throughout their life cycles" [1]. Green IT is becoming an important concept for outsourcing vendor organizations, however, not every organization is completely ready to apply Green IT initiatives. Therefore, it is necessary to develop an easy-to-use but comprehensive self-assessment to evaluate organizational readiness to implement Green IT. The primary goal of the emerging discipline of IT sustainability is to enable firms to use computing resources more efficiently while maintaining or increasing overall performance. The first wave of these efforts is commonly identified as "green computing" where the emphasis has been primarily minimizing power usage for datacenters and technical equipments (such as desktops, projectors) [2]. The benefits of green computing in terms of reducing power consumption and corporate carbon footprints are direct and relatively rapid to achieve [2]. However, to move beyond internally focused green-computing initiatives to the realm of competitive advantage and corporate sustainability, more attention needs to be directed to how a second wave of sustainable IT practices can align with and enable corporate sustainability strategy.

Software behavior can significantly affect computer energy efficiency in everything from small devices up to servers in data centers. However, if Green IT software's can use the software developers, then it can reduce the energy consumption of drivers and applications [3].

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Several companies have recently announced plans to build "green" datacenters, i.e. datacenters partially or completely powered by renewable energy. These datacenters will either generate their own renewable energy or draw it directly from an existing nearby plant [4]. Besides reducing carbon footprints, renewable energy can potentially reduce energy costs; reduce peak power costs, or both. However, certain renewable fuels are intermittent, which requires approaches for tackling the energy supply variability [4]. One approach is to use batteries and/or the electrical grid as a backup for the renewable energy [4]. For highest benefits, green datacenter operators must intelligently manage their workloads and the sources of energy at their disposal [4].

However, Green IT outsourcing has several benefits but it faces different challenges such as: Recently, energy efficiency or Green IT has become a hot issue for many IT infrastructures as they attempt to utilize energy efficient strategies in their enterprise IT systems in order to minimize operational costs. Networking devices are shared resources connecting important IT infrastructures, especially in a data center network they are always operated 24/7 which consumes a huge amount of energy, and it has been obviously shown that this energy consumption is largely independent of the traffic through the devices. As a result, power consumption in networking devices is becoming more and more a critical problem, which is of interest for both research community and general public [5].

The environmental impact of the healthcare sector has become an important factor globally and is continuing to draw the attention of regulators. The energy use of the healthcare sector (whose largest sub-segment is `hospitals') has been growing due to many factors. These factors include the rapid growth and adoption of Information and Communication Technology (ICT) in healthcare [6]. Outsourcing, co-location and cloud computing, energy and cost-efficiency, environmental and legislative issues: running a data center in a constantly changing world is a huge challenge [7]. Looking at high power data center applications on medium voltage level, the biggest challenge was to find a green (SF6 free) switchgear with a small footprint, a maintenance free mechanism with a high number of operations, high reliability and that is easy to scale [7]. As a natural evolution of services computing, cloud computing has provided a promising way, for delivering flexible and cheap computing resources, via cloud platform. However, the computing resources held by a cloud platform are usually limited, compared with the nearly unlimited resource requirements from various end users [8]. As the primary energy source, coal is widely used for power generation, which causes serious concerns related to the environment and sustainability. Thus, renewable energy sources have been developed significantly over the past decade. Unfortunately renewable energy tends to be variable and uncertain because of the prime movers (i.e., wind and solar) and the dependence on natural and neurological conditions [9].

We have formulated the following research questions in order to understand the motivators for Green IT outsourcing from vendor's perspective.

RQ 1: What are the motivators, as identified in the literature, for green IT outsourcing?

RQ 2: Do the identified motivators vary based on study strategies used?

RQ 3: Do the identified motivators vary with the passage of time?

2. BACKGROUND

Green IT or Green Computing has emerged as a fast growing business paradigm in recent years in order to develop sustainable softwares and energy-efficient peripheral devices. With the constant evolution of technology and the world critical environmental status, all private and public information technology (IT) businesses are moving towards sustainability [10]. Green IT simply means using technology efficiently, while taking into account the triple bottom line: "economic viability, social responsibility and environmental impact" [10]. Reduction in energy consumption over the full equipment life cycle as the prime motivator for "green" application design; with energy reduction as the best measure of "green-ness" [11]. Organizations are now actively pursuing Green IT solutions for a multitude of reason and benefits, including reduce power consumption, lower costs, lower carbon emissions and environmental impact, improved systems performance and use, increased collaboration and interaction amid constituents, space savings, and an agile workforce [12]. IT virtualization reduces costs for hardware,

improves software testing and deployment, reduces energy and physical space use, and increases the flexibility of hardware investments [12]. Similarly, automation of business processes through virtualization (e.g., online electronic commerce) enables increased utilization of business resources, time and space flexibility for the participants and operations of the process along with reduction in cost and improved service delivery to customers [12]. Sensory readiness, relationship readiness, synchronism readiness, identification and control readiness, champion support, resource commitment, firm size, regulatory support and competition intensity are important antecedents of undertaking Green IT initiatives via virtualization [12]. Advantages of Green IT to the environment clearly grip reduce the cost, effectiveness of energy, improved the interaction of stakeholders, reduction in reuse, improvement, variety of image and devotion [13].

From the experience of several industrial trials on smart grid with communication infrastructures, we expect that the traditional carbon fuel based power plants can cooperate with emerging distributed renewable energy such as wind, solar, etc, to reduce the carbon fuel consumption and consequent green house gas such as carbon dioxide emission [14]. As a new generation green lighting source, the light emitting diode (LED) is rapidly replacing traditional incandescent and fluorescent light sources [15]. Apart from providing energy savings, the use of LED lighting technology creates scope for an innovative optical wireless communication technology known as visible light communication (VLC), which takes advantage of the superior modulation capability of LEDs to transmit data through a wireless channel [15]. VLC is capable of concurrently providing communication as well as illumination [15]. Hitec Power Protection, a leading manufacturer of rotary Uninterruptible Power Supply (UPS) systems using flywheel energy storage is active in the data center business for over 20 years [7]. One of their integrated solutions operates on medium voltage level and is the perfect green and efficient solution for high power data centers [7]. Attribute-based encryption (ABE) with outsourced decryption not only enables fine-grained sharing of encrypted data, but also overcomes the efficiency drawback (in terms of cipher text size and decryption cost) of the standard ABE schemes [16].

IT energy consumption sustainability is important from an economic, societal and environmental perspective for organizations [17]. These three dimensions are overlapping factors for sustainability, but very often the economic and societal are ultimately constrained by the environment [17]. Energy efficient software can play an important role in these three overlapping spheres of sustainability [17]. Green IT infrastructures are responsible for 2% of the CO_2 world emissions and for the greenhouse effect, which is the first reason of the global warming [17].

A number of researchers have highlighted the importance of Green IT in the context of outsourcing, e.g.:

- Energy saving can be achieved by adopting renewable energy resources or improving design of certain hardware (e.g., power amplifier) to make it more energy-efficient, the cost of purchasing, replacing, and installing new equipment (including manpower, transportation, disruption to normal operation, as well as associated energy and direct cost) is often prohibitive [18].
- In the recent years, many organizations have started to consider the importance of green manufacturing and take steps towards this new responsibility. Manufacturers try to achieve an end of life, a cost-effective and a green solution, enabling them to answer effectively to the future legislation regarding responsibility of their products. The authors [19] argue that context re-manufacturing, as a solution for green manufacturing, can be defined as the reconstructing or reassembling of a used product by using a combination of process. In order to be feasible, re-manufacturing should answer not only to the legislation, but also it should be profitable for the companies [19]. So, from the phase of conception, it is necessary to design the products such that they can be easily disassembled, cleaned, tested, and re-assembled to have a new life and a new value [19].
- In the current business environment, the competitiveness of companies and their supply chain depends not only on lowest cost, high quality, reduced lead time, and high service level, but also on their ability to avoid and

overcome the numerous disturbances that jeopardize their performance [20].

- Gu et al [21], argue that "having greener software practices can have advantages for hosting service providers and for their customers. For example, green practices for developing green software in data centers could be investigated".
- Recent changes in environmental legislation have focused company thinking on business practices, particularly concerning the importance of integrating environmental concerns like outsourcing and procurement in supply chain networks [22]. Supply chain partners are becoming progressively more accountable not only for their internal practices, but also for their suppliers' behavior because implementing a "greener" supply chain is far from an individual strategy [22].
- Al-Hasib and Sharmin [23] argue that Green IT paradigm also necessitates experts in the industry and the academia to focus on green software development in order to develop high quality and energy efficient softwares. They have proposed energy efficient software development framework to design energy efficient software.
- Li and Zhou [24] have conducted survey on state of the art in green computing and have identified various generic issues. These include the modeling and evaluation of energy efficiency, energy-awareness and green networking.
- Information Technology (IT) is at the heart of every successful modern business. IT is so pervasive, that energy efficiency through the implementation of Green IT has moved to center stage for many companies in their pursuit of helping make a difference for the environment [25].
- For mobile operators in particular, another motivation and objective of "green" approaches is to gain extra commercial benefits, mainly by reducing operating expenses related to energy cost [26].
- Ambtman [13] has worked on Green IT auditing to assess the greenness of an organization through auditing process and proposed Green IT control framework. The research is based on findings from the literature through ordinary literature review and questionnaire survey in Green IT industry. The results conclude that "In order to achieve Green IT objectives, Green IT initiatives within organizations should include Green management, procurement, use and disposal. For providing a certain level of assurance on the effectiveness of these mechanisms, performing a Green IT audit is considered useful. However, in practice the level of auditor involvement and hence level of assurance in Green IT audits is limited" [13].

We have used systematic literature review (SLR) as a research methodology to dig-out the motivators in Green IT outsourcing from vendor's perspective. Till date, no SLR study in this domain has been published for identification of motivators that have positive impact in Green IT-outsourcing, which shows the novelty of our research. The findings of the paper will assist vendor organizations to be aware about all possible motivators that have positive impact in Green IT outsourcing. Timely awareness about the motivators in Green IT outsourced project will also necessitate towards findings of the strategies, practices and tools, for addressing the challenges faced to Green IT outsourcing vendors, which we plan to do in future.

3. RESEARCH METHODOLOGY

A Systematic Literature Review (SLR) [27] process was used for data collection, because it is more thorough, less biased, rigorous and open as compared to ordinary literature review [27]. In finding, evaluating and summarizing all available evidences on a specific research question, a systematic review may provide a greater level of validity in its findings than ordinary literature review. A number of researchers [28-31] have used the SLR approach for reviewing the literature. Protocol development is the first phase of the SLR process and it describes planning of the review. In this connection a systematic review protocol was written first to describe the plan for the review. Details of the various steps in our SLR methodology are discussed in the following sub sections:

3.1 Research/Problem Identification

Identification of motivators in Green IT outsourcing from vendor perspectives is the main goal of this research. We have formulated the research questions given in the Introduction section of the paper.

3.2 Searching of the Literature

A trial search string was initially used in different digital libraries in this phase. Based on the available access, the digital libraries IEEE Explore (http://ieeexplore.ieee.org), ScienceDirect (http://www.sciencedirect.com), Google Scholar (http://scholar.google.com.pk), and CiteSeer (http://citeseerx.ist.psu.edu) were used to carry out the search phase of the SLR. The final list of sources searched, their search terms, and the number of publications found for each resources are listed in Table 1. We have selected these resources based on our previous SLRs [29-32] experiences and discussions with our colleagues at the University.

("Green Software" OR "Energy-efficient software" OR "sustainable software") OR ("Green computing" OR "Green IT-outsourcing")

Table 1 presents the overall final list of resources that we have searched during this phase and found number of publications in each digital libraries. A similar approach has been used by other researchers [29-31, 33-35].

3.3 Literature Selection

3.3.1 Inclusion Criteria

The following inclusion criteria have been used for the selection of relevant papers:

- Research work that describe web Green IT from vendors point of view.
- Research papers that describe challenges in Green IT-outsourcing.
- Research papers that describe motivators or success factors for Green IT-outsourcing.
- Research work that describe the practices/solutions for Green IT-outsourcing.
- Research work that describe strategies/ paradigms for developing Green softwares.
- Research work that describe tools/ technologies for the development of Green softwares.
- Research papers that describe the design and prototype development of distributed data-intensive service-oriented architectures – a key technology for Green IT.

3.3.2 Exclusion Criteria

The following exclusion criteria have been used for the selection of relevant papers:

- Papers/articles/books etc not following the inclusion criteria have been excluded.

3.4 Publication Quality Assessment

We have performed the publication quality assessment after the final selection of publications. During the selection process of studies, some questions were check listed to ensure the quality of the selected studies. The reason of applying these quality criteria is to facilitate the studies selection process and to ensure that only relevant papers are being selected. The questions used in our settled quality criteria were:

- Is it clear how the motivators were measured in Green IT-outsourcing relationship? (Yes/No/Partially)
- Is it clear how the motivators in the selection of Green IT-outsourcing vendor were identified? (Yes/No/Partially)

For validation purpose a secondary reviewer scored a small subset of the selected publications. By using publication quality assessment questions, studies that are not scholarly reviewed were excluded. Only those studies are selected that aims the motivators in Green IT-outsourcing relationships. Similarly, studies that do not provide persuasive results in motivators in the aspects of Green-IT outsourcing relationships were excluded.

During the search phase of the SLR, initially we found 1676 papers as shown in Table 1. After reviewing through title and abstract of each of the identified paper we selected 210 papers as our primary selection. After applying the aforementioned criteria, we have selected 82 papers as our final selection. Thus we found 82 papers in total as our final sample for the data extraction phase, as shown in Table1.

3.5 Data Extraction and Synthesis

In the data extraction phase of the SLR, the data was extracted from each paper of our final sample of publication on a pre-defined data extraction

Table 1 Data sources and search strategy for motivators.

Name of Digital Library	Search Strategy	Date of Search	Year Covered by Search	No. of Publication Found	Initial Selection Decision	Final Selection Decision
IEEE Explore	("Green Software" OR "Energy-efficient software" OR "sustainable software") OR ("Green computing" OR "Green IT-outsourcing")	1st July 2013	All	1,167	68	28
Science Direct				397	104	23
Google Scholar		27-Nov-13		87	31	25
CiteseerX				25	07	06
			Total	1676	210	82

Table 2 List of motivators in green IT outsourcing identified through SLR.

S. No.	Motivators	Paper-ids (details are given at the Table 5)	Total Papers = 82	
			Freq	%
1	Energy efficiency	P-1, P-2, P-3, P-4, P-5, P-6, P-7, P-8, P-9, P-10, P-11, P-12, P-13, P-14, P-15, P-16, P-17, P-18, P-19, P-20, P-21, P-22, P-23, P-24, P-27, P-28, P-29, P-30, P-34, P-35, P-36, P-37, P-38, P-39, P-40, P-41, P-42, P-43, P-44, P-45, P-47, P-48, P-49, P-50, P-51, P-52, P-53, P-54, P-55, P-56, P-57, P-58, P-59, P-60, P-61, P-62, P-63, P-64, P-65, P-66, P-67, P-68, P-69, P-70, P-71, P-72, P-73, P-74, P-75, P-76, P-77, P-78, P-79, P-80, P-81, P-82	76	93
2	Development and use of environment-friendly softwares	P-1, P-2, P-3, P-4, P-5, P-6, P-7, P-8, P-9, P-10, P-11, P-12, P-13, P-14, P-15, P-16, P-17, P-18, P-19, P-20, P-21, P-22, P-23, P-24, P-25, P-26, P-27, P-28, P-29, P-30, P-31, P-32, P-36, P-37, P-38, P-39, P-40, P-41, P-43, P-44, P-45, P-46, P-49, P-50, P-51, P-52, P-53, P-54, P-55, P-56, P-57, P-58, P-59, P-60, P-61, P-62, P-63, P-64, P-65, P-66, P-67, P-68, P-69, P-70, P-71, P-72, P-73, P-75, P-76, P-77, P-78, P-79, P-80, P-81, P-82	75	91
3	Overall business cost reduction	P-1, P-2, P-3, P-4, P-5, P-6, P-7, P-8, P-9, P-10, P-11, P-12, P-13, P-14, P-15, P-16, P-17, P-18, P-19, P-20, P-21, P-24, P-25, P-28, P-29, P-30, P-31, P-33, P-36, P-37, P-40, P-41, P-43, P-44, P-45, P-47, P-49, P-50, P-51, P-52, P-53, P-54, P-55, P-56, P-57, P-58, P-59, P-60, P-62, P-63, P-64, P-65, P-66, P-68, P-70, P-71, P-72, P-75, P-76, P-77, P-78, P-79, P-81, P-82	64	78
4	Promoting reusability and sustainability both in terms of hardwares and softwares	P-1, P-3, P-4, P-5, P-6, P-8, P-13, P-24, P-25, P-27, P-31, P-32, P-33, P-36, P-37, P-38, P-39, P-43, P-46, P-49, P-50, P-51, P-52, P-53, P-54, P-55, P-56, P-57, P-58, P-59, P-60, P-65, P-67, P-68, P-69, P-70, P-71, P-72, P-75, P-76, P-77, P-78, P-79, P-80, P-82	45	55
5	Improving quality of services	P-1, P-3, P-4, P-6, P-7, P-8, P-9, P-19, P-20, P-21, P-22, P-24, P-27, P-28, P-31, P-33, P-36, P-37, P-38, P-41, P-46, P-49, P-50, P-52, P-53, P-55, P-56, P-59, P-65, P-66, P-67, P-68, P-69, P-70, P-71, P-73, P-75, P-76, P-77, P-79, P-80, P-81, P-82	43	52
6	Deployment of Virtualization strategies with respect to hardwares, softwares and processes	P-1, P-2, P-4, P-5, P-6, P-7, P-8, P-10, P-12, P-15, P-16, P-18, P-20, P-21, P-24, P-25, P-26, P-29, P-36, P-37, P-38, P-40, P-41, P-43, P-50, P-52, P-53, P-54, P-55, P-56, P-57, P-58, P-59, P-60, P-61, P-62, P-63, P-64, P-65, P-66, P-67, P-68, P-69, P-70, P-71, P-72, P-73, P-74, P-75, P-76, P-77, P-78, P-79, P-80, P-81, P-82	69	84
7	Use of cloud based resources	P-4, P-8, P-10, P-12, P-17, P-18, P-20, P-21, P-25, P-31, P-36, P-39, P-40, P-41, P-49, P-50, P-51, P-52, P-54, P-55, P-56, P-57, P-58, P-60, P-61, P-62, P-63, P-64, P-65, P-66, P-69, P-70, P-71, P-72, P-75, P-78, P-79, P-80	39	48
8	Lowering CO_2 emission	P-1, P-4, P-6, P-10, P-26, P-38, P-55, P-56, P-57, P-58, P-59, P-60, P-65, P-67, P-68, P-70, P-77, P-79, P-82	19	23
9	Use of green IT equipments	P-4, P-24, P-37, P-45, P-55, P-57, P-58, P-59, P-60, P-62, P-65, P-66, P-68, P-70, P-77	15	18

form. The data extraction form contains the following areas: Date of review, Title, Authors, Reference, Database, motivators: factors that have a positive impact on Green IT-outsourcing relationship, Methodology (interview, case study, ordinary literature review, systematic literature review, report, survey etc), Target Population, Sample Population, Publication Quality Description, Organization Type (software house, university, research institute etc), Company size(small, medium, large), Country/location of the Analysis and Year.

The data was extracted on the pre-defined extraction form from each of the finally selected paper. Our final selection includes the sample size of 82 papers as shown in the Table 5. After the data extraction phase, the data synthesis was performed for the identification of the motivators from the extracted data.

The data synthesis phase was done by the primary reviewer (the primary author) with the help of secondary reviewer (the co-author). After a thorough review with external reviewer, we have identified 9 motivators from the sample of 82 papers in Green IT-outsourcing from vendor's perspectives as shown in Table 2.

After identifying motivators for Green IT-outsourcing from vendor's perspectives through SLR, we classified few motivators in different tables as shown in results section. The criteria for selection of significant motivator as, that motivator will be considered as significant motivator whose frequency was ≥50. The identified significant motivators are 'energy efficiency', 'development and use of environment-friendly softwares', 'overall business cost reduction', 'promoting reusability and sustainability both in terms of hardwares and softwares', 'improving quality of services' and 'deployment of virtualization strategies with respect to hardwares, softwares and processes'.

4. RESULTS

For answering RQ1 Table 2 presents a list of 9 motivators, in total, in Green IT-outsourcing from vendor's perspective. In the following table 'P' represents paper id, such as 'P1' means paper 1, details listed in the Table 5. We have classified six motivators as significant motivators. The classification of significant motivators is based upon the criteria, such as: those motivators are considered as significant motivators whose frequency ≥50. The identified significant motivators are 'energy efficiency – 93%', 'development and use of environment-friendly softwares – 91%', 'overall business cost reduction – 78%', 'promoting reusability and sustainability both in terms of hardwares and softwares – 55%', 'improving quality of services – 52%' and 'deployment of virtualization strategies with respect to hardwares, softwares and processes – 84%'.

On these identified motivators, we have then performed statistical analysis based on different variables. They are, study strategy/methodology and year. The aim is to recognize whether these motivators remain stable /consistent in each study strategy and year respectively or vice versa. These analyses are presented in the sub sequent sections.

4.1 Analysis of the Motivators for Green IT-Outsourcing Vendors Based on Study Strategy

In order to answer RQ2, Table 3 explains the analysis based on various study strategy used. The number of articles with respect to various study strategies used in these articles are shown in Fig. 1. In the SLR process we have found the sample size of 82 papers as shown in the Table 5. From this sample size we have then extracted the information regarding the study strategies for each paper. We have identified five study strategies used in our sample during synthesizing the extracted data. These include ordinary literature review (OLR), case studies, empirical study, formal methods and systematic literature review (SLR). According to our findings, the majority of motivators have been reported through OLR, case studies and formal methods, as given in Table 3.

Among the list of 9 motivators in both OLR and case studies, our results shows 7 significant motivators have been cited in ≥50 of the articles. These motivators are identified through OLR are 'energy efficiency – 96% ', 'development and use of environment-friendly softwares – 96%', 'overall business cost reduction – 68%', 'promoting reusability and sustainability both in terms of hardwares and softwares – 72%' , 'improving quality of services – 60%', –

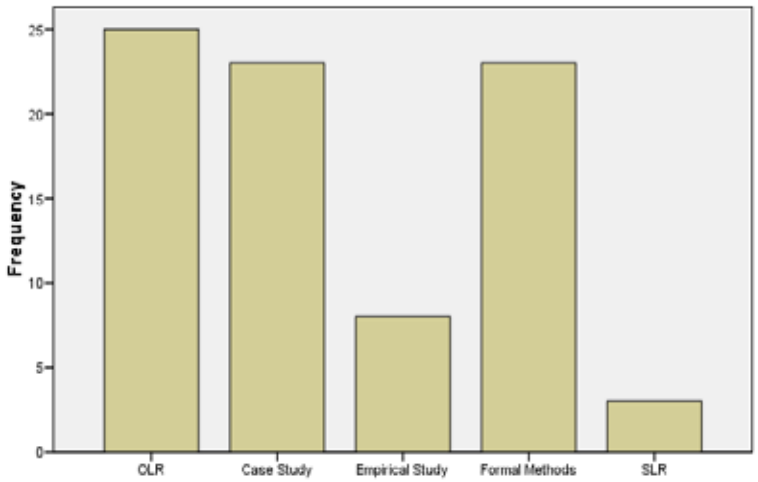

Fig. 1. Distribution of motivators, identified through the SLR, based on study strategy.

Table 3. Distribution of motivators, identified through the SLR, based on study strategies.

S. No	Motivators	Occurrence in SLR (N=82)										Chi-square Test (Linear-by-Linear Association α=0.05), df=1	
		OLR (N=25)		Case Study (N=23)		Empirical Study (N=8)		Formal Methods (N=23)		SLR (N=3)			
		Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	X^2	P
1	Energy efficiency	24	96	19	83	8	100	22	96	0	0	0.344	0.558
2	Development and use of environment-friendly softwares	24	96	21	91	7	88	20	87	0	0	0.715	0.398
3	Overall business cost reduction	17	68	20	87	7	88	17	74	0	0	0.479	0.489
4	Promoting reusability and sustainability both in terms of hardwares and softwares	18	72	15	65	4	50	8	35	0	0	10.551	0.001
5	Improving quality of services	15	60	12	52	3	38	11	48	1	33	1.229	0.268
6	Deployment of Virtualization strategies with respect to hardwares, softwares and processes	20	80	13	56	6	75	15	65	2	67	0.530	0.467
7	Use of cloud based resources	14	56	12	52	2	25	8	35	2	67	1.712	0.191
8	Lowering CO_2 emission	8	32	5	22	2	25	4	17	0	0	1.912	0.167
9	Use of green IT equipments	3	12	4	18	4	50	4	17	1	33	0.806	0.369

'deployment of Virtualization strategies with respect to hardwares, softwares and processes 80%' and 'use of cloud based resources – 56%'. Similarly the significant motivators identified through case studies are 'energy efficiency – 83%', 'development and use of environment-friendly softwares – 91%', 'overall business cost reduction – 87%', 'promoting reusability and sustainability both in terms of hardwares and softwares – 65%' , 'improving quality of services – 52%', 'deployment of Virtualization strategies with respect to hardwares, softwares and processes – 56%' and 'use of cloud based resources – 52%'.

Table 3 shows that 6 significant motivators have been cited in ≥ 50% of the articles among the list of 9 motivators in empirical study. These are 'energy efficiency – 100%', 'development and use of environment-friendly softwares – 88%', 'overall business cost reduction – 88%', 'promoting reusability and sustainability both in terms of hardwares and softwares – 50%' , 'deployment of Virtualization strategies with respect to hardwares, softwares and processes – 75%' and 'use of Green IT equipments – 50%'. Similarly we have identified 4 significant

Table 4 Distribution of motivators, identified through the SLR, based on period of years.

S. No	Motivators	Occurrence in SLR (N=82)				Chi-square test (Linear-by-Linear association α=0.05) df=1	
		Period-1 (N=8)		Period-2 (N=74)			
		Freq	%	Freq	%	X^2	P
1	Energy efficiency	6	75	70	95	4.038	0.044
2	Development and use of environment-friendly softwares	8	100	67	91	0.817	0.366
3	**Overall business cost reduction**	**6**	**75**	**58**	**78**	**0.048**	**0.827**
4	Promoting reusability and sustainability both in terms of hardwares and softwares	7	88	38	51	3.763	0.052
5	**Improving quality of services**	**4**	**50**	**38**	**51**	**0.005**	**0.942**
6	Deployment of Virtualization strategies with respect to hardwares, softwares and processes	7	88	49	65	1.492	0.222
7	Use of cloud based resources	5	63	33	45	0.919	0.338
8	Lowering CO_2 emission	3	38	16	22	1.010	0.315
9	Use of green IT equipments	2	25	13	18	0.264	0.608

motivators through formal methods, these are 'energy efficiency – 96% ', 'development and use of environment-friendly softwares – 87%', 'overall business cost reduction – 74%', and 'deployment of Virtualization strategies with respect to hardwares, softwares and processes – 65%.

We have identified 2 significant motivators from Table 3 that have been cited in ≥ 50% of the articles among the list of 9 motivators in SLR. These are 'deployment of Virtualization strategies with respect to hardwares, softwares and processes – 67%' and 'use of cloud based resources – 67%'. Our analysis reveals that these motivators are important to be noted by the Green IT-outsourcing vendors for successful outsourcing relationship with their clients.

We have used Linear-by-linear Chi-Square test for the identification of statistically significant differences among the various study strategies used. According to literature study this test is more powerful and preferred best as compare to Pearson chi-square test [36], when testing the differences between ordinal variables. The following hypotheses have been examined:

Null Hypothesis (H_0): To expose the motivators, there is no significant difference among the various study strategies used for a particular motivator.

Alternative Hypothesis (H_1): To expose the motivators, there is a significant difference among the various study strategies used for a particular motivator.

We will consider H_0, if the value of 'p' is greater than 0.05 for motivators otherwise H_1 will be considered. In our analysis maximum motivators have no significant differences; it means that there are no big differences for the motivators across the various study strategies used.

4.2 Analysis of the Motivators, Identified through the SLR, Based on the Period of Years

In order to answer RQ 3, Table 4 shows the analysis of our results based on the two periods of years used. Period-1 contains papers from 1999 to 2009 and Period-2 contains papers onwards to 2009. Fig. 2 shows the number of articles with respect to the two periods of year used in these articles. We have extracted the information regarding these periods in the sample size of 82 papers.

According to our findings, as given in Table 4, the majority of motivators have been reported through period-2. In our results 7 significant motivators have been cited in ≥ 50% of the articles among the list of 9 motivators in period-1. These six motivators are 'energy efficiency – 75%', 'development and use of environment-friendly softwares – 100%', 'overall business cost reduction – 75%', 'promoting reusability and sustainability both in terms of hardwares and

Table 5. List of finally selected publications in the SLR.

Paper-id	Papers
P1	S. Naumann, M. Dick, E. Kern, and T. Johann, "The greensoft model: A reference model for green and sustainable software and its engineering," *Sustainable Computing: Informatics and Systems*, vol. 1, pp. 294-304, 2011.
P2	G. Agosta, M. Bessi, E. Capra, and C. Francalanci, "Automatic memoization for energy efficiency in financial applications," *Sustainable Computing: Informatics and Systems*, vol. 2, pp. 105-115, 2012.
P3	A. Kipp, T. Jiang, M. Fugini, and I. Salomie, "Layered green performance indicators," *Future Generation Computer Systems*, vol. 28, pp. 478-489, 2012.
P4	L. Ardito and M. Morisio, "Green IT - Available data and guidelines for reducing energy consumption in IT systems," *Sustainable Computing: Informatics and Systems*, 2013.
P5	Q. Gua, P. Lago, H. Muccini, and S. Potenza, "A categorization of green practices used by Dutch data centers," *Procedia Computer Science*, vol. 19, pp. 770-776, 2013.
P6	C. Colicchia, G. Marchet, M. Melacini, and S. Perotti, "Building environmental sustainability: empirical evidence from Logistics Service Providers," *Journal of Cleaner Production*, vol. 59, pp. 197-209, 2013.
P7	E. Capra, C. Francalanci, and S. A. Slaughter, "Is software "green"? Application development environments and energy efficiency in open source applications," *Information and Software Technology*, vol. 54, pp. 60-71, 2012.
P8	D. Aikema, R. Simmonds, and H. Zareipour, "Delivering ancillary services with data centres," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 172-182, 2013.
P9	L. Ardito, M. Torchiano, M. Marengo, and P. Falcarin, "gLCB: an energy aware context broker," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 18-26, 2013.
P10	L. M. Zhang, K. Li, D. C.-T. Lo, and Y. Zhang, "Energy-efficient task scheduling algorithms on heterogeneous computers with continuous and discrete speeds," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 109-118, 2013.
P11	T. Sheltami, A. J. Siddiqui, H. I. Abbasi, U. Baroudi, and L. Ghouti, "Implementation of Rank Based Sleep Scheduling (RBSS) Protocol for WSNs in a Fixed Grid Topology," *Procedia Computer Science*, vol. 19, pp. 348-355, 2013.
P12	A. E. Trefethen and J. Thiyagalingam, "Energy-aware software: Challenges, opportunities and strategies," *Journal of Computational Science*, vol. 4, pp. 444-449, 2013.
P13	A. El Kouche, L. Al-Awami, and H. Hassanein, "Dynamically Reconfigurable Energy Aware Modular Software (DREAMS) Architecture for WSNs in Industrial Environments," *Procedia Computer Science*, vol. 5, pp. 264-271, 2011.
P14	Y.-W. Kwon and E. Tilevich, "The impact of distributed programming abstractions on application energy consumption," *Information and Software Technology*, vol. 55, pp. 1602-1613, 2013.
P15	M. Kazandjieva, B. Heller, O. Gnawali, P. Levis, and C. Kozyrakis, "Measuring and analyzing the energy use of enterprise computing systems," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 218-229, 2013.
P16	L. Wang, S. U. Khan, D. Chen, J. KoÅ‚odziej, R. Ranjan, C.-z. Xu, and A. Zomaya, "Energy-aware parallel task scheduling in a cluster," *Future Generation Computer Systems*, vol. 29, pp. 1661-1670, 2013.
P17	C. De Alfonso, M. Caballer, F. Alvarruiz, and V. HernÃ¡ndez, "An energy management system for cluster infrastructures," *Computers & Electrical Engineering*, vol. 39, pp. 2579-2590, 2013.
P18	B. Dougherty, J. White, and D. C. Schmidt, "Model-driven auto-scaling of green cloud computing infrastructure," *Future Generation Computer Systems*, vol. 28, pp. 371-378, 2012.
P19	X. Zheng and Y. Cai, "CMDP based adaptive power management in server clusters," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 70-79, 2013.
P20	T. GuÃ©rout, T. Monteil, G. Da Costa, R. Neves Calheiros, R. Buyya, and M. Alexandru, "Energy-aware simulation with DVFS," *Simulation Modelling Practice and Theory*, vol. 39, pp. 76-91, 2013.
P21	M. Marzolla and R. Mirandola, "Dynamic power management for QoS-aware applications," *Sustainable Computing: Informatics and Systems*, vol. 3, pp. 231-248, 2013.
P22	Y. Woo, S. Y. Park, and E. Seo, "Virtual Battery: A testing tool for power-aware software," *Journal of Systems Architecture*, vol. 59, pp. 794-800, 2013.
P23	M. Witkowski, A. Oleksiak, T. Piontek, and J. WÄ™glarz, "Practical power consumption estimation for real life HPC applications," *Future Generation Computer Systems*, vol. 29, pp. 208-217, 2013.
P24	S. S. Shenoy and R. Eeratta, "Green software development model: An approach towards sustainable software development," presented at 2011 Annual IEEE India Conference (INDICON), 2011.
P25	Y. Sun and Y. Song, "Beyond Green: Evolution to Adaptability and Recyclability," presented at 2011 International Conference on Green Computing and Communications (GreenCom) IEEE/ACM, 2011.
P26	P. Gupta and G. Singh, "User centric framework of power schemes for minimizing energy consumption by computer systems," presented at 2012 International Conference on Radar, Communication and Computing (ICRCC),, 2012.
P27	M. Dick, J. Drangmeister, E. Kern, and S. Naumann, "Green software engineering with agile methods," presented at 2013 2nd International Workshop on Green and Sustainable Software (GREENS), 2013.

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Table 5 (Contd.)

Paper-id	Papers
P28	R. Zhang, Z. Zilic, and K. Radecka, "Energy efficient software-based self-test for wireless sensor network nodes," presented at Proceedings of the 24th IEEE VLSI Test Symposium (VTS'06), 2006.
P29	S. Bhattacharya, K. Gopinath, K. Rajamani, and M. Gupta, "Software Bloat and Wasted Joules: Is Modularity a Hurdle to Green Software?," *IEEE Computer Society*, vol. 44, pp. 97-101, 2006.
P30	A. Sivasubramaniam, M. J. Irwin, M. Kandemir, and N. Vijaykrishnan, "Designing energy-efficient software," presented at Proceedings of the International Parallel and Distributed Processing Symposium (IPDPS'02), 2002.
P31	N. Khamis, A. M. Misfian, and R. Md Noor, "Towards sustainable software criteria: Rescue operation and disaster management system model," presented at 10th IEEE International Conference on Networking, Sensing and Control (ICNSC), 2013, 2013.
P32	A. Larab, E. Conchon, R. m. Bastide, and N. Singer, "A sustainable software architecture for home care monitoring applications," presented at 6th IEEE International Conference on Digital Ecosystems Technologies (DEST), 2012, 2012.
P33	H. Koziolek, D. Domis, T. Goldschmidt, P. Vorst, and R. J. Weiss, "MORPHOSIS: A Lightweight Method Facilitating Sustainable Software Architectures," presented at 2012 Joint Working Conference on Software Architecture & 6th European Conference on Software Architecture, 2012.
P34	G. Konduri, J. Goodman, and A. Chandrakasan, "Energy efficient software through dynamic voltage scheduling," presented at Proceedings of the 1999 IEEE International Symposium on Circuits and Systems, 1999. ISCAS'99., 1999.
P35	M. R. Sabharwal, "Software power optimization: Analysis and optimization for energy-efficient software," presented at 2011 International Symposium on Low Power Electronics and Design (ISLPED), 2011.
P36	N. S. Chauhan and A. Saxena, "A Green Software Development Life Cycle for Cloud Computing," *IT Professional*, vol. 15, pp. 28-34, 2013.
P37	K. Erdelyi, "Special factors of development of green software supporting eco sustainability," presented at IEEE 11th International Symposium on Intelligent Systems and Informatics (SISY), 2013, 2013.
P38	T. Johann, M. Dick, E. Kern, and S. Naumann, "Sustainable development, sustainable software, and sustainable software engineering: An integrated approach," presented at 2011 International Symposium on Humanities, Science & Engineering Research (SHUSER), 2011.
P39	S. Dustdar, F. Li, H.-L. Truong, S. Sehic, S. Nastic, S. Qanbari, M. Vogler, and M. Claesens, "Green software services: From requirements to business models," presented at 2nd International Workshop on Green and Sustainable Software (GREENS), 2013, 2013.
P40	M. Liangli, Y. Chen, Y. Sun, and Q. Wu, "Virtualization Maturity Reference Model for Green Software," presented at Proceedings of the 2012 International Conference on Control Engineering and Communication Technology, 2012.
P41	C. Sahin, F. Cayci, J. Clause, F. Kiamilev, L. Pollock, and K. Winbladh, "Towards power reduction through improved software design," presented at Energytech, 2012 IEEE, 2012.
P42	D. Q. Ren, E. Bracken, S. Polstyanko, N. Lambert, R. Suda, and D. D. Giannacopulos, "Power Aware Parallel 3-D Finite Element Mesh Refinement Performance Modeling and Analysis With CUDA/MPI on GPU and Multi-Core Architecture," *IEEE Transactions on Magnetics*, vol. 48, pp. 335-338, 2012.
P43	G. Scanniello, U. Erra, G. Caggianese, and C. Gravino, "Using the GPU to Green an Intensive and Massive Computation System," presented at 2013 17th European Conference on Software Maintenance and Reengineering (CSMR),, 2013.
P44	A. Noureddine, A. Bourdon, R. Rouvoy, and L. Seinturier, "Runtime monitoring of software energy hotspots," presented at Proceedings of the 27th IEEE/ACM International Conference on Automated Software Engineering(ASE), 2012, 2012.
P45	M. A. Khan, C. Hankendi, A. K. Coskun, and M. C. Herbordt, "Software optimization for performance, energy, and thermal distribution: Initial case studies," presented at 2011 International Green Computing Conference and Workshops (IGCC), 2011.
P46	B. Penzenstadler, H. Femmer, and D. Richardson, "Who is the advocate? Stakeholders for sustainability," presented at 2nd International Workshop on Green and Sustainable Software (GREENS),2013, 2013.
P47	Y. S. Shao and D. Brooks, "Energy characterization and instruction-level energy model of Intel's Xeon Phi processor," presented at 2013 IEEE International Symposium on Low Power Electronics and Design (ISLPED), 2013.
P48	S. Alawnah and A. Sagahyroon, "Modeling smartphones power," presented at 2013 IEEE EUROCON, 2013.
P49	Z. Durdik, B. Klatt, H. Koziolek, K. Krogmann, J. Stammel, and R. Weiss, "Sustainability guidelines for long-living software systems," presented at 28th IEEE International Conference on Software Maintenance (ICSM), 2012, 2012.
P50	V. G. Moshnyaga, "An assessment of software lifecycle energy," presented at 23rd International Workshop on Power and Timing Modeling, Optimization and Simulation (PATMOS), 2013, 2013.
P51	J. Arnoldus, J. Gresnigt, K. Grosskop, and J. Visser, "Energy-efficiency indicators for e-services," presented at 2nd International Workshop on Green and Sustainable Software (GREENS), 2013.
P52	S. K. Garg, S. C. Yeo, and R. Buyya, "Green Cloud Framework for Improving Carbon Efficiency of Clouds," *Springer-Verlag Berlin Heidelberg 2011*, pp. 491–502, 2011.
P53	N. Amsel, Z. Ibrahim, A. Malik, and B. Tomlinson, "Toward Sustainable Software Engineering (NIER Track)," in ICSE'11, Waikiki, Honolulu, HI, USA, 2011.
P54	Q. Gu, P. Lago, and S. Potenza, "Aligning Economic Impact with Environmental Benefits: A Green Strategy Model," in IEEE

Contd.....

Table 5 (Contd.)

Paper-id	Papers
	2012, GREENS 2012, Zurich, Switzerland, 2012, pp. 62-68.
P55	G. Sissa, "Utility Computing: Green Oppertunities and Risks," *CEPIS UPGRADE The European Journal for the Informatics Professionals,* vol. 12, no. 4, October,, 2011.
P56	A. Ozturk, K. Umit, I. T. Medeni, B. Ucuncu, M. Caylan, F. Akba, and T. D. Medeni, "Green ict (information and communication technologies): A review of academic and practitioner perspectives " *International journal of ebusiness and egovernment studies,* vol. 3, no. 1, pp. 1-16, 2011.
P57	G. Sissa, "Green Software," *CEPIS UPGRADE The European Journal for the Informatics Professionals,* vol. 11, no. 3, pp. 53-63, June,, 2010.
P58	S. Murugesan, G. R. Gangadharan, R. R. Harmon, and N. Godbole, "Fostering Green IT," in IEEE Computer Society, 2013.
P59	S. S. Mahmoud, and I. Ahmad, "Green Performance Indicators for Energy Aware IT Systems: Survey and Assessment," *Journal of Green Engineering,* vol. 3, pp. 33–69, 2012.
P60	A. Harbla, D. P. Dimri, D. Negi, and D. Y. S. Chauhan, "Green Computing Research Challenges: A Review," *International Journal of Advanced Research in Computer Science and Software Engineering* vol. 3, no. 10, pp. 1075-1077, October,, 2013.
P61	M. Sen, A. Dutt, J. Shah, S. Agarwal, and A. Nath, "Smart Software and Smart Cities: A study on Green Software and Green Technology to develop a smart urbanized world," *International Journal of Advanced Computer Research* vol. 2, no. 6, pp. 373-380, December, 2012.
P62	J. S. Jacob, and K. G. Preetha, "Edpac : Event-driven power aware pervasive computing for effective power utilization in green computing," *International Journal of Advanced Information Technology (IJAIT),* vol. 2, no. 1, pp. 55-64, Febraury, 2012.
P63	M. Liangli, and C. Yanshen, "Virtualization Maturity Reference Model for Green Software," *2012 International Conference on Control Engineering and Communication Technology*, pp. 573-576, 2012.
P64	Y.-s. Liu, and G.-z. Cao, "Development of Green Energy-saving Web Atoms Component " *2011 International Conference of Information Technology, Computer Engineering and Management Sciene*, pp. 246-249, 2011.
P65	P. Bozzelli, Q. Gu, and P. Lago, "A systematic literature review on green software metrics."
P66	Y. SUN, Y. ZHAO, Y. SONG, Y. YANG, H. FANG, H. ZANG, Y. LI, and Y. GAO, "Green challenges to system software in data centers," *Front. Comput. Sci. China 2011,* vol. 5, no. 3, pp. 353 – 368, 2011.
P67	C. Siebra, P. Costa, R. Miranda, F. Q B Silva, and A. Santos, "The Software Perspective for Energy-Efficient Mobile Applications Development," in MoMM 2012, Bali, Indonesia., 2012, pp. 143-150.
P68	G. Bekaroo, and C. Bokhoree, "Towards Emerging Green Information and Communication Technologies: A Review ".
P69	D. Schien, P. Shabajee, S. G. Wood, and C. Preist, "A Model for Green Design of Online News Media Services," in International World Wide Web Conference Committee (IW3C2), Brazil, 2013, pp. 1111-1121.
P70	L. Ardito, and M. Morisio, "Green IT – Available data and guidelines for reducing energy consumption in IT systems," *Sustainable Computing: Informatics and Systems*, pp. 9, 2013.
P71	S. S. Mahmoud, and I. Ahmad, "A Green Model for Sustainable Software Engineering " *International Journal of Software Engineering and Its Applications,* vol. 7, no. 4, pp. 55-74, July, 2013.
P72	Y. SUN, and Y. SONG, "Beyond Green: Evolution to adaptability and recyclability," in 2011 IEEE/ACM International Conference on Green Computing and Communications, 2011, pp. 234-237.
P73	L. Ponciano, A. Brito, L. ı. Sampaio, and F. Brasileiro, "Energy Efficient Computing through Productivity-Aware Frequency Scaling," in 2012 Second International Conference on Cloud and Green Computing, 2012, pp. 191-198.
P74	G. Agosta, M. Bessi, E. Capra, and C. Francalanci, "Dynamic Memoization for Energy Efficiency in Financial Applications," in 2011 IEEE, 2011, pp. 8.
P75	N. S. Chauhan, and A. Saxena, "A Green Software Development Life Cycle for Cloud Computing," *IEEE Computer Society*, January/February, 2013.
P76	M. N. Razali, H. R. Loindin, L. Y. Beng, and R. Hanapi, *Ecological Monitoring and Feedback Approach for Green and Sustainable IT*, pp. 131-137.
P77	G.Bekaroo & C.Bokhoree, "Towards Emerging Green Information and Communication Technologies: A Review",
P78	Schahram Dustdar, Fei Li, Hong-Linh Truong, Sanjin Sehic, Stefan Nastic, Soheil Qanbari, Michael V¨ogler and Markus Claeßens, "Green Software Services: From Requirements to Business Models",
P79	Chiranjeeb Roy Chowdhury, Arindam Chatterjee, Alap Sardar, Shalabh Agarwal And Asoke Nath, "International Journal of Advanced Computer Research", Volume-3 Number-1 Issue-8 March-2013, pp. 78-85
P80	Steve Easterbrook, "Climate Change: A Grand Software Challenge",
P81	Julius T. Nganji and Mike Brayshaw, "IS GREEN IT AN ANTIDOTE TO E-WASTE PROBLEMS?", ITALICS Volume 9 Issue 2 November 2010, pp. 1-9
P82	Capra, Eugenio, Politecnico di Milano, "THE IMPACT OF MIS SOFTWARE ON IT ENERGY CONSUMPTION", 18th European Conference on Information Systems, pp. 1-13

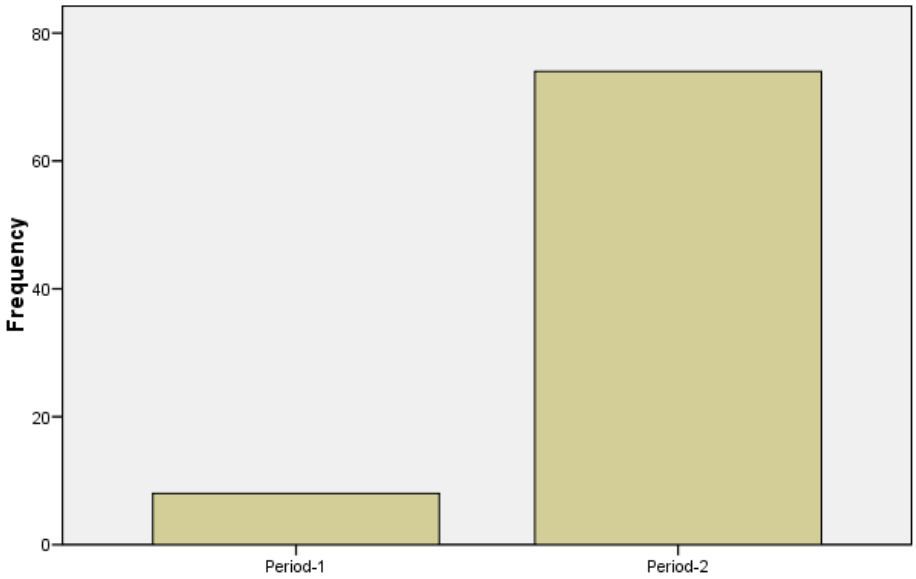

Fig. 2. Distribution of motivators, identified through the SLR, based on period of years.

softwares – 88%' , 'improving quality of services – 50%', 'deployment of Virtualization strategies with respect to hardwares, softwares and processes – 88%' and 'use of cloud based resources – 63%'. Similarly we have identified 6 significant motivators in period-2, these are 'energy efficiency – 95%', 'development and use of environment-friendly softwares – 91%', 'overall business cost reduction – 78%', 'promoting reusability and sustainability both in terms of hardwares and softwares – 51%' , 'improving quality of services – 51%', 'deployment of virtualization strategies with respect to hardwares, softwares and processes – 65%'.

Our analysis reveals that these motivators are important to be noted by the Green IT-outsourcing vendors for successful outsourcing relationship with their clients. We have used a similar Linear-by-linear Chi-Square test also for the analysis of the identification of statistically significant differences among the various study strategies used. In our analysis maximum motivators have no significant differences; it means that there are no big differences for the motivators across the two periods used. Table 4 reveals that only two motivators('overall business cost reduction' and 'improving quality of services') posses' value of 'p' less than 0.05 and exposes statistical significance difference for these motivators only across the two periods.

5. SUMMARY AND DISCUSSION

This study has identified a total of nine motivators for Green IT-outsourcing vendors. For Green IT software outsourcing our findings represents some basic consideration. To develop better Green IT outsourcing idea and plan, the motivators presents some basic key areas which need management's attention and awareness. The IT outsourcing vendor organization can also get help from these findings in order to know that what their clients actually want.

In order to answer RQ1, we identified 9 motivators, in total, for Green IT-outsourcing vendors. Out of these 9 motivators, some have occurrences of greater than or equal to 50% as shown in "Table 2". The identified significant motivators are 'energy efficiency', 'development and use of environment-friendly softwares', 'overall business cost reduction', 'promoting reusability and sustainability both in terms of hardwares and softwares ', 'improving quality of services' and 'deployment of virtualization strategies with respect to hardwares, softwares and processes'.

A similar criterion was also used for addressing the RQ2 and we found, 'energy efficiency', 'development and use of environment-friendly softwares', 'overall business cost reduction', and 'deployment of Virtualization

strategies with respect to hardwares, softwares and processes', most cited. Motivators for each of the research methodologies used respectively as shown in "Table 3".

A similar criterion was also used for addressing the RQ4 and we found, 'energy efficiency', 'development and use of environment-friendly softwares', 'overall business cost reduction', 'promoting reusability and sustainability both in terms of hardwares and softwares', 'improving quality of services', and 'deployment of virtualization strategies with respect to hardwares, softwares and processes', most cited motivators on the bases of period of years respectively as shown in "Table 4".

6. STUDY LIMITATIONS

By using our systematic literature review, we extracted data about the motivators in Green IT-outsourcing from vendor's perspective, but how valid are our findings? To internal validity one possible threat is that for any specific reporting article in the SLR, which may have not in fact described underlying reasons to report motivators in Green IT-outsourcing. In these studies the authors would not be supposed to give the original reasons for a particular motivator. Regarding threats to external validity, in many studies such as case studies, empirical studies and self-reported experience reports may have a propensity to report particular kinds of motivators in Green IT-outsourcing. There may be a chance of publication bias in these studies. By using our SLR process, we may have missed out some relevant papers, due to the increasing number of papers in Green IT- outsourcing domain. Though, like other researchers of SLR, thi**s** is not a systematic omission [37].

7. CONCLUSION AND FUTURE WORK

We have identified 9 motivators, in total, through SLR in Green IT-outsourcing from vendor's perspective. Our results reveal that focusing on these motivators can help vendor organizations in the adoption of Green IT initiatives in the context of IT outsourcing relationships with their clients. The objective of our research is to provide Green IT-outsourcing vendors with a body of knowledge that can assist them to implement and design successful Green IT-outsourcing initiatives. Our results suggest that Green IT-outsourcing vendors should adopt all of the identified motivators in order to be 'Green' in IT-outsourcing relationship.

On the basis of finding of this study, we have identified the following goals to plan our future studies:

- To identify various strategies and software tools that could be used by outsourcing to vendors, to engineer and maintain Green and sustainable software.
- To identify challenges in the adoption of Green IT-outsourcing from vendor's perspective.
- To identify the real-world practices for implementation of the identified motivators.

Our future work will focused on the development of Green IT-Outsourcing Assurance Model (GITAM) that will assist outsourcing vendor organizations to assure greenness of the organizations and to engineer energy-efficient softwares at low cost. The structure of the proposed model has already been published [38].

8. ACKNOWLEDGMENTS

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9. REFERENCES

1. Mohiddin, S.K. & Y.S. Babu. Green computing an eco-friendly IT environment for upcoming technologies. *International Journal of Advanced Research in Computer Science* 6: 28-32 (2015).
2. Harmon, R., H. Demirkan & N.M. Auseklis. From green computing to sustainable IT: Developing a sustainable service orientation. In: *43rd Hawaii International Conference System Sciences (HICSS)*, Honolulu, Hawaii, p. 1-10 (2010).
3. Sabharwal, M. & A. Agrawal. Enabling green IT through energy-aware software. *IT Professional* 15: 19 – 27 (2012).
4. Goiri, I., W. Katsak. L.N. Kien. D. Thu & B. Ricardo. Parasol and green switch: Managing datacenters powered by renewable energy. *ACM SIGARCH Computer Architecture News - ASPLOS* 13: 51-64 (2013).

5. Cheocherngngarn, T., J. Andrian & P. Deng. Deployment of a hybrid multicast switch in energy-aware data center network: a case of fat-tree topology. *Journal ISRN Communications and Networking Archive* 2012: 1-10 (2012) doi:10.5402/2012/209573.
6. Godbole, N. S. & J. Lamb. Calculating a hospital's IT energy efficiency and determining cost effective ways for improvement. In: *11th International Conference on Emerging Technologies for a Smarter World (CEWIT)*, Melville, New York, p. 1-6 (2014).
7. Lacina, R., F. Besseling & B. T. Hedde. An integrated data center solution on medium voltage level. *22nd International Conference and Exhibition on Electricity Distribution (CIRED 2013)*, Stockholm, Sweden, p. 1-4 (2013).
8. Jiancheng, N., Q. Lianyong & Y. Chao. A selection strategy supporting service outsourcing in cloud platform. In: *Second International Conference Cloud and Green Computing (CGC)*, Xiangtan, China, p. 726 –729 (2012).
9. Marija, I., L. Xie & L. Qixing. Sustainable electricity: A look at the green Azores islands, *IEEE Power & Energy Magazine* 12(5): 88-90 (2014).
10. Bachour, N. & L. Chasteen. Optimizing the value of green it projects within organizations. In: *Green Technologies Conference*, Grapevine, TX- USA, p. 1 – 10 (2010).
11. Yamini. B. & S.D.V. Selvi. Cloud virtualization: A potential way to reduce global warming. In: *Recent Advances in Space Technology Services and Climate Change (RSTSCC)*, Chennai, India, p. 55 – 57 (2010).
12. Bose, R. & L. Xin. Integrative framework for assessing firms' potential to undertake Green IT initiatives via virtualization – A theoretical perspective. *Journal of Strategic Information Systems* 20(1): 38-54 (2011).
13. Ambtman, E.L. & R. Christiaanse. *Green IT Auditing*. MS Thesis, Vrije University in Brussels, Belgium (2011).
14. Yan, Y., Y. Qian, H. Sharif & D. Tipper. A Survey on smart grid communication infrastructures: Motivations, requirements and challenges. *Communications Surveys & Tutorials* 15(1): 5-20 (2013).
15. Zafar, F., D. Karunatilaka & R. Parthiban. Dimming schemes for visible light communication: the state of research. *Wireless Communications* 22(2): 29–35 (2015).
16. Baodong, Q., R.H. Deng., L. Shengl. & M. Siqi. Attribute-based encryption with efficient verifiable outsourced decryption, information forensics and security. *IEEE Transactions* 10 (7): 1384–1393 (2015).
17. Capra, E., G. Formenti., C. Francalanci & S. Gallazzi. The impact of MIS software on IT energy consumption. *18th European Conference on Information Systems*, Pretoria, South Africa, p. 1-13 (2010).
18. Wu, J., Y. Zhang, M. Zukerman & E. Yung. Energy-efficient base-stations sleep-mode techniques in green cellular networks: A survey. *IEEE Communication Surveys & Tutorials* 17(2): 803-826 (2015).
19. Taghipour, A., M. Abed & N. Zoghlami. Design for remanufacturing respecting reverse logistics processes: A review. In: *4th International Conference in Advanced Logistics and Transport (ICALT)*, Valenciennes, France, p. 299-304 (2015).
20. Azevedo, S., M. Cruz, V. Hofstetter, J. Elizabeth, A. Cudney & T. Yihui. Importance of green and resilient SCM practices to the competitiveness of the automotive industry: A multinational perspective. In: *Reverse Supply Chains: Issues and Analysis*, S.M. Gupta (Ed.), CRC Press, p. 229-252 (2013).
21. Gu, Q.L., P. Muccini & S.A. Potenza. A categorization of green practices used by Dutch data centers. In: *3rd International Conference on Sustainable Energy Information Technology (SEIT 2013)*, Nova Scotia, Canada, p. 1-7(2013).
22. Dorsaf, Z. Inter-organizational issues toward a "Greener" supply chain. In: *International Logistics and Supply Chain*, *CERAG 2011-01 E5* <halshs-00659348>, p. 1-9 (2011).
23. Al Hasib, A. & S. Sharmin. Green software: recent trends and techniques for software design and development, *International Journal of Advanced Research in Computer Engineering & Technology (IJARCET)* 2(1): 77-89 (2013).
24. Li, Q. & M. Zhou. The survey and future evolution of green computing. In: *IEEE/ACM International Conference on Green Computing and Communications, IEEE Computer Society,* Sichuan, China, p. 230-233 (2011).
25. Lamb, J. Green IT and use of private cloud computing in South Africa. In: *8th International Conference & Expo on in Emerging Technologies for a Smarter World (CEWIT)*, New York, USA, p. 1-6 (2011).
26. Bianzino, A.P., C. Chaudet, D. Rossi & J.L. Rougier. A survey of green networking research. *IEEE Communication, Surveys Tutorials* 14(1): 3-20 (2012).
27. Kitchenham. B. & S. Chartless. *Guidelines for Performing Systematic Literature Reviews in Software Engineerin*g. Keele University and Durham University Joint Report (2007).
28. Mahmood, N. Do systematic literature reviews outperform informal literature reviews in the software engineering domain? An initial case

study. *Arabian Journal for Science and Engineering* 40: 845-855 (2015).
29. Khan, S.U. & M.I. Azeem. Intercultural challenges in offshore software development outsourcing relationships: an exploratory study using a systematic literature review. *IET Software* 8(4): 161-173 (2014).
30. Khan. R.A. & S.U. Khan. Communication and coordination challenges in offshore software development outsourcing relationship from vendors' perspective: Preliminary results. *ISoRIS2014 Malaysia, Special ed., Journal of Science International, Lahore* 26: 1425-1429 (2014).
31. Mahmood. N., S. Mahmood, M. Alshayeb & M. Hroub. Empirical investigation of the challenges of the existing tools used in global software development project. *IET Software* 9(5): 135-143 (2015).
32. Khan, S.U. & M. Salam. Web usability management model (Web-Umm) from customer perspective. In: *Software Engineering Post Graduate Workshop SEPoW*, Melaka, Malaysia, p. 2747-2758 (2013).
33. Ali, S., & S.U. Khan. Software outsourcing partnership model. *Journal of Science International* (Lahore) 26: 1437-144 (2014).
34. Verner. J.M., O.P. Brereton, B. Kitchenham, M. Turner & M. Niazi. Risks and risk mitigation in global software development: A tertiary study. *Information and Software Technology* 56: 54–78 (2014).
35. Khan, S.U., M. Niazi & A. Rashid. Critical barriers for offshore software development outsourcing vendors: A systematic literature review. In: *IEEE International Asia-Pacific Software Engineering Conference, APSEC '09*, Asia-Pacific, p. 79 – 86, (2009).
36. Bland, M. *An Introduction to Medical Statistics,* 3rd ed. Oxford Medical Publications (2000).
37. Hossain, E., M.A. Babar & H.Y. Paik. Using Scrum in global software development: A systematic literature review. In: *IEEE International Conference on Global Software Engineering, ICGSE09*, Lero, Limerick, Ireland, p. 175-184 (2009).
38. Khan, R.U. & S.U. Khan. Green IT-outsourcing assurance model. In: *IEEE International Conference of Global Software Engineering Workshop, ICSGSEW13*, Bari, Italy, p. 84-87 (2013).

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Research Article

Effect of Drying Temperature and Natural Preservatives on Reducing Aflatoxins in Solar Dried Persimmon (*Diospyros kaki* L)

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Abstract: Experiments were conducted in 2013-14 to study the effect of drying temperature and natural preservatives for reducing aflatoxins in dried persimmons. Persimmons were dried using a flat plate solar collector connected to a drying chamber at a temperature range of 40 to 60°C and less than 10% relative humidity. Persimmons dried at 40°C took 22, at 50°C 19 and at 60°C 17 hours to minimize moisture from 76% to less than 9% under an average drying rates of 0.11, 0.12 and 0.13 $g_{H2O}.g_{d.m}^{-1}$ respectively. After drying the samples were studied for aflatoxins. Analysis of variance was done using two factorial completely randomized design. The analysis of variance (ANOVA) showed that both the factors significantly ($\alpha \leq 0.01$) affect the aflatoxin deposition in dried persimmons. Maximum aflatoxins of 31.7 µg kg−1 was recorded in persimmons treated with Aloe vera and dried under a temperature of 40°C with B1 and G1 type of aflatoxins while minimum of 7.1 µg kg−1 was recorded in persimmons treated with honey and dried under a temperature of 60°C with G1 type. It was concluded that increase in temperature of the drying system decreased the contamination by fungus up to 22%. Also the overall performance of honey was good as compared to aloe vera gel used as natural preservatives for reducing aflatoxins in dried persimmons.

Keywords: Persimmon, solar drying, moisture, drying rate, aflatoxins

1. INTRODUCTION

Persimmon (*Diospyros kaki* L) is native to China and is a seasonal fruit grown in many countries like Korea, India, Iran and Pakistan. They have a good demand and good market all over the world, but due to its short availability time and shelf life they are dried and preserved for future consumption. The dried persimmons are delicious and rich in nutrients and are popular worldwide [1]. They are in high demand due to their aroma, taste and medicinal values. Quality dried Persimmons have high demand in international market. The best quality indicator is the minimum pathogenic attacks on the persimmons during drying that is, the minimum amount of aflatoxins in the dried persimmons [2].

In most cases the quality of dried persimmons is affected by the presence of afltoxins in the product. Aflatoxins (AFs) are chemicals generated by fungus [3]. They are a serious threat to human health if its consumption exceeds the minimum limit (ML). The ML for AFs in dried fruits is 10 µg kg−1. If the ML level increases, it may be dangerous for health and may cause serious problems like ulcer, cancer and other diseases [4]. Once aflatoxins are deposited on the dried food it is very hard to remove. The best option is preserved before drying [5]. Many chemical preservatives are used to stop fungus attacks during drying, but as we know, chemicals are not good for human health so natural or bio preservatives are the best choice for preserving dried fruits [6].

Several genera of fungus attack on dried persimmons during and after drying, causing AFs deposition in which Aspergillus is the main fungus attacking the persimmons during drying. Scientists

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and researchers are using different natural preservatives to minimize the fungal attacks on the dried fruits [7]. Aloe Vera (*Aloe barbadensis* M) gel at different concentrations can minimize the AFs in dried persimmons. If used, Aloe Vera gel is best remedy for the problem of AFs contamination in dried fruits [7, 8]. Honey is a natural preservative for dried persimmons which can reduce AFs significantly [2]. The purpose of the study was to examine the effect of temperature and natural preservatives (Aloe Vera gel and honey) for minimizing AFs in dried persimmons for quality assurance.

2. MATERIAL AND METHODS

2.1 Preparation of Solutions

The gel was extracted from fresh leaves of Aloe Vera. The gel extracted was mixed with distilled water to prepare 5% solution. Good quality honey was added to distilled water to prepare 5% solution. Also a mixture of both Aloe Vera gel solution and honey solution were mixed to prepare a mixture of both the solutions [2, 6].

2.2 Persimmon Processing and Drying

Persimmons were peeled by the peeler, blanched for two minutes in water having a temperature of 80^{o}C and were soaked in the prepared solutions for two minutes. After soaking they were put on a clean tray and were dried in the solar collector (Fig. 1) till their moisture became less than 9% (recommended to minimize microbial attacks). The temperature of the solar collector was regulated using a regulator to control the flow of the fan. The drying curves were then developed for the persimmons [9, 11].

2.3 Extraction and Determination of Aflotoxins

Each of the samples was tested to extract and determine the presence of AFs [6, 7].

2.4 Statistical Analysis

The experiment was laid out as a completely randomized design The factors and their levels are as given below:

Factor A (Temperature of drying chamber) T1 = 40^{o}C, T2 = 50^{o}C and T3 = 60^{o}C

Factor B (Natural Preservatives) P1= 5% Honey solution, P2= 5% Aloe Vera solution, P3= P1+P2 combination

The data collection was replicated three times. ANOVA was computed using 18 treatments by the standard procedure. The means of AFs were compared using Least Significant Difference (LSD) test [12].

3. RESULTS AND DISCUSSION

3.1 The Drying Curves of Persimmons

The drying curves of persimmons on wet basis are shown in Fig. 2 and on dry basis are shown in Fig. 3. The data in the Fig. 2 show that increase in drying temperature decreases the drying time. The initial moisture content of persimmons was 76%, which was reduced to less than 9%. Persimmons dried at 40^{o}C took 22, at 50^{o}C took 19 and at 60^{o}C took 17 hours to minimize moisture content from 76% to less than 9% respectively. Two term exponential model was applied to the curves to find the correlation between drying time and moisture lost. The results showed a strong correlation between drying time and moisture lost with an R^2 value of -0.993. These results are in accordance with the findings of Kim et al [2] and Hanif et al [10]. The data in Fig. 3 shows a persistent constant and falling rate drying by the persimmons. The drying rate increased with increase in drying temperature. The average drying rate of Persimmons dried at 40^{o}C was 0.11, at 50^{o}C was 0.12, and at 60^{o}C was 0.13 g H_2Og dm^{-1}, respectively. Two term exponential model was applied on the drying rate curves to find the correlation between with drying time. The results showed a strong correlation between drying time and drying rate with an R^2 value of -0.995. These results are in argument with the findings of Hyun and Woo [1], Yong- Seo et al [13] and Hanif et al [10, 11].

3.2 Aflatoxins Deposition on Dried Persimmon

The analysis of variance (Table 1) showed that drying temperature and natural preservatives as well as their interaction have a significant effect ($\alpha \leq 0.01$) on reducing the AFs in dried persimmons. The data of AFs extracted on each sample are given in Table 2. The maximum amount of 31.7 μg kg^{-1} AFs was found in samples treated with aloe vera gel and dried under a temperature of 40^{o}C while the minimum of 7.1 μg kg^{-1} was found in samples treated with honey and dried under a

temperature of 60°C. The means comparison of temperature taken as a main factor showed that the higher AFs of 21.5 µg kg^{-1} was recorded under a temperature of 40°C followed by 16.8 µg kg^{-1} recorded under a temperature of 50°C and the lowest in 9.7µg kg^{-1} was recorded under a temperature of 60°C. The means comparison of natural preservatives showed that the higher AFs of 23.8 µg kg^{-1} was recorded in persimmons treated with aloe vera followed by 14.8 µg kg^{-1} recorded in the dried persimmons treated with aloe vera and honey while the low of 9.4 µg kg^{-1} AFs was recorded for persimmons treated with honey. This is due to the reason that the honey was having strong concentration and power reduced mycoflora germination. That is why the samples treated with honey showed reduced AFs levels as compared to aloe vera. Fungal growth was minimum on dried persimmons treated with honey. These results are in accordance with the findings of Hyun and Woo [1], Kim et al [2], Najmus [5] and Karina et al [7] who reported the same results for aloe vera and honey for reducing AFs in dried persimmons.

Fig. 1. Front view of the flat plate solar air heater used in the experiment.

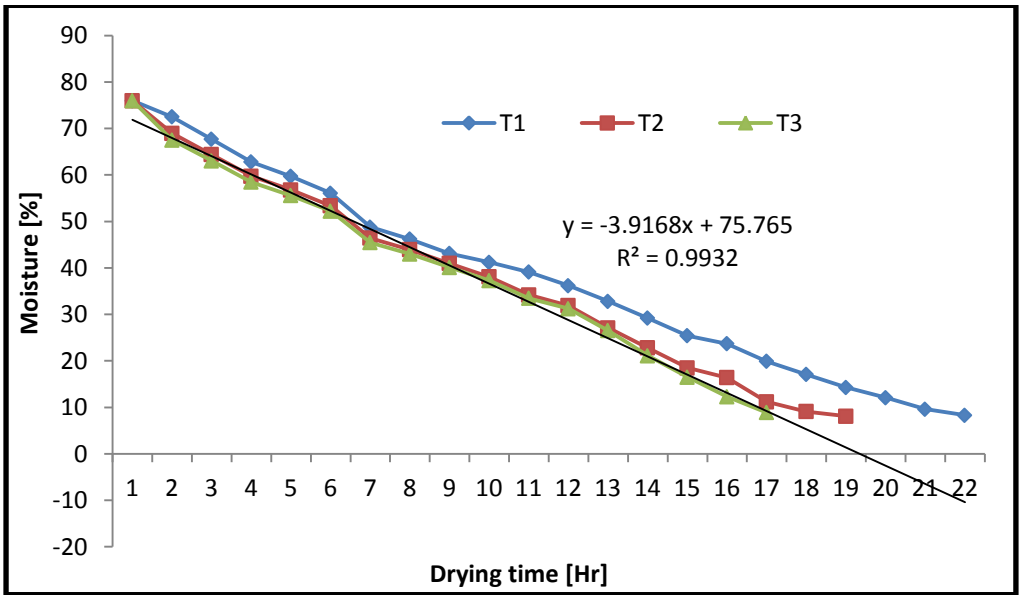

Fig. 2. Moisture losses on wet basis.

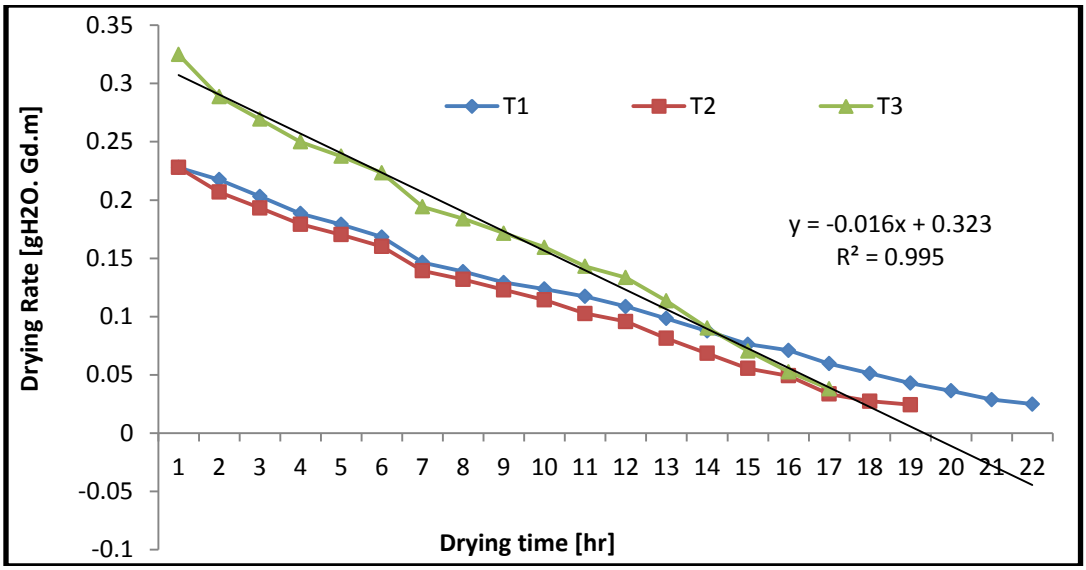

Fig. 3 Moisture losses on dry basis.

Table.1. Analysis of Variance Table for AFs.

Source	DF	MS	P	
A	2	107.053	5.99	**
B	2	159.303	9.55	**
AXB	4	58.0753	12.93	**
Error	9	71.547	17.887	
Total	17	604.260		

Table. 2. Means Comparison of Drying Temperature and natural preservatives on the aflatoxins (µg kg−1) deposition on dried persimmons @ 1% confidence interval **.**

Natural Preservative	Drying Temperature			
	T1	**T2**	**T3**	**Mean**
P1	$31.7^{\alpha,\beta}$	$27.1^{\alpha,\beta}$	12.6^{α}	**23.8a**
P2	11.3^{α}	9.7^{α}	7.1^{α}	**9.4b**
P3	21.6^{α}	13.6^{α}	9.3^{α}	**14.8ab**
Mean	**21.5a**	**16.8ab**	**9.7b**	

LSD= 2.103
α stands for AFs G1 type and β for B1
Means followed by different alphabets are significantly different from each other ($P \leq 0.01$)

4. CONCLUSIONS

It was concluded from the results that drying persimmons at 60°C treated with honey solution to minimize the Mycoflora attacks on them so that minimum AFs is deposited on them. It is recommended that AFs contamination in persimmons requires further investigation, monitoring and routine analysis. Proper harvesting, drying, handling, storage and transport conditions need to be employed on persimmons to reduce AFs in persimmons. Furthermore, different concentrations of honey and aloe vera gel must be tested to achieve more appropriate results.

6. REFERENCES

1. Hyun, K.J. & K.K. Woo. Studies on the population of toxigenic fungi in dried persimmons. *Korean Journal of Applied Micro Biology and Bioengineering* 8(4): 237-245 (1980).
2. Kim, D., J.W. Lee, S.J. Kim, J.W. Choo, S.K. Dungana, Y.S. Lim & D.H. Shin. Exogenous Application of natural extracts of persimmon can help in maintaining nutritional and mineral composition of dried persimmons. *African Journal of Biotechnology* 13(22): 2231-2239 (2014).
3. Lee, S.R. Present studies of mycotoxin in Korea. *Korean Journal of Toxicology* 1(1): 17-30 (1985).
4. Karakasova, L., F. Milenkovska, M. Lazov, B. Karakasov & M. Stojanova. Quality properties of solar dried persimmon (*Diospyros kaki* L). *Journal of Hygienic Engineering and Design* 2(1): 54-59 (2012).
5. Najmus, S., M. Ahmed, Z. Parveen, A. Ilyas & A. Bhutto. Screening of mycotoxins in wheat, fruits and vegetables grown in Sindh, Pakistan. *Pakistan Journal of Botany* 41(1): 337-341 (2009).
6. Sahib, A., H. Shah, H. Khan & N. Magan. The Effect of Substrate, Season, and Agroecological Zone on Mycoflora and Aflatoxin Contamination of Poultry Feed from Khyber Pakhtunkhwa, Pakistan. *Mycopathologia* 174 (4): 341-349 (2012).
7. Karina, D., A. Vega-Gálvez, K. Ah-Hen, Y. Nuñez-Mancilla, G. Tabilo-Munizaga, M. Pérez-Won & C. Giovagnoli. Chemical and physical properties of aloe vera (*Aloe barbadensis* Miller) gel is stored after high hydrostatic pressure processing. *Journal of Food Science and Technology- Campinas* 33(1): 52-59 (2013).
8. Rajeswari, R., M. Umadevi, C.S. Rahale, R. Pushpa, S. elvavenkadesh, D. Kumar & K.P.S. Bhowmik. Aloe vera: The miracle plant, its medicinal and traditional uses in India. *Journal of Pharmacognosy and Phytochem* 1(4): 118-125 (2012).
9. Giray, B., G. Girgin, A. Basak-Engin, S. Aydin & G. Sahin. Aflatoxin levels in wheat samples consumed in some regions of Turkey. *Journal of Food Control* 18 (1): 23–29 (2007).
10. Hanif, M., M. Rahman, M. Amin, M.K. Khattak, M. Ramzan, S. Din & K. Khan. Convective Drying and Mathematical Modeling of Onions Dried Using a Flat Plate Solar Air Heater. *Pakistan Journal of Nutrition* 12 (11): 1003-1007 (2013).
11. Hanif, M., M.K. Khattak, M. Khan & M. Rahman. Evaluation of existing thin layer drying models on drying kinetics of persimmon (*Diospyros kaki* L.). *Journal of Agricultural Research*. 53(2): 269-275 (2015).
12. Steel, R.G.D., J.H. Torrie & D. Dickey. *Principles and Procedures Of Statistics. A Biometrical Approach*, 3rd ed. McGraw Hill Book Co. New York (1997).
13. Yong-Seo. P., S. Jung, S. Kang, E. Delgado-Licon, E. Leticia, M. Ayala, S. Maria, S. Tapia, O.M. Belloso, S. Trakhtenberg & S. Gorinstein. Drying of persimmons (*Diospyros kaki* L.) and the following changes in the studied bioactive compounds and the total radical scavenging activities. *Journal of Food Science and Technology* 39(4): 748–755 (2006).

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Research Article

Hermite-Hadamard Type Inequalities for GA-convex Functions on the Co-ordinates with Applications

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Abstract: In this paper, the concept of GA-convex functions on the co-ordinates is introduced. By using the concept of GA-convex functions on the co-ordinates, theHölder's integral inequality and a new identity established for twice differentiable functions, Hermite-Hadamard type inequalities for this class of functions are established. Finally, applications to special means of positive numbers are given.

Keywords and Phrases: Convex function, Hermite-Hadamard type inequality, co-ordinated GA-convex function, Hölder's integral integral inequality

(AMS SUBJECT CLASSFICATION: Primary: 26A15, 26A51; Secondary 52A30.**)**

1. INTRODUCTION

A function $\boldsymbol{f}: \boldsymbol{I} \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if

$$\boldsymbol{f}(\boldsymbol{\lambda x} + (\boldsymbol{1} - \boldsymbol{\lambda})\boldsymbol{y}) \leq \boldsymbol{\lambda f}(\boldsymbol{x}) + (\boldsymbol{1} - \boldsymbol{\lambda})\boldsymbol{f}(\boldsymbol{y})$$

for all $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{I}$ and $\boldsymbol{\lambda} \in [\boldsymbol{0}, \boldsymbol{1}]$.

One of the most famous inequalities for convex functions is Hermite-Hadamard inequality. This double inequality is stated as follows:

Let $\boldsymbol{f}: \boldsymbol{I} \to \mathbb{R}$be a convex function on some nonempty interval $\boldsymbol{I}$of the set of real numbers $\mathbb{R}$. If $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{I}$with $\boldsymbol{a} < \boldsymbol{b}$. Then

$$\boldsymbol{f}\left(\frac{\boldsymbol{a}+\boldsymbol{b}}{\boldsymbol{2}}\right) \leq \frac{\boldsymbol{1}}{\boldsymbol{b}-\boldsymbol{a}} \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx} \leq \frac{\boldsymbol{f}(\boldsymbol{a})+\boldsymbol{f}(\boldsymbol{b})}{\boldsymbol{2}}.$$

The Hermite-Hadamard inequality has received renewed attention in recent years and a number of papers have been written which provides noteworthy refinements, generalizations and new proofs of the Hermite-Hadamard inequality, see for instance [3, 5, 10, 20], and the references therein.

The classical convexity has been generalized in many ways and one of them is the so called GA-convexity, which is stated in the definition below:

Definition 1 *([14], [15]) A function* $f: I \subseteq \mathbb{R}_0 = [0, \infty) \to \mathbb{R}$ *is said to be GA-convex function on* I *if*

$$f\left(x^{\lambda} y^{1-\lambda}\right) \leq \lambda f(x) + (1-\lambda) f(y)$$

holds for all $x, y \in I$ and $\lambda \in [0,1]$, where $x^{\lambda} y^{1-\lambda}$ and $\lambda f(x) + (1-\lambda) f(y)$ are respectively the weighted geometric mean of two positive numbers x and y and the weighted arithmetic mean of $f(x)$ and $f(y)$.

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In what follows we will use the following notations of means:

For positive numbers $\alpha > 0$ and $\beta > 0$ with $\alpha \neq \beta$

$$A(\alpha,\beta) = \frac{\alpha+\beta}{2}, \qquad L(\alpha,\beta) = \frac{\beta-\alpha}{\ln\beta - \ln\alpha}$$

and

$$L_p(\alpha,\beta) = \begin{cases} \left[\dfrac{\beta^{p+1}-\alpha^{p+1}}{(p+1)(\beta-\alpha)}\right]^{\frac{1}{p}}, & p \neq -1,0 \\ L(\alpha,\beta), & p = -1 \\ \dfrac{1}{e}\left(\dfrac{\beta^{\beta}}{\alpha^{\alpha}}\right)^{\frac{1}{\beta-\alpha}}, & p = 0 \end{cases}$$

are the arithmetic mean, the logarithmic mean and the generalized logarithmic mean of order $p \in \mathbb{R}$ respectively. For further information on means, we refer the readers to [4], [22], [23] and the references therein.

In a very recent paper, Zhang et al. in [26] established the following Hermite-Hadamard type integral inequalities for GA-convex function.

Theorem 1 *[26] Let $f: I \subseteq \mathbb{R}_+ = (0,\infty) \to \mathbb{R}$ be a function differentiable function on I° and $a, b \in I^\circ$ with $a < b$ and $f' \in L([a,b])$. If $|f'|^q$ is GA-convex on $[a,b]$ for $q \geq 1$, we have the following inequality:*

$$\left|bf(b) - af(a) - \int_a^b f(x)dx\right| \leq \frac{[(b-a)A(a,b)]^{1-1/q}}{2^{1/q}}$$
$$\times \left\{[L(a^2,b^2) - a^2]|f'(a)|^q + [b^2 - L(a^2,b^2)]|f'(b)|^q\right\}^{1/q}. \tag{1}$$

Theorem 2 *[26] Let $f: I \subseteq \mathbb{R}_+ = (0,\infty) \to \mathbb{R}$ be a function differentiable function on I° and $a, b \in I^\circ$ with $a < b$ and $f' \in L([a,b])$. If $|f'|^q$ is GA-convex on $[a,b]$ for $q > 1$, we have the following inequality:*

$$\left|bf(b) - af(a) - \int_a^b f(x)dx\right|$$
$$\leq (\ln b - \ln a)\left[L\left(a^{\frac{2q}{(q-1)}}, b^{\frac{2q}{(q-1)}}\right)\right]^{1-\frac{1}{q}}\left[A\left(|f'(a)|^q |f'(b)|^q\right)\right]^{\frac{1}{q}}. \tag{2}$$

Theorem 3 *[26] Let $f: I \subseteq \mathbb{R}_+ = (0,\infty) \to \mathbb{R}$ be a function differentiable function on I° and $a, b \in I^\circ$ with $a < b$ and $f' \in L([a,b])$. If $|f'|^q$ is GA-convex on $[a,b]$ for $q \geq 1$, we have the following inequality:*

$$\left|bf(b) - af(a) - \int_a^b f(x)dx\right| \leq \frac{(\ln b - \ln a)^{1-1/q}}{(2q)^{1/q}}\left[L\left(a^{2q/(q-1)}, b^{2q/(q-1)}\right)\right]^{1-1/q}$$
$$\times \left\{[L(a^{2q},b^{2q}) - a^{2q}]|f'(a)|^q + [b^{2q} - L(a^{2q},b^{2q})]|f'(b)|^q\right\}^{1/q}. \tag{3}$$

Theorem 4 *[26] Let $f: I \subseteq \mathbb{R}_+ = (0,\infty) \to \mathbb{R}$ be a function differentiable function on I° and $a, b \in I^\circ$ with $a < b$ and $f' \in L([a,b])$. If $|f'|^q$ is GA-convex on $[a,b]$ for $q > 1$ and $2q > p > 0$. Then*

$$\left|bf(b)-af(a)-\int_a^b f(x)dx\right| \le \frac{(\ln b-\ln a)^{1-1/q}}{p^{1/q}}\left[L\left(a^{(2q-p)/(q-1)},b^{(2q-p)/(q-1)}\right)\right]^{1-1/q}$$

$$\times\left\{[L(a^p,b^p)-a^p]\left|f'(a)\right|^q+[b^p-L(a^p,b^p)]\left|f'(b)\right|^q\right\}^{1/q}. \quad (4)$$

For more recent results on the class of (α,m)-GA convex functions we refere the interested readers to [9] and the references therein.

We now recall some basic concepts about convex functions on the co-ordinates on rectangle from the plane.

Let $\Delta =: [a,b]\times[c,d]$ in $\mathbb{R}^2$ with $a<b$ and $c<d$ be a bidimensional interval. A mapping $f:\Delta\to\mathbb{R}$ is said to be convex on Δ if the inequality

$$f(\lambda x+(1-\lambda)z,\lambda y+(1-\lambda)w)\le\lambda f(x,y)+(1-\lambda)f(z,w)$$

holds for all $(x,y),(z,w)\in\Delta$ and $\lambda\in[0,1]$.

A modification for convex functions on Δ, known as co-ordinated convex functions, was introduced by Dragomir [6] as follows:

A function $f:\Delta\to\mathbb{R}$ is said to be convex on the co-ordinates on Δ if the partial mappings $f_y:[a,b]\to\mathbb{R}$, $f_y(u)=f(u,y)$ and $f_x:[c,d]\to\mathbb{R}$, $f_x(v)=f(x,v)$ are convex where defined for all $x\in a,b]$, $y\in c,d]$.

Remark 1 *It is clear that if a function $f:\Delta\to\mathbb{R}$ is convex on the co-ordinates on Δ. Then*

$$f(tx+(1-t)z,sy+(1-s)w)$$

$$\le tsf(x,y)+t(1-s)f(x,w)+s(1-t)f(z,y)+(1-t)(1-s)f(z,w),$$

holds for all $(t,s)\in[0,1]\times[0,1]$ and $x,z\in[a,b],y,w\in[c,d]$.

Clearly, every convex mapping $f:\Delta\to\mathbb{R}$ is convex on the co-ordinates but converse may not be true see for instance [6].

The following Hermite-Hadamard type inequalities for co-ordinated convex functions on the rectangle from the plane $\mathbb{R}^2$ were established in [6, Theorem 1, page778]:

Theorem 5 *[6] Suppose that $f:\Delta\to\mathbb{R}$ is co-ordinated convex on Δ, then*

$$f\left(\frac{a+b}{2},\frac{c+d}{2}\right)\le\frac{1}{2}\left[\frac{1}{b-a}\int_a^b f\left(x,\frac{c+d}{2}\right)dx+\frac{1}{d-c}\int_c^d f\left(\frac{a+b}{2},y\right)dy\right]$$

$$\le\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(x,y)dydx$$

$$\le\frac{1}{4}\left[\frac{1}{b-a}\int_a^b[f(x,c)+f(x,d)]dx+\frac{1}{d-c}\int_c^d[f(a,y)+f(b,y)]dy\right]$$

$$\le\frac{f(a,c)+f(a,d)+f(b,c)+f(b,d)}{4}. \quad (5)$$

The above inequalities are sharp.

Most recently, the concept of co-ordinated convexity has been generalized in a diverse manner. A number of papers have been written on Hermite-Hadamard type inequalities for the classes of co-ordinated s-convex functions, co-ordinated m-convex functions, (α,m)-convex functions, co-ordinated h-convex

functions and co-ordinated quasi-convex functions see for example [1]-[3], [6]-[8], [11]-[19], [24] and [25] and the references therein.

Motivated by the above results for GA-convex functions, we first introduce the notion of GA-convex functions on the co-ordinates and establish Hermite-Hadamard type inequalities for this class of functions in Section 2. We will also present applications of our results to special means of positive real numbers in Section

2. MAIN RESULTS

In this section we first give the notion of GA-convex functions on the co-ordinates and then we prove inequalities of Hermite-Hadamard type for this class of functions.

Definition 2 *A function* $f:\Delta \subseteq (0,\infty)\times(0,\infty)\to\mathbb{R}$ *is GA-convex on* Δ *if*

$$f\left(x^{\lambda}z^{1-\lambda}, y^{\lambda}w^{1-\lambda}\right) \le \lambda f(x,y) + (1-\lambda)f(z,w)$$

holds for all $(x,y),(z,w)\in\Delta$ and $\lambda\in[0,1]$.

Definition 3 *A function* $f:\Delta \subseteq (0,\infty)\times 0,\infty)\to\mathbb{R}$ *is said to be GA-convex on the co-ordinates on* Δ *if the partial mappings* $f_y:[a,b]\subseteq(0,\infty)\to\mathbb{R}$, $f_y(u)=f(u,y)$ *and* $f_x:[c,d]\subseteq(0,\infty)\to\mathbb{R}$, $f_x(v)=f(x,v)$ *are GA-convex where defined for all* $x\in[a,b]$, $y\in[c,d]$.

Remark 2 *If a function* $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ *is GA-convex on the co-ordinates on* Δ. *Then*

$$f(x^{t}z^{1-t}, y^{s}w^{1-s}) \le tf(x,y^{s}w^{1-s}) + (1-t)f(z,y^{s}w^{1-s})$$
$$\le t[sf(x,y)+(1-s)f(x,w)] + (1-t)[sf(z,y)+(1-s)f(z,w)]$$
$$\le tsf(x,y) + t(1-s)f(x,w) + s(1-t)f(z,y) + (1-t)(1-s)f(z,w)$$

holds for all $(t,s)\in[0,1]\times[0,1]$ and $x,z\in[a,b], y,w\in[c,d]$.

The following Lemma will be used to establish our main results:

Lemma 1 *Let* $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ *be a twice differentiable mapping on* Δ° *and* $[a,b]\times[c,d]\subseteq\Delta^{\circ}$ *such that* $\frac{\partial^2 f}{\partial s\,\partial t}\in L([a,b]\times[c,d])$. *Then*

$$acf(a,c) - adf(a,d) - bcf(b,c) + bdf(b,d) - b\int_c^d f(b,y)dy$$
$$+a\int_c^d f(a,y)dy - d\int_a^b f(x,d)dx + c\int_a^b f(x,c)dx + \int_a^b\int_c^d f(x,y)dydx$$
$$= (\ln b-\ln a)(\ln d-\ln c)\int_0^1\int_0^1 b^{2t}a^{2(1-t)}d^{2s}c^{2(1-s)}\frac{\partial^2 f(b^{t}a^{1-t}, d^{s}c^{1-s})}{\partial s\,\partial t}dsdt. \qquad (6)$$

Proof. By the change of the variables $x=b^{t}a^{1-t}$, $y=d^{s}c^{1-s}$ and by integration by parts with respect to y and then with respect to x, we have

$$(\ln b-\ln a)(\ln d-\ln c)\int_0^1\int_0^1 b^{2t}a^{2(1-t)}d^{2s}c^{2(1-s)}\frac{\partial^2 f(b^{t}a^{1-t}, d^{s}c^{1-s})}{\partial s\,\partial t}dsdt$$
$$= \int_a^b\int_c^d xy\frac{\partial^2 f(x,y)}{\partial y\,\partial x}dydx = \int_a^b x\left[d\frac{\partial f(x,d)}{\partial x} - c\frac{\partial f(x,c)}{\partial x} - \int_c^d \frac{\partial f(x,y)}{\partial x}dy\right]dx$$

$$= d\int_a^b x\frac{\partial f(x,d)}{\partial x}dx - c\int_a^b x\frac{\partial f(x,c)}{\partial x}dx - \int_c^d\left[\int_a^b x\frac{\partial f(x,y)}{\partial x}dx\right]dy$$

$$= bdf(b,d) - adf(a,d) - bcf(b,c) + acf(a,c) - d\int_a^b f(x,d)dx$$

$$+c\int_a^b f(x,c)dx - b\int_c^d f(b,y)dy + a\int_c^d f(a,y)dy + \int_a^b\int_c^d f(x,y)dydx. \tag{7}$$

Which is the desired identity. This completes the proof of the lemma.

Theorem 6 *Let $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ be a twice differentiable mapping on Δ° and $[a,b]\times[c,d]\subseteq\Delta^\circ$ such that $\frac{\partial^2 f}{\partial s\,\partial t}\in L([a,b]\times[c,d])$. If $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ for $q\geq 1$, is GA-convex on the co-ordinates on $[a,b]\times[c,d]$, we have*

$$\left|acf(a,c) - adf(a,d) - bcf(b,c) + bdf(b,d) - b\int_c^d f(b,y)dy\right.$$

$$\left.+a\int_c^d f(a,y)dy - d\int_a^b f(x,d)dx + c\int_a^b f(x,c)dx + \int_a^b\int_c^d f(x,y)dydx\right|$$

$$\leq \frac{[(b-a)(d-c)A(a,b)A(c,d)]^{1-1/q}}{2^{2/q}}\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q[b^2 - L(a^2,b^2)][d^2 - L(c^2,d^2)]\right.$$

$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q[b^2 - L(a^2,b^2)][L(c^2,d^2) - c^2] + \left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q[L(a^2,b^2) - a^2][d^2 - L(c^2,d^2)]$$

$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q[L(a^2,b^2) - a^2][L(c^2,d^2) - c^2]\right\}^{1/q}. \tag{8}$$

Proof. From Lemma 1, Hölder's inequality for double integrals and By the GA-convexity of $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ for $q\geq 1$ on the co-ordinates on $[a,b]\times[c,d]$, we have

$$\left|acf(a,c) - adf(a,d) - bcf(b,c) + bdf(b,d) - b\int_c^d f(b,y)dy\right.$$

$$\left.+a\int_c^d f(a,y)dy - d\int_a^b f(x,d)dx + c\int_a^b f(x,c)dx + \int_a^b\int_c^d f(x,y)dydx\right|$$

$$\leq a^2c^2(\ln b - \ln a)(\ln d - \ln c)\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}\left|\frac{\partial^2 f(b^t a^{1-t}, d^s c^{1-s})}{\partial s\,\partial t}\right|dsdt$$

$$\leq a^2c^2(\ln b - \ln a)(\ln d - \ln c)\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}dsdt\right]^{1-1/q}$$

$$\times\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}\left|\frac{\partial^2 f(b^t a^{1-t}, d^s c^{1-s})}{\partial s\,\partial t}\right|dsdt\right]^{1/q} \leq a^2c^2(\ln b - \ln a)(\ln d - \ln c)$$

$$\times\left[\frac{(b^2-a^2)(d^2-c^2)}{4a^2c^2(\ln b - \ln a)(\ln d - \ln c)}\right]^{1-1/q}\left[\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}stdsdt\right.$$

$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q \int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} t(1-s)dsdt + \left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q \int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} (1-t)sdsdt$$

$$+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q \int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} (1-t)(1-s)dsdt\Bigg]^{1/q}. \tag{9}$$

Since

$$\int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} stdsdt$$

$$= \left[\frac{2b^2(\ln b-\ln a)-b^2+a^2}{4a^2(\ln b-\ln a)^2}\right]\left[\frac{2d^2(\ln d-\ln c)-d^2+c^2}{4c^2(\ln d-\ln c)^2}\right] = \frac{[b^2-L(a^2,b^2)][d^2-L(c^2,d^2)]}{4a^2c^2(\ln b-\ln a)(\ln d-\ln c)}.$$

Similarly

$$\int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} t(1-s)dsdt = \frac{[b^2-L(a^2,b^2)][L(c^2,d^2)-c^2]}{4a^2c^2(\ln b-\ln a)(\ln d-\ln c)}$$

$$\int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} s(1-t)dsdt = \frac{[L(a^2,b^2)-a^2][d^2-L(c^2,d^2)]}{4a^2c^2(\ln b-\ln a)(\ln d-\ln c)},$$

and

$$\int_0^1\int_0^1 \left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s} t(1-s)dsdt = \frac{[L(a^2,b^2)-a^2][L(c^2,d^2)-c^2]}{4a^2c^2(\ln b-\ln a)(\ln d-\ln c)}.$$

Using the above four equalities in (9) and simplifying, we get the required inequality (8).

Corollary 1 *Under the assumptions of Theorem 6, if* $q = 1$. *Then*

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$

$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$

$$=\frac{1}{4}\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|[b^2-L(a^2,b^2)][d^2-L(c^2,d^2)]\right.$$

$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|[b^2-L(a^2,b^2)][L(c^2,d^2)-c^2]+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|\left[L\left(a^2,b^2\right)-a^2\right]\left[d^2-L\left(c^2,d^2\right)\right]$$

$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|[L(a^2,b^2)-a^2][L(c^2,d^2)-c^2]\right\}. \tag{10}$$

Theorem 7 *Let* $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ *be a twice differentiable mapping on* Δ° *and* $[a,b]\times[c,d]\subseteq\Delta^\circ$ *such that* $\frac{\partial^2 f}{\partial s\,\partial t}\in L([a,b]\times[c,d])$. *If* $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ *for* $q>1$, *is GA-convex on the co-ordinates on* $[a,b]\times[c,d]$, *we have*

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$
$$\leq(\ln b-\ln a)(\ln d-\ln c)\left[L\left(a^{2q/(q-1)},b^{2q/(q-1)}\right)L\left(c^{2q/(q-1)},d^{2q/(q-1)}\right)\right]^{1-1/q}$$
$$\times\left[A\left(\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q,\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q,\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q,\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q\right)\right]^{1/q}. \tag{11}$$

Proof. From Lemma 1, Hölder's inequality for double integrals and By the GA-convexity of $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ for $q>1$ on the co-ordinates on $[a,b]\times[c,d]$, we have

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$
$$\leq a^2c^2(\ln b-\ln a)(\ln d-\ln c)\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}\left|\frac{\partial^2 f(b^ta^{1-t},d^sc^{1-s})}{\partial s\,\partial t}\right|dsdt$$
$$\leq a^2c^2(\ln b-\ln a)(\ln d-\ln c)\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt/(q-1)}\left(\frac{d}{c}\right)^{2qs/(q-1)}dsdt\right]^{1-1/q}$$
$$\times\left[\int_0^1\int_0^1\left|\frac{\partial^2 f(b^ta^{1-t},d^sc^{1-s})}{\partial s\,\partial t}\right|dsdt\right]^{1/q}\leq(\ln b-\ln a)(\ln d-\ln c)$$
$$\times\left[\frac{(q-1)^2\left(b^{2q/(q-1)}-a^{2q/(q-1)}\right)\left(d^{2q/(q-1)}-c^{2q/(q-1)}\right)}{4q^2(\ln b-\ln a)(\ln d-\ln c)}\right]^{1-1/q}$$
$$\times\left[\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1(1-t)(1-s)dsdt+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1 t(1-s)dsdt\right.$$
$$\left.+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1(1-t)sdsdt+\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1 stdsdt\right]^{1/q}. \tag{12}$$

Since

$$\int_0^1\int_0^1 stdsdt=\int_0^1\int_0^1 t(1-s)dsdt=\int_0^1\int_0^1 s(1-t)dsdt=\int_0^1\int_0^1 t(1-s)dsdt=\frac{1}{4}.$$

Hence from (12), we get the required result.

Theorem 8 *Let $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ be a twice differentiable mapping on Δ° and $[a,b]\times[c,d]\subseteq\Delta^\circ$ such that $\frac{\partial^2 f}{\partial s\,\partial t}\in L([a,b]\times[c,d])$. If $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ for $q\geq 1$, is GA-convex on the co-ordinates on $[a,b]\times[c,d]$, we have*

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$
$$\leq\frac{[(\ln b-\ln a)(\ln d-\ln c)]^{1-1/q}}{(2q)^{2/q}}\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q[b^{2q}-L(a^{2q},b^{2q})][d^{2q}-L(c^{2q},d^{2q})]\right.$$
$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q[b^{2q}-L(a^{2q},b^{2q})][L(c^{2q},d^{2q})-c^{2q}]$$
$$+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q[L(a^{2q},b^{2q})-a^{2q}][d^{2q}-L(c^{2q},d^{2q})]$$
$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q[L(a^{2q},b^{2q})-a^{2q}][L(c^{2q},d^{2q})-c^{2q}]\right\}^{1/q}. \tag{13}$$

Proof. From Lemma 1, Hölder's inequality for double integrals and By the GA-convexity of $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ for $q\geq 1$ on the co-ordinates on $[a,b]\times[c,d]$, we have

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$
$$\leq a^2c^2(\ln b-\ln a)(\ln d-\ln c)\left[\int_0^1\int_0^1 1dsdt\right]^{1-1/q}$$
$$\times\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2t}\left(\frac{d}{c}\right)^{2s}\left|\frac{\partial^2 f(b^t a^{1-t},d^s c^{1-s})}{\partial s\,\partial t}\right|^q dsdt\right]^{1/q}$$
$$\leq a^2c^2(\ln b-\ln a)(\ln d-\ln c)\left[\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}stdsdt\right.$$
$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}t(1-s)dsdt+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}(1-t)sdsdt$$
$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}(1-t)(1-s)dsdt\right]^{1/q}. \tag{14}$$

Since

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}stdsdt=\frac{[b^{2q}-L(a^{2q},b^{2q})][d^{2q}-L(c^{2q},d^{2q})]}{4q^2a^{2q}c^{2q}(\ln b-\ln a)(\ln d-\ln c)}.$$

Similarly

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}t(1-s)dsdt=\frac{[b^{2q}-L(a^{2q},b^{2q})][L(c^{2q},d^{2q})-c^{2q}]}{4q^2a^{2q}c^{2q}(\ln b-\ln a)(\ln d-\ln c)}$$

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}s(1-t)dsdt=\frac{[L(a^{2q},b^{2q})-a^{2q}][d^{2q}-L(c^{2q},d^{2q})]}{4q^2a^{2q}c^{2q}(\ln b-\ln a)(\ln d-\ln c)},$$

and

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{2qt}\left(\frac{d}{c}\right)^{2qs}t(1-s)dsdt=\frac{[L(a^{2q},b^{2q})-a^{2q}][L(c^{2q},d^{2q})-c^{2q}]}{4q^2a^{2q}c^{2q}(\ln b-\ln a)(\ln d-\ln c)}.$$

Using the above four in (14) and simplifying, we get the required inequality (13).

Theorem 9 *Let* $f:\Delta\subseteq(0,\infty)\times(0,\infty)\to\mathbb{R}$ *be a twice differentiable mapping on* Δ° *and* $[a,b]\times[c,d]\subseteq\Delta^\circ$ *such that* $\frac{\partial^2 f}{\partial s\,\partial t}\in L([a,b]\times[c,d])$. *If* $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ *is GA-convex on the co-ordinates on* $[a,b]\times[c,d]$ *for* $q>1$ *and* $q>p>1$. *Then*

$$\begin{aligned}&\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.\\&\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|\\&\le\frac{[(\ln b-\ln a)(\ln d-\ln c)]^{1-1/q}}{p^{2/q}}\\&\times\left[L\left(a^{(2q-p)/(q-1)},b^{(2q-p)/(q-1)}\right)L\left(c^{(2q-p)/(q-1)},d^{(2q-p)/(q-1)}\right)\right]^{1-1/q}\\&\times\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q[b^p-L(a^p,b^p)][d^p-L(c^p,d^p)]+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q[b^p-L(a^p,b^p)][L(c^p,d^p)-c^p]\right.\\&+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q[L(a^p,b^p)-a^p][d^p-L(c^p,d^p)]\\&\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q[L(a^p,b^p)-a^p][L(c^p,d^p)-c^p]\right\}^{1/q}.\end{aligned}\tag{15}$$

Proof. From Lemma 1, Hölder's inequality for double integrals and by the GA-convexity of $\left|\frac{\partial^2 f}{\partial s\,\partial t}\right|^q$ on the co-ordinates on $[a,b]\times[c,d]$ for $q>1$ and $2q>p>1$, we have

$$\begin{aligned}&\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.\\&\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|\\&\le a^2c^2(\ln b-\ln a)(\ln d-\ln c)\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{(2q-p)t/(q-1)}\left(\frac{d}{c}\right)^{(2q-p)s/(q-1)}dsdt\right]^{1-1/q}\\&\times\left[\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}\left|\frac{\partial^2 f(b^ta^{1-t},d^sc^{1-s})}{\partial s\,\partial t}\right|^q dsdt\right]^{1/q}\le a^2c^2(\ln b-\ln a)(\ln d-\ln c)\\&\left[\frac{(q-1)^2\left(b^{(2q-p)/(q-1)}-a^{(2q-p)/(q-1)}\right)\left(d^{(2q-p)/(q-1)}-c^{(2q-p)/(q-1)}\right)}{a^{(2q-p)/(q-1)}c^{(2q-p)/(q-1)}(2q-p)^2(\ln b-\ln a)(\ln d-\ln c)}\right]^{1-1/q}\end{aligned}$$

$$\times\left[\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}stdsdt+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}t(1-s)dsdt\right.$$
$$+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}(1-t)sdsdt$$
$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}(1-t)(1-s)dsdt\right]^{1/q}. \tag{16}$$

Since

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}stdsdt=\frac{[b^p-L(a^p,b^p)][d^p-L(c^p,d^p)]}{p^2a^pc^p(\ln b-\ln a)(\ln d-\ln c)}.$$

Similarly

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}t(1-s)dsdt=\frac{[b^p-L(a^p,b^p)][L(c^p,d^p)-c^p]}{p^2a^pc^p(\ln b-\ln a)(\ln d-\ln c)}$$

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}s(1-t)dsdt=\frac{[L(a^p,b^p)-a^p][d^p-L(c^p,d^p)]}{p^2a^{2q}c^{2q}(\ln b-\ln a)(\ln d-\ln c)},$$

and

$$\int_0^1\int_0^1\left(\frac{b}{a}\right)^{pt}\left(\frac{d}{c}\right)^{ps}t(1-s)dsdt=\frac{[L(a^p,b^p)-a^p][L(c^p,d^p)-c^p]}{p^2a^pc^p(\ln b-\ln a)(\ln d-\ln c)}.$$

Using the above four in (16) and simplifying, we get the required inequality (15).

Corollary 2 *Under the assumptions of Theorem 9, if $p=q$, we have the inequality*

$$\left|acf(a,c)-adf(a,d)-bcf(b,c)+bdf(b,d)-b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy-d\int_a^b f(x,d)dx+c\int_a^b f(x,c)dx+\int_a^b\int_c^d f(x,y)dydx\right|$$
$$\le\frac{[(\ln b-\ln a)(\ln d-\ln c)]^{1-1/q}}{q^{2/q}}\left[L\left(a^{q/(q-1)},b^{q/(q-1)}\right)L\left(c^{q/(q-1)},d^{q/(q-1)}\right)\right]^{1-1/q}$$
$$\times\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q[b^q-L(a^q,b^q)][d^q-L(c^q,d^q)]+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q[b^q-L(a^q,b^q)][L(c^q,d^q)-c^q]\right.$$
$$+\left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q[L(a^q,b^q)-a^q][d^q-L(c^q,d^q)]$$
$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q[L(a^q,b^q)-a^q][L(c^q,d^q)-c^q]\right\}^{1/q}. \tag{17}$$

3. APPLICATIONS TO SPECIAL MEAN

In this section we apply our results to establish inequalities for special means.

Theorem 10 *For* $b > a > 0,\ d > c > 0,\ s_1,\ s_2 > 0,\ q \geq 1$ *and* $s_1 q \neq 1,\ s_2 q \neq 1$, *we have*

$$\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1} \leq \frac{[(a+b)(c+d)]^{1-1/q}}{4}$$
$$\times\left[\left\{(s_1q+2)\left[L_{s_1q+1}(a,b)\right]^{s_1q+1} - s_1qL(a^2,b^2)\left[L_{s_1q-1}(a,b)\right]^{s_1q-1}\right\}\right.$$
$$\left.\times\left\{(s_2q+2)\left[L_{s_2q+1}(c,d)\right]^{s_2q+1} - s_2qL(c^2,d^2)\left[L_{s_2q-1}(c,d)\right]^{s_2q-1}\right\}\right]^{1/q}. \tag{18}$$

Proof. Let

$$f(x,y) = \frac{x^{s_1+1}y^{s_2+1}}{(s_1+1)(s_2+1)}, (x,y) \in (0,\infty)\times(0,\infty), s_1, s_2 > 0.$$

Then $\left|\frac{\partial^2 f(x,y)}{\partial y\,\partial x}\right|^q = x^{s_1q}y^{s_2q}$ is GA-convex on the co-ordinates on $(0,\infty)\times(0,\infty)$ and both sides of the inequality (8) in Theorem 6 become

$$\left|acf(a,c) - adf(a,d) - bcf(b,c) + bdf(b,d) - b\int_c^d f(b,y)dy\right.$$
$$\left.+a\int_c^d f(a,y)dy - d\int_a^b f(x,d)dx + c\int_a^b f(x,c)dx + \int_a^b\int_c^d f(x,y)dydx\right|$$
$$= (b-a)(d-c)\left[\frac{b^{s_1+2}-a^{s_1+2}}{(s_1+2)(b-a)}\right]\left[\frac{d^{s_2+2}-c^{s_2+2}}{(s_2+2)(d-c)}\right]$$
$$= (b-a)(d-c)\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1}.$$
$$\frac{[(b-a)(d-c)A(a,b)A(c,d)]^{1-1/q}}{2^{2/q}}\left\{\left|\frac{\partial^2 f(b,d)}{\partial s\,\partial t}\right|^q [b^2 - L(a^2,b^2)][d^2 - L(c^2,d^2)]\right.$$
$$+\left|\frac{\partial^2 f(b,c)}{\partial s\,\partial t}\right|^q [b^2 - L(a^2,b^2)][L(c^2,d^2) - c^2] + \left|\frac{\partial^2 f(a,d)}{\partial s\,\partial t}\right|^q [L(a^2,b^2) - a^2][d^2 - L(c^2,d^2)]$$
$$\left.+\left|\frac{\partial^2 f(a,c)}{\partial s\,\partial t}\right|^q [L(a^2,b^2) - a^2][L(c^2,d^2) - c^2]\right\}^{1/q} = \frac{(b-a)(d-c)[(a+b)(c+d)]^{1-1/q}}{4}$$
$$\times\left[\left\{(s_1q+2)\left[L_{s_1q+1}(a,b)\right]^{s_1q+1} - s_1qL(a^2,b^2)\left[L_{s_1q-1}(a,b)\right]^{s_1q-1}\right\}\right.$$
$$\left.\times\left\{(s_2q+2)\left[L_{s_2q+1}(c,d)\right]^{s_2q+1} - s_2qL(c^2,d^2)\left[L_{s_2q-1}(c,d)\right]^{s_2q-1}\right\}\right]^{1/q}.$$

A combination of the above two equalities gives us the desired inequality (18).

Corollary 3 *Under the assumptions of Theorem 10, if* $q = 1$ *and* $s_1,\ s_2 \neq 1$. *Then*

$$\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1} \leq \frac{1}{4}\left[\left\{(s_1+2)\left[L_{s_1+1}(a,b)\right]^{s_1+1} - s_1L(a^2,b^2)\left[L_{s_1-1}(a,b)\right]^{s_1-1}\right\}\right.$$
$$\left.\times\left\{(s_2+2)\left[L_{s_2+1}(c,d)\right]^{s_2+1} - s_2L(c^2,d^2)\left[L_{s_2-1}(c,d)\right]^{s_2-1}\right\}\right]. \tag{19}$$

Theorem 11 *Let* $b > a > 0,\ s_1, s_2 > 0$ *and* $q > 1$. *Then*

$$[L(a,b)][L(c,d)]\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1}$$
$$\leq \left[L\left(a^{2q/(q-1)},b^{2q/(q-1)}\right)L\left(c^{2q/(q-1)},d^{2q/(q-1)}\right)\right]^{1-1/q}$$
$$\times [A(a^{s_1q},b^{s_2q})A(c^{s_1q},d^{s_2q})]^{1/q}. \quad (20)$$

Proof. The proof follows from Theorem 7 and using the following GA- convex function on the co-ordinates on $(0,\infty)\times(0,\infty)$

$$f(x,y)=\frac{x^{s_1+1}y^{s_2+1}}{(s_1+1)(s_2+1)},(x,y)\in(0,\infty)\times(0,\infty),s_1,s_2>0.$$

The following interesting inequalities of means can be obtained using the results of Theorem 8 and Theorem 9 and the GA-convex function on the co-ordinates on $(0,\infty)\times(0,\infty)$ as defined in Theorem 10, however the details are left to the interested reader.

Theorem 12 *Let* $b>a>0,\ s_1,s_2>0$ *and* $s_1q,s_2q\neq 1$. *Then*

$$[L(a,b)L(c,d)]^{1-1/q}\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1}\leq\frac{1}{(2q)^{2/q}}$$
$$\times\left\{(s_1+2)q\left[L_{(s_1+2)q-1}(a,b)\right]^{(s_1+2)q-1}-s_1qL(a^{2q},b^{2q})\left[L_{s_1q-1}(a,b)\right]^{s_1q-1}\right\}^{1/q}$$
$$\times\left\{(s_2+2)q\left[L_{(s_2+2)q-1}(c,d)\right]^{(s_2+2)q-1}-s_2qL(c^{2q},d^{2q})\left[L_{s_2q-1}(c,d)\right]^{s_2q-1}\right\}^{1/q}. \quad (21)$$

Theorem 13 *Let* $b>a>0,\ s_1,s_2>0,\ q>1,2q>p>0$ *and* $s_1q,s_2q\neq 1$. *Then*

$$[L(a,b)L(c,d)]^{1-1/q}\left[L_{s_1+1}(a,b)\right]^{s_1+1}\left[L_{s_2+1}(c,d)\right]^{s_2+1}$$
$$\leq\frac{1}{(p)^{2/q}}\left[L\left(a^{(2q-p)/(q-1)},b^{(2q-p)/(q-1)}\right)L\left(c^{(2q-p)/(q-1)},d^{(2q-p)/(q-1)}\right)\right]^{1-1/q}$$
$$\left\{(p+s_1q)\left[L_{p+s_1q-1}(a,b)\right]^{p+s_1q-1}-s_1qL(a^p,b^p)\left[L_{s_1q-1}(a,b)\right]^{s_1q-1}\right\}^{1/q}$$
$$\times\left\{(s_2+2)q\left[L_{(s_2+2)q-1}(c,d)\right]^{(s_2+2)q-1}-s_2qL(c^{2q},d^{2q})\left[L_{s_2q-1}(c,d)\right]^{s_2q-1}\right\}^{1/q}. \quad (22)$$

4. CONLUSIONS

In our paper, a new notion of GA-convex fucntions on the co-ordinates on the rectangle from the plane is introduced and some of the properties of this class of functions are discussed. A new integral inequality for twice differitiable mappings of two variables which aredefined on a rectangle from the plane is established. By using the notion of GA convexity of the mappings on the co-ordinates, Holder inequality and mathematical analysis, some new inequalities of Hermite-Hadamard type are established. Applications of our results to special means of positive real numbers are given as well. We believe that by using the notion GA conveity on the co-ordinates introduced in this article and some identitities for functions of two variables, many other intersting inequalities of Hermite-Hadamard type can be investigated. Moreover, some weighted generilazations can also be proved by using some appropriate choice of the weight function.

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6. REFERENCES

1. Alomari, M. & M. Darus. Co-ordinated s-convex function in the first sense with some Hadamard-type inequalities. *International Journal of Contemporary Mathematical Sciences* 32 (3):1557-1567 (2008).
2. Alomari, M. &M. Darus. The Hadamard's inequality for s-convex function of 2-variables on the co-ordinates. *International Journal of Mathematical Analysis* 2 (13): 629–638 (2008).
3. Alomari, M., M. Darus& S.S. Dragomir. Inequalities of Hermite-Hadamard's type for functions whose derivatives absolute values are quasi-convex. *Research Group of Mathematical Inequalities and Applications, Research Report Collection* 12 (suppl. 14) (2009).
4. Bullen, P.S. *Handbook of Means and Their Inequalities*. Kluwer Academic Publishers, Dordrecht (2003).
5. Dragomir, S.S. & R. P. Agarwal. Two inequalities for differentiable mappings and applications to special means of real numbers and to Trapezoidal formula. *Applied Mathematics Lett*ers 11(5): 91-95 (1998).
6. Dragomir, S. S. On Hadamard's inequality for convex functions on the co-ordinates in a rectangle from the plane.*Taiwanese Journal of Mathematics* 4: 775-788 (2001).
7. Dragomir, S. S. & C.E.M. Pearce.*Selected Topics on Hermite-Hadamard Inequalities and Applications*. Research Group of Mathematical Inequalities and Applications, Monographs, Victoria University, 2000. Online: [http://www.staff.vu.edu.au/RGMIA/monographs/hermite_hadamard.html].
8. Hwang, D. –Y., K. L. Tseng & G.S. Yang. Some Hadamard's inequalities for co-ordinated convex functions in a rectangle from the plane. *Taiwanese Journal of Mathematics* 11: 63-73 (2007).
9. Ji, A.-P., T.-Y. Zhang & F. Qi. Integral inequalities of Hermite-Hadamard type for (α, m)-GA-convex functions.*Journal of Function Spaces and Applications* 2013: Article ID 823856, 8 pages (2013).
10. Kirmaci, U.S. Inequalities for differentiable mappings and applications to special means of real numbers to midpoint formula. *Applied Mathematics and Comput*ation 147: 137-146 (2004).
11. Latif, M.A. & M. Alomari. Hadamard-type inequalities for product two convex functions on the co-ordinates. *International Mathematical Forum* 4(47): 2327-2338 (2009).
12. Latif, M.A. & M. Alomari. On the Hadamard-type inequalities for h-convex functions on the co-ordinates. *International Journal of Mathematical Analysis* 3(33): 1645-1656 (2009).
13. Latif, M.A. & S.S. Dragomir. On Some New inequalities for differentiable co-ordinated convex functions.*Journal of Inequalities and Applications* 2012: Article 28 (2012).
14. Niculescu, C. P. Convexity according to the geometric mean. *Mathematical Inequalities and Appl*ications 3 (2) 155-167 (2000).
15. Niculescu, C.P. Convexity according to means. *Mathematical Inequalities and Appl*ications 6 (4) 571-579 (2003).
16. Özdemir, M.E., E. Set & M.Z. Sarikaya. New some Hadamard's type inequalities for coordinated m-convex and (α,m)-convex functions. *Research Group of Mathematical Inequalities and Applications, Research Report Collection* 13 (Supplement): Article 4 (2010).
17. Özdemir, M.E., H.Kavurmac, A. O. Akdemir & M. Avc. Inequalities for convex and s-convex functions on Δ=[a,b]×[c,d].*Journal of Inequalities and Applications* 2012: Article 20 (2012).
18. Özdemir, M.E., M. A. Latif & A. O.Akdemir, On some Hadamard-type inequalities for product of two s-convex functions on the co-ordinates.*Journal of Inequalities and Applications* 2012: Article 21 (2012).
19. Özdemir, M E., A. O. Akdemir & M. Tunc. On the Hadamard-type inequalities for co-ordinated convex functions.*arXiv*: 1203.4327v1[math.CA] (2012).
20. Pearce, C.M. E. & J. E. Pečarić. Inequalities for differentiable mappings with applications to special means and quadrature formula. *Applied Mathematics Letters* 13: 51-55 (2000).
21. Pečarić, J.E., F. Proschan& Y. L. Tong.*Convex Functions, Partial Ordering and Statistical Applications*, Academic Press, New York(1991).
22. Qi, F. Generalized abstracted mean values. *Journal of Inequalities in Pure Applied Mathematics* 1 (1): Article 4 (2000).
23. Qi, F. Generalized weighted mean values with two parameters. *Proceedings of the Royal Society A: Mathematical, Physical & Engineering Sciences* 454(1998): 2723–2732 (1998).
24. Sarikaya, M.Z., E. Set, M.E. Özdemir & S.S. Dragomir. New some Hadamard's type inequalities for co-ordinated convex functions.*arXiv*:1005.0700v1 [math.CA] (2010).
25. Sarikaya, M.Z. On the Hermite-Hadamard-type inequalities for co-ordinated convex function via fractional integrals.*Integral Transforms and Special Functions*25(2): 134-147 (2014).
26. Zhang, T.–Y., A.-P. Ji & F. Qi. Some inequalities of Hermite-Hadamard type for GA-Convex functions with applications to means. *Le Matematiche* 48(1): 229-239 (2013).

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Research Article

An Efficient Algorithm for Nonlinear Fractional Partial Differential Equations

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Abstract: In this paper, we used a newly developed fractional complex transform to convert the given fractional partial differential equations into corresponding partial differential equations and subsequently Variational Iteration Method is applied on the transformed partial differential equations. Three examples are illustrated to show the effectiveness and applicability of the transform. The obtained results are very audacious and hence the same can be applied to other mathematical problems.

Keywords: Fractional differential equation; Fractional complex transform; Variational iteration method
AMS Mathematics Subject Classification: 35R11.

1. INTRODUCTION

Fractional calculus of arbitrary order [1-2] has been used to model much physical and engineering process that are found to be best described by fractional differential equations. Considerable attention has been given to the solution of fractional differential equations, integral equations and system of fractional partial differential equations of physical interest. Most fractional differential equations do not have exact analytic solutions, analytic and numerical techniques, therefore, are used extensively for the solutions of such problems. The detailed study of literature reflects the implementation of wide range of numerical and analytical techniques (including Finite Difference [3-5], Adomian's Decomposition [6-10], Exp function [11-12], Homotopy Perturbation [13], Reduce Differential Transform [14], Homotopy Analysis [15], and Variational Iteration [16-17] for the solutions of linear and nonlinear equations of fractional-order. Inspired and motivated by the ongoing research in this area, we use a fractional complex transform (FCT) [18-19] in order to convert the given fractional partial differential equations (FPDEs) into corresponding partial differential equations (PDEs); subsequently Variational Iteration Method (VIM) is applied on the transformed PDEs and inverse transformation yields the results it in terms of original variables. Computational work re-confirms that the proposed algorithm is highly efficient, fully compatible, and extremely appropriate for factional PDEs arising in mathematical physics and hence can be extended to other problems of diversified nonlinear nature. In particular, we focus our attention on three very important equations which are called Benjamin–Bona–Mahoney (BM) Equation [20], Cahn-Hilliard equation [21] and Gardner equation [22]. Numerical results are very encouraging.

2. FRACTIONAL COMPLEX TRANSFORM

The fractional complex transform was first proposed [23] and is defined as

$$\begin{cases} T = \dfrac{p\,t^{\alpha}}{\Gamma(\alpha+1)} \\ X = \dfrac{q\,x^{\beta}}{\Gamma(\beta+1)} \\ Y = \dfrac{k\,y^{\gamma}}{\Gamma(1+\gamma)} \\ Z = \dfrac{l\,z^{\delta}}{\Gamma(1+\delta)} \end{cases} \tag{1}$$

where p, q, k, and l are unknown constants, $0 < \alpha \le 1, \quad 0 < \beta \le 1, \quad 0 < \gamma \le 1, \quad 0 < \delta \le 1.$

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3. VARIATIONAL ITERATION METHOD

To illustrate the basic concept of variational iteration method, we consider the following general nonlinear differential equation given in the form:

$$Lu(t)+Nu(t)=g(t), \tag{2}$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a known analytical function. We can construct a correction functional as

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda(Lu_n(\xi)+N\tilde{u}_n(\xi)-g(\xi))d\xi \tag{3}$$

where λ is a general Lagrange multiplier, which can be identified optimally via variational theory, the subscript n denotes the nth approximation, and $\tilde{u}_n$ is considered as a restricted variation, namely $\delta\tilde{u}_n=0$. It is obvious that the successive approximation u_j, $j\geq 0$ can be established by determining general Lagrange's multiplier λ, which can be identified optimally via the variational theory. Therefore, we first determine Lagrange's multiplier that will be identified optimally via integration by parts. The successive approximation of the $u_{n+1}(x,t)$, $n\geq 0$ solution $u(x,t)$ will be readily obtained upon using the Lagrange's multiplier and by using any selective function u_0. The initial values are usually used for selecting the zeroth approximation u_0. With λ determined, several approximations u_j, $j\geq 0$ follows immediately. Consequently, the exact solution may be obtained by using

$$u(x,t)=\lim_{x\to\infty} u_n(x,t). \tag{4}$$

4. SOLUTION PROCEDURE

4.1 Benjamin–Bona–Mahoney (BM) Equation

$$\frac{\partial^\alpha u}{\partial t^\alpha}-\frac{\partial^3 u}{\partial^2 x\,\partial t}+\frac{\partial u}{\partial x}+u\left(\frac{\partial u}{\partial x}\right)=0, \tag{5}$$

with the initial condition $u(x,0)=\sec h^2\left(\frac{x}{4}\right)$.

Applying the transformation [23], we get the following partial differential equation

$$\frac{\partial u}{\partial T}-\frac{\partial^3 u}{\partial^2 x\,\partial t}+\frac{\partial u}{\partial x}+u\left(\frac{\partial u}{\partial x}\right)=0, \tag{6}$$

The correction functional can be written in the form:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^T \lambda(s)\left(\frac{\partial u}{\partial T}-\frac{\partial^3 u}{\partial^2 x\,\partial t}+\frac{\partial u}{\partial x}+u\left(\frac{\partial u}{\partial x}\right)\right)ds. \tag{7}$$

The stationary conditions yield

$$1+\lambda=0,\quad \lambda'=0.$$

This in turn gives

$$\lambda=-1. \tag{8}$$

Hence (7) takes the form

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^T\left(\frac{\partial u}{\partial T}-\frac{\partial^3 u}{\partial^2 x\,\partial t}+\frac{\partial u}{\partial x}+u\left(\frac{\partial u}{\partial x}\right)\right)ds. \tag{9}$$

Consequently,

$$u_0(x,T)=\sec h^2\left(\frac{x}{4}\right),\ u_1(x,T)=\sec h^2\left(\frac{x}{4}\right)+\frac{1}{2}\sec h^2\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)T-\frac{1}{4}\sec h^4\left(\frac{x}{4}\right)\tanh^2\left(\frac{x}{4}\right)T,$$

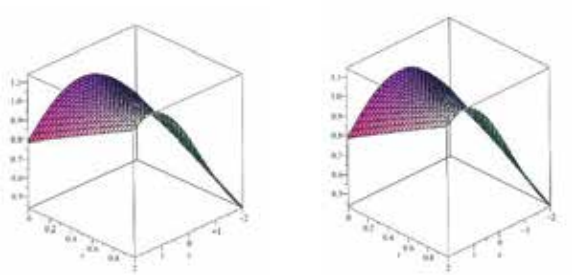
$$u_2(x,T)=\sec h^2\left(\frac{x}{4}\right)+\frac{1}{2}\sec h^2\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)T$$
$$-\frac{1}{2}\begin{pmatrix}\frac{1}{2}\sec h^2\left(\frac{x}{4}\right)\left(\frac{1}{4}-\frac{1}{4}\tanh^2\left(\frac{x}{4}\right)\right)+\frac{1}{4}\sec h^4\left(\frac{x}{4}\right)\tanh^3\left(\frac{x}{4}\right)\\-\frac{1}{2}\sec h^4\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)\left(\frac{1}{4}-\frac{1}{4}\tanh^2\left(\frac{x}{4}\right)\right)-\frac{1}{4}\sec h^2\left(\frac{x}{4}\right)\tanh^2\left(\frac{x}{4}\right)\end{pmatrix}T^2+\frac{1}{2}\sec h^4\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right),$$

$\vdots$.

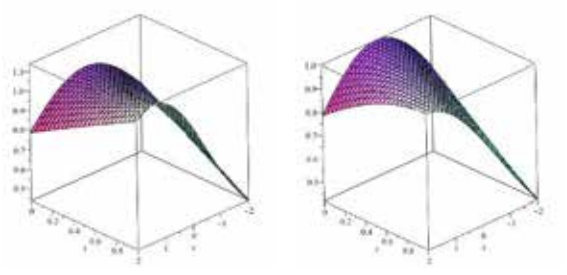
The inverse transformation will yield

$$u_2(x,t)=\sec h^2\left(\frac{x}{4}\right)+\frac{1}{2}\sec h^2\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)\frac{t^\alpha}{\Gamma(\alpha+1)}$$
$$-\frac{1}{2}\begin{pmatrix}\frac{1}{2}\sec h^2\left(\frac{x}{4}\right)\left(\frac{1}{4}-\frac{1}{4}\tanh^2\left(\frac{x}{4}\right)\right)+\frac{1}{4}\sec h^4\left(\frac{x}{4}\right).\\\tanh^3\left(\frac{x}{4}\right)-\frac{1}{2}\sec h^4\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)\left(\frac{1}{4}-\frac{1}{4}\tanh^2\left(\frac{x}{4}\right)\right)\\-\frac{1}{4}\sec h^2\left(\frac{x}{4}\right)\tanh^2\left(\frac{x}{4}\right)\end{pmatrix}\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}\qquad(10)$$
$$+\frac{1}{2}\sec h^4\left(\frac{x}{4}\right)\tanh\left(\frac{x}{4}\right)\frac{t^\alpha}{\Gamma(\alpha+1)}.$$

Graphical representation of approximate and exact solutions of (5) for different values of α, using only three iterations of the VIM solution



$(a)\ \alpha=0.4$ $(b)\ \alpha=0.8$



$(c)\ \alpha=1$ (d) *Exact solution*

4.2 Cahn-Hilliard (CH) Equation

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} - \frac{\partial^2 u}{\partial x^2} + u^3 - u = 0, \tag{11}$$

with initial condition $u(x,0) = \dfrac{1}{1+e^{\frac{x}{\sqrt{2}}}}$.

Applying the transformation [23], we get the following partial differential equation

$$\frac{\partial u}{\partial T} - \frac{\partial^2 u}{\partial x^2} + u^3 - u = 0, \tag{12}$$

The correction functional can be written in the form

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^T \lambda(s)\left(\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + u^3 - u\right) ds. \tag{13}$$

The stationary conditions yields

$$1 + \lambda = 0, \ \lambda' = 0.$$

This in turn gives

$$\lambda = -1. \tag{14}$$

Hence (13) takes the form:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^T \left(\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + u^3 - u\right) ds. \tag{15}$$

Consequently,

$$u_0(x,T) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}}, \ u_1(x,T) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} + \frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} T - \frac{1}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2} T - \frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} T + \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} T,$$

$$u_2(x,T) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} + \frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} T - \frac{1}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2} T - \frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} T + \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} T$$

$$-\frac{1}{2}\left[\begin{array}{l} -\dfrac{11}{2}\dfrac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} + 9\dfrac{\left(e^{\frac{x}{\sqrt{2}}}\right)^3}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^4} + \dfrac{5}{4}\dfrac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2} + 6\dfrac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^5} - \dfrac{3}{2}\dfrac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^4} - 6\dfrac{\left(e^{\frac{x}{\sqrt{2}}}\right)^4}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^5} \\ \dfrac{3}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2}\left(\dfrac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} - \dfrac{1}{2}\dfrac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2} - \dfrac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} + \dfrac{1}{1+e^{\frac{x}{\sqrt{2}}}}\right) + \dfrac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} - \dfrac{1}{1+e^{\frac{x}{\sqrt{2}}}} \end{array}\right] T^2,$$

$\vdots$

The inverse transformation will yield

$$u_2(x,t) = \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} + \frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} \frac{t^{\alpha}}{\Gamma(\alpha+1)} - \frac{1}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2} \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

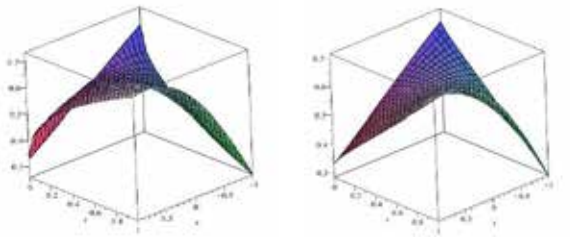
$$-\frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3} \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{1}{1+e^{\frac{x}{\sqrt{2}}}} \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

$$u_2(x,t)=\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}+\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}\frac{t^\alpha}{\Gamma(\alpha+1)}-\frac{1}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2}\frac{t^\alpha}{\Gamma(\alpha+1)}$$

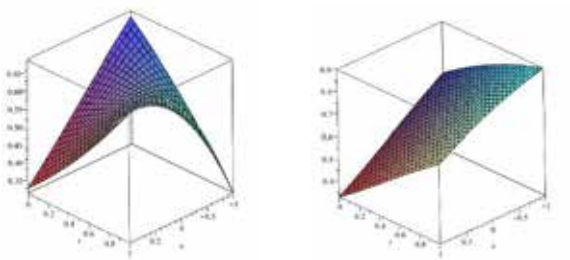
$$-\frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}\frac{t^\alpha}{\Gamma(\alpha+1)}+\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}\frac{t^\alpha}{\Gamma(\alpha+1)}$$

$$-\frac{1}{2}\left(\begin{array}{l}-\frac{11}{2}\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}+9\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^3}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^4}+\frac{5}{4}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2}+6\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^5}\\ -\frac{3}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^4}-6\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^4}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^5}\frac{3}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2}\left(\begin{array}{l}\frac{\left(e^{\frac{x}{\sqrt{2}}}\right)^2}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}-\frac{1}{2}\frac{e^{\frac{x}{\sqrt{2}}}}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^2}\\ -\frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}+\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}\end{array}\right)\\ +\frac{1}{\left(1+e^{\frac{x}{\sqrt{2}}}\right)^3}-\frac{1}{1+e^{\frac{x}{\sqrt{2}}}}\end{array}\right)\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}.$$

Graphical representation of approximate and exact solutions of (11) for different values of α, using only three iterations of the VIM solution



$(a)\,\alpha=0.4$ $(b)\,\alpha=0.8$



$(c)\,\alpha=1$ $(d)\,Exact\ solution$

4.3 Gardner Equation

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} - \frac{\partial^{3} u}{\partial x^{3}} - 6u^{2}\left(\frac{\partial u}{\partial x}\right) - 6u = 0, \tag{16}$$

with initial condition

$$u(x,0) = -\frac{1}{2}\left(1 - \tanh\left(\frac{x}{2}\right)\right).$$

Applying the transformation [23], we get the following partial differential equation

$$\frac{\partial u}{\partial T} - \frac{\partial^{3} u}{\partial x^{3}} - 6u^{2}\left(\frac{\partial u}{\partial x}\right) - 6u = 0, \tag{17}$$

The correction functional can be written in the form

$$u_{n+1}(x,t) = u_{n}(x,t) + \int_{0}^{T} \lambda(s)\left(\frac{\partial u}{\partial T} - \frac{\partial^{3} u}{\partial x^{3}} - 6u^{2}\left(\frac{\partial u}{\partial x}\right) - 6u\right) ds. \tag{18}$$

The stationary conditions yields

$$1 + \lambda = 0, \qquad \lambda' = 0.$$

This in turn gives

$$\lambda = -1, \tag{19}$$

Hence (18) takes the form

$$u_{n+1}(x,t) = u_{n}(x,t) - \int_{0}^{T}\left(\frac{\partial u}{\partial T} - \frac{\partial^{3} u}{\partial x^{3}} - 6u^{2}\left(\frac{\partial u}{\partial x}\right) - 6u\right)(ds). \tag{20}$$

Consequently,

$$u_{0}(x,T) = -\frac{1}{2}\left(1 - \tanh\left(\frac{x}{2}\right)\right),$$

$$u_{1}(x,T) = -\frac{1}{2}\left(1 - \tanh\left(\frac{x}{2}\right)\right) - 3T - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\tanh^{2}\left(\frac{x}{2}\right)\right)^{2} T + \frac{1}{2}\tanh^{2}\left(\frac{x}{2}\right)\left(\frac{1}{2} - \frac{1}{2}\tanh^{2}\left(\frac{x}{2}\right)\right)T$$

$$+3\tanh\left(\frac{x}{2}\right)T + 6\left(-\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{2}\right)\right)^{2}\left(\frac{1}{4} - \frac{1}{4}\tanh^{2}\left(\frac{x}{2}\right)\right)^{2} T,$$

$$u_{2}(x,T) = -\frac{9}{4}T^{3}\tanh^{9}\left(\frac{x}{2}\right) + \frac{63}{16}T^{3}\tanh^{8}\left(\frac{x}{2}\right) + \frac{1}{16}\left(90T^{2} + 45\,T^{3}\right).$$

$$\tanh^{7}\left(\frac{x}{2}\right) + \frac{21}{8}T^{3}\tanh^{16}\left(\frac{x}{2}\right) + \frac{1}{16}\left(-270T^{2} - 275\,T^{3}\right)\tanh^{5}\left(\frac{x}{2}\right)$$

$$+\frac{1}{16}\left(-6T + 60\,T^{3}\right)\tanh^{4}\left(\frac{x}{2}\right) + \frac{1}{16}\left(-12T + 211T^{3} + 200T^{2}\right).$$

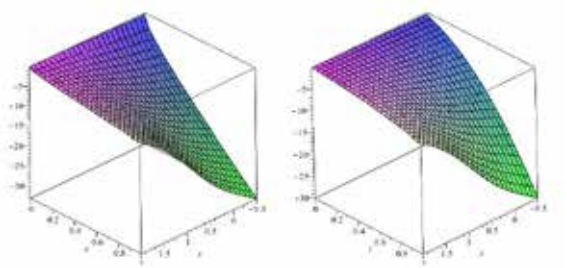
$$\tanh^{3}\left(\frac{x}{2}\right) + \frac{1}{16}\left(48T^{2} + 20T + 54\,T^{3}\right)\tanh^{2}\left(\frac{x}{2}\right) +$$

$$\frac{1}{16}\left(124T^{2} + 60T + 8 + 37\,T^{3}\right)\tanh\left(\frac{x}{2}\right) - \frac{31}{8}T - 12T^{2} - \frac{1}{2} - \frac{99}{16}T^{3}$$

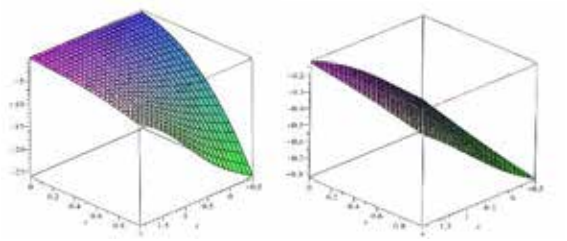
The inverse transformation will yield

$$u_1(x,t) = -\frac{9}{4}\tanh^9\left(\frac{x}{2}\right)\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)} + \frac{63}{16}\tanh^8\left(\frac{x}{2}\right)\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}$$
$$+\frac{1}{16}\left(90\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)} + 45\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}\right)\tanh^7\left(\frac{x}{2}\right) + \frac{21}{8}\tanh^{16}\left(\frac{x}{2}\right).$$
$$\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)} + \frac{1}{16}\left(-270\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)} - 275\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}\right)\tanh^5\left(\frac{x}{2}\right) +$$
$$\frac{1}{16}\left(-6\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 60\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}\right)\tanh^4\left(\frac{x}{2}\right) +$$
$$\frac{1}{16}\left(-12\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 211\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)} + 200\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}\right)\tanh^3\left(\frac{x}{2}\right)$$
$$+\frac{1}{16}\left(48\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)} + 20\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 54\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}\right)\tanh^2\left(\frac{x}{2}\right)$$
$$+\frac{1}{16}\left(124\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)} + 60\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 8 + 37\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}\right)\tanh\left(\frac{x}{2}\right)$$
$$-\frac{31}{8}\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 12T\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)} - \frac{1}{2} - \frac{99}{16}\frac{t^{3\alpha}}{\Gamma^3(\alpha+1)}.$$

Graphical representation of approximate and exact solutions of (16) for different values of α, using only three iterations of the VIM solution



$(a)\,\alpha = 0.4$ $(b)\,\alpha = 0.8$



$(c)\,\alpha = 0.4$ $(d)\,Exact\ solution$

5. CONCLUSIONS

In this paper, variational iteration method (VIM) has been successfully implemented to find approximate solutions for nonlinear partial differential equations of fractional order by considering a change of variable to a new variable.

Three different physical models were tested and the results were in excellent agreement with the exact solution by considering third approximation terms of the variational iteration method. The method is extremely simple, easy to use and is very accurate for solving nonlinear differential difference equation. Also, the method is a powerful tool to search for solutions of various linear/nonlinear problems of fractional order in science and engineering.

6. REFERENCES

1. Podlubny, I. *Fractional Differential Equations*. Academic Press, New York (1999).
2. He, J.H. Nonlinear oscillation with fractional derivative and its applications. In: *Proceedings International Conference on Vibrating Engineering Dalian,* China, p. 288-291 (1998).
3. Tadjeran, C. & M.M. Meerschaert. A second order accurate numerical method for the two-dimensional fractional diffusion equation. *Journal of Computational Physics* 220: 813-823 (2007).
4. Meerschaert, M.M. & C. Tadjeran, Finite difference approximations for two-sided space-fractional partial differential equations. *Applied Numerical Mathematics* 56: 80-90 (2006).
5. Lynch, V.E. et al. Numerical methods for the solution of partial differential equations of fractional order. *Journal of Computational Physics* 192: 406-421 (2003).
6. Momani, S. & K. Al-Khaled. Numerical solutions for systems of fractional differential equations by the decomposition method. *Applied Mathematics and Computation* 162(3): 1351-1365 (2005).
7. Momani, S. An explicit and numerical solutions of the fractional KdV equation. *Mathematics and Computer in Applications* 70(2): 110-118 (2005).
8. Momani, S. Non-perturbative analytical solutions of the space- and time-fractional Burger's equations. *Chaos, Solitons & Fractals* 28(4): 930-937 (2006).
9. Momani, S. & Z. Odibat. Analytical solution of a time-fractional Navier-Stokes equation by Adomian's decomposition method. *Applied Mathematics and Computation* 177: 488-494 (2006).
10. Odibat, Z. & S. Momani. Approximate solutions for boundary value problems of time-fractional wave equation. *Applied Mathematics and Computation* 181: 1351-1358 (2006).
11. Mohyud-Din, S.T., M.A. Noor, K.I. Noor. & M.M. Hosseini. Variational iteration method for re-formulated partial differential equations. *Communications in Nonlinear Science and Numerical Simulation* 11(2): 87-92 (2010).
12. Mohyud-Din, S.T., M.A. Noor & K.I. Noor. Some relatively new techniques for nonlinear problems/ *Mathematical Problems in Engineering;* Article ID 234849 (2009).
13. Odibat, Z. & S. Momani. Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order. *Chaos, Solitons & Fractals* 36(1): 167-174 (2008).
14. Ahmad, J., S.T. Mohyud-Din. An efficient algorithm for some highly nonlinear fractional PDEs in mathematical physics. *PLOS ONE* 9(12): (2014), e109127. doi:10.1371/journal.pone.0109127.
15. Hashim, I., O. Abdulazi & S. Momani. Homotopy analysis method for fractional IVPs. *Communications in Nonlinear Science and Numerical Simulation* 14(3): 674-684 (2009).
16. He, J.H. Variational iteration method-a kind of non-linear analytical technique: Some examples. *International Journal of Non-Linear Mechanics* 34: 699-708 (1999).
17. Jamshad, A. & S.T. Mohyud-Din.An efficient algorithm to some highly nonlinear fractional PDEs. *European Journal of Scientific Research* 117(2): 242-251 (2014).
18. He, J.H. & Z.B. Li. Converting Fractional differential equations into partial differential equations. *Thermal Science* 16(2): 331–334 (2012).
19. Li, Z.B. & J.H. He. Fractional complex transform for fractional differential equations. *Applied Mathematics and Computation* 15(5): 970-973 (2010).
20. Wazwaz, A.M. Exact solutions with compact and noncompact structures for the one-dimensional generalized Benjamin Bona Mahony equation. *Communications in Nonlinear Science and Numerical Simulation* 10(8): 855-67 (2005c).
21. Ugurlu, Y. & D. Kaya. Solutions of the Cahn-Hilliard equation. *Computer & Mathematics with Applications* 56: 3038-45 (2008).
22. Wazwaz, A.M. New solitons and kink solutions for the Gardner equation. *Communications in Nonlinear Science and Numerical Simulation* 12: 1395-404 (2007).
23. Li, Z.B. & J.H. He. Fractional complex transform for fractional differential equations. *Applied Mathematics and Computation* 15(5): 970-973 (2010).

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Research Article

Approximations for Linear Tenth-order Boundary Value Problems through Polynomial and Non-polynomial Cubic Spline Techniques

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Abstract: Higher order differential equations have always been a tedious problem to solve for the mathematicians and engineers. Different numerical techniques were carried out to obtain numerical approximations to such problems. This research work presented and illustrated a novel numerical technique to approximate the tenth-order boundary value problems (BVPs). The techniques developed in this research were based on the principle of employing non-polynomial cubic spline method (NPCSM) and polynomial cubic spline method (PCSM) along with the decomposition procedure. The decomposition procedure was used to reduce the tenth-order BVPs into the corresponding system of second-order BVPs. The NPCSM and PCSM schemes were constructed for each second-order ordinary differential equation (ODE), whereas the first-order derivatives were approximated by finite central differences. The performance of the new developed numerical techniques was illustrated by numerical tests that involved comparing numerical solutions with the exact solution on a collection of test problems.

Keywords: Linear tenth-order ordinary differential equation, non-polynomial cubic spline, polynomial cubic spline, boundary value problems, finite central difference, system of algebraic equations

1. INTRODUCTION

A numerical approximation of tenth-order BVPs and the linked characteristic value problem is hardly addressed in the literature. When heating an endless flat film of fluid from below, under the supposition that fluid is subject to the action of rotation and uniform magnetic field across the fluid is implemented in the same direction as gravity, unsteadiness begins. When unsteadiness sets in as usual convection, then it can be modeled by tenth-order BVP, Chandarasekhar [1]. Pervaiz et al [2] discussed the numerical approximations of twelfth-order BVPs by employing non-polynomial cubic spline technique. Omotayo et al [3] proposed non-polynomial spline method for the fourth order BVPs by reducing it to a system of second order BVPs. Usmani [4] presented his work to approximate the fourth order BVPs by utilizing the quartic spline method. Twizell and Boutayeb [5] developed and illustrated the numerical approximations for eighth, tenth, and twelfth-order characteristic value problems emerging in thermal unsteadiness. The approximation of second order BVPs was presented by Alberg and Ito [6]. Siraj-ul-Islam et al [7] presented a non-polynomial spline method to approximate the sixth-order BVPs. Papamichael and Worsey [8] successively proposed the cubic spline algorithm for solving linear fourth-order BVPs. Siddiqui and Twizell [9, 10] developed numerical approximations of tenth and twelfth-order BVPs using tenth and twelfth order splines, respectively.

2. MATERIALS AND METHODS

The primary objective of this paper was to construct a novel cubic spline algorithm using numerical approximations of linear tenth-order BVPs. In this case, non-polynomial and polynomial cubic spline approaches are employed to develop numerical approximations of tenth-order BVPs, which is of the form

$$u^{(x)}(x) + a_1(x)u^{(ix)}(x) + a_2(x)u^{(viii)}(x) + a_3(x)\,u^{(vii)}(x) + a_4(x)u^{(vi)}(x) + a_5(x)u^{(v)}(x) + a_6(x)u^{(iv)}(x) + a_7(x)u'''(x) + a_8(x)u''(x) + a_9(x)u'(x) + a_{10}(x)\,u(x) = f(x)\,,\ x \in [a,b], \qquad (1)$$

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with boundary conditions:

$$u(a) = \alpha_0, \qquad u(b) = \beta_0, \tag{2}$$

$$u''(a) = \alpha_1, \qquad u''(b) = \beta_1, \tag{3}$$

$$u^{(iv)}(a) = \alpha_2, \qquad u^{(iv)}(b) = \beta_2, \tag{4}$$

$$u^{(vi)}(a) = \alpha_3, \qquad u^{(vi)}(b) = \beta_3, \tag{5}$$

$$u^{(viii)}(a) = \alpha_4, \qquad u^{(viii)}(b) = \beta_4, \tag{6}$$

where, $\alpha_j, \beta_j,\ j = 0, 1, 2, 3, 4$ are arbitrary fixed real constants, $a_j(x),\ j = 1, 2, \ldots, 10$ and $f(x)$ are continuous functions defined on $[a, b]$. On employing appropriate substitutions, $u''(x) = v(x),\ v''(x) = w(x),\ w''(x) = r(x)$, and $r''(x) = t(x)$ along with the boundary conditions (2-6), equation (1) was expressed in second-order ODEs form as under:

$$t''(x) + a_1(x)t'(x) + a_2(x)\,t(x) + a_3(x)r'(x) + a_4(x)\,r(x) + a_5(x)\,w'(x) + a_6(x)\,w(x) + a_7(x)v'(x) + a_8(x)\,v(x) + a_9(x)u'(x) + a_{10}(x)\,u(x) = f(x), \quad a \le x \le b, \tag{7}$$

$$u''(x) - v(x) = 0, \tag{8}$$

$$v''(x) - w(x) = 0, \tag{9}$$

$$w''(x) - r(x) = 0, \tag{10}$$

$$r''(x) - t(x) = 0. \tag{11}$$

The boundary conditions can be written in reduced form as:

$$u(a) = \alpha_0, \qquad u(b) = \beta_0, \tag{12}$$

$$v(a) = \alpha_1, \qquad v(b) = \beta_1, \tag{13}$$

$$w(a) = \alpha_2, \qquad w(b) = \beta_2, \tag{14}$$

$$r(a) = \alpha_3, \qquad r(b) = \beta_3, \tag{15}$$

$$t(a) = \alpha_4, \qquad t(b) = \beta_4. \tag{16}$$

Equations (7-11) along with the boundary conditions in equations (12-16) form a complete system of ODEs. This system of ODEs can be solved by simple numerical algorithms.

3. CONSTRUCTION OF NON-POLYNOMIAL CUBIC SPLINE SCHEME

To derive non-polynomial cubic spline approximation S for equations (7-11) with boundary conditions in equations (12-16), the period $[a, b]$ was distributed into n like subintervals:

$\mathrm{x_j} = \mathrm{a} + \mathrm{jh}, \mathrm{j} = 0,1, \ldots, \mathrm{n}$, where $\mathrm{a} = \mathrm{x_0},\ \mathrm{b} = \mathrm{x_n},$ and $\mathrm{h} = \frac{\mathrm{b}-\mathrm{a}}{\mathrm{n}}$.

Suppose the approximation to the exact solution $\mathrm{u}(\mathrm{x_j})$ was considered to be $\mathrm{u_j}$, which was obtained using non-polynomial cubic spline $\mathrm{S_j(x)}$, passing by the points $(\mathrm{x_j}, \mathrm{u_j})$ and $(\mathrm{x_{j+1}}, \mathrm{u_{j+1}})$. Then $\mathrm{S_j(x)}$was required to satisfy the interpolating conditions at $(\mathrm{x_j}, \mathrm{x_{j+1}})$, the boundary conditions in equations (12-16), and also the continuity condition of first derivative at the grid point $(\mathrm{x_j}, \mathrm{u_j})$. For every section $[\mathrm{x_j}, \mathrm{x_{j+1}}]$, $\mathrm{j} = 0, 1, \ldots, \mathrm{n} - 1$, the non-polynomial spline $\mathrm{S_j(x)}$ can be written in the form

$$S_j(x) = a_j + b_j(x - x_j) + c_j \sin\tau\,(x - x_j) + d_j \cos\tau\,(x - x_j), \quad j = 0, 1, \ldots, n - 1, \tag{17}$$

here, $a_j, b_j, c_j,$ and d_j are arbitrary constant values and τ represents the free parameter. The non-polynomial function $S(x)$, chosen from class $C^2[a, b]$, interpolates $u(x)$ at the common knots $x_j,\ j =$

$0, 1, \ldots, n$, rely on the parameter τ and then converted to an ordinary cubic spline $S(x)$ over $[a, b]$ when $\tau \to 0$.

Let, $S_j(x_j) = u_j,\ S_j(x_{j+1}) = u_{j+1},\ S_j(x_{j-1}) = u_{j-1},\ S_j''(x_j) = L_j,\ S_j''(x_{j+1}) = L_{j+1},\ S_j''(x_{j-1}) = L_{j-1}$.

By using the conditions of continuity on first and second derivatives at the grid points (x_j, y_j) and through simple algebraic manipulation, the arbitrary constants in equation (17) can be obtained in the form

$a_j = u_j + \frac{L_j}{\tau^2},\ b_j = \frac{u_{j+1}-u_j}{h} + \frac{L_{j+1}-L_j}{\tau\theta},\ c_j = \frac{L_j\cos\theta - L_{j+1}}{\tau^2\sin\theta},\ d_j = -\frac{L_j}{\tau^2}$, where, $\theta = \tau h,\ j = 0, 1, \ldots, n-1$. Further at the grid points (x_j, y_j), we obtained the following consistency relation

$$\alpha L_{j+1} + 2\beta L_j + \alpha L_{j-1} = \frac{1}{h^2}(u_{j+1} - 2u_j + u_{j-1}),\ j = 1, \ldots, n-1, \tag{18}$$

where, $\alpha = \frac{1}{\theta\sin\theta} - \frac{1}{\theta^2},\ \beta = -\frac{1}{\theta^2} - \frac{\cos\theta}{\theta}$, and $\theta = \tau h$.

Similarly, relations for $v(x), w(x), r(x)$, and $t(x)$, respectively, can be obtained as follows

$$\alpha M_{j+1} + 2\beta M_j + \alpha M_{j-1} = \frac{1}{h^2}(v_{j+1} - 2v_j + v_{j-1}) \tag{19}$$

$$\alpha N_{j+1} + 2\beta N_j + \alpha N_{j-1} = \frac{1}{h^2}(w_{j+1} - 2w_j + w_{j-1}), \tag{20}$$

$$\alpha P_{j+1} + 2\beta P_j + \alpha P_{j-1} = \frac{1}{h^2}(r_{j+1} - 2r_j + r_{j-1}), \tag{21}$$

$$\alpha W_{j+1} + 2\beta W_j + \alpha W_{j-1} = \frac{1}{h^2}(t_{j+1} - 2t_j + t_{j-1}), \tag{22}$$

where we have substituted

$$u''(x) = L\ ,\quad v''(x) = M\ ,\quad w''(x) = N\ ,\quad r''(x) = P\ ,\quad \text{and } t''(x) = W.$$

To illustrate the applications of the developed scheme, we discretized equations (7-11) at the knots (x_j, u_j), (x_j, v_j), (x_j, w_j), (x_j, r_j), and (x_j, t_j), we had

$$t_j'' + a_1(x_j)t_j' + a_2(x_j)t_j + a_3(x_j)r_j' + a_4(x_j)r_j + a_5(x_j)w_j' + a_6(x_j)w_j + a_7(x_j)v_j' + a_8(x_j)v_j + a_9(x_j)u_j' + a_{10}(x_j)u_j = f(x_j) \tag{23}$$

$$u_j'' = v_j, \tag{24}$$

$$v_j'' = w_j, \tag{25}$$

$$w_j'' = r_j, \tag{26}$$

$$r_j'' = t_j. \tag{27}$$

Take $u_j'' = L_j, v_j'' = M_j, w_j'' = N_j, r_j'' = P_j$, and $t_j'' = W_j$. The equations (23-27) were as under

$$W_j = f(x_j) - a_1(x_j)t_j' - a_2(x_j)t_j - a_3(x_j)r_j' - a_4(x_j)r_j - a_5(x_j)w_j' - a_6(x_j)w_j - a_7(x_j)v_j' - a_8(x_j)v_j - a_9(x_j)u_j' + a_{10}(x_j)u_j, \tag{28}$$

$$L_j = v_j, \tag{29}$$

$$M_j = w_j, \tag{30}$$

$$N_j = r_j, \tag{31}$$

$$P_j = t_j. \tag{32}$$

Approximating the first derivatives of u, v, w, r, and t in (28) by the central finite differences at j, $j+1$, $j-1$, we had

$$u_j' \cong \frac{u_{j+1}-u_{j-1}}{2h}, \quad u_{j+1}' \cong \frac{3u_{j+1}-4u_j+u_{j-1}}{2h}, \quad u_{j-1}' \cong \frac{-u_{j+1}+4u_j-3u_{j-1}}{2h},$$

$$v_j' \cong \frac{v_{j+1}-v_{j-1}}{2h}, \quad v_{j+1}' \cong \frac{3v_{j+1}-4v_j+v_{j-1}}{2h}, \quad v_{j-1}' \cong \frac{-v_{j+1}+4v_j-3v_{j-1}}{2h},$$

$$w_j' \cong \frac{w_{j+1}-w_{j-1}}{2h}, \quad w_{j+1}' \cong \frac{3w_{j+1}-4w_j+w_{j-1}}{2h}, \quad w_{j-1}' \cong \frac{-w_{j+1}+4w_j-3w_{j-1}}{2h},$$

$$r_j' \cong \frac{r_{j+1}-r_{j-1}}{2h}, \quad r_{j+1}' \cong \frac{3r_{j+1}-4r_j+r_{j-1}}{2h}, \quad r_{j-1}' \cong \frac{-r_{j+1}+4r_j-3r_{j-1}}{2h},$$

$$t_j' \cong \frac{t_{j+1}-t_{j-1}}{2h}, t_{j+1}' \cong \frac{3t_{j+1}-4t_j+t_{j-1}}{2h}, \quad t_{j-1}' \cong \frac{-t_{j+1}+4t_j-3t_{j-1}}{2h}.$$

Substitute the above approximations in equation (28), we had

$$W_j = f(x_j) - a_1(x_j)\frac{t_{j+1}-t_{j-1}}{2h} - a_2(x_j)t_j - a_3(x_j)\frac{r_{j+1}-r_{j-1}}{2h} - a_4(x_j)r_j - a_5(x_j)\frac{w_{j+1}-w_{j-1}}{2h} - a_6(x_j)w_j - a_7(x_j)\frac{v_{j+1}-v_{j-1}}{2h} - a_8(x_j)v_j - a_9(x_j)\frac{u_{j+1}-u_{j-1}}{2h} - a_{10}(x_j)u_j. \tag{33}$$

Similarly,

$$W_{j+1} = f(x_{j+1}) - a_1(x_{j+1})\frac{3t_{j+1}-4t_j+t_{j-1}}{2h} - a_2(x_{j+1})t_j - a_3(x_{j+1})\frac{3r_{j+1}-4r_j+r_{j-1}}{2h} - a_4(x_{j+1})\,r_j - a_5(x_{j+1})\frac{3w_{j+1}-4w_j+w_{j-1}}{2h} - a_6(x_{j+1})w_j - a_7(x_{j+1})\frac{3v_{j+1}-4v_j+v_{j-1}}{2h} - a_8(x_{j+1})v_j - a_9(x_{j+1})\frac{3u_{j+1}-4u_j+u_{j-1}}{2h} - a_{10}(x_{j+1})u_j, \tag{34}$$

$$W_{j-1} = f(x_{j-1}) - a_1(x_{j-1})\frac{-t_{j+1}+4t_j-3t_{j-1}}{2h} - a_2(x_{j-1})t_j - a_3(x_{j-1})\frac{-r_{j+1}+4r_j-3r_{j-1}}{2h} - a_4(x_{j-1})\,r_j - a_5(x_{j-1})\frac{-w_{j+1}+4w_j-3w_{j-1}}{2h} - a_6(x_{j-1})w_j - a_7(x_{j-1})\frac{-v_{j+1}+4v_j-3v_{j-1}}{2h} - a_8(x_{j-1})v_j - a_9(x_{j-1})\frac{-u_{j+1}+4u_j-3u_{j-1}}{2h} - a_{10}(x_{j-1})u_j. \tag{35}$$

Substituting equations (33-35) in equation (22) and on simplifying, we obtained

$$\left(\frac{1}{h^2} - \frac{3\alpha a_1(x_{j-1})}{2h} - \frac{\beta a_1(x_j)}{h} + \frac{\alpha a_1(x_{j+1})}{2h} + \alpha a_2(x_{j-1})\right)t_{j-1} + \left(-\frac{2}{h^2} + \frac{2\alpha a_1(x_{j-1})}{h} - \frac{2\alpha a_1(x_{j+1})}{h} + 2\beta a_2(x_j)\right)t_j + \left(\frac{1}{h^2} - \frac{\alpha a_1(x_{j-1})}{2h} + \frac{\beta a_1(x_j)}{h} + \frac{3\alpha a_1(x_{j+1})}{2h} + \alpha a_2(x_{j+1})\right)t_{j+1} + \left(-\frac{3\alpha a_3(x_{j-1})}{2h} - \frac{\beta a_3(x_j)}{h} + \frac{\alpha a_3(x_{j+1})}{2h} + \alpha a_4(x_{j-1})\right)r_{j-1} + \left(\frac{2\alpha a_3(x_{j-1})}{h} - \frac{2\alpha a_3(x_{j+1})}{h} + 2\beta a_4(x_j)\right)r_j + \left(-\frac{\alpha a_3(x_{j-1})}{2h} + \frac{\beta a_3(x_j)}{h} + \frac{3\alpha a_3(x_{j+1})}{2h} + \alpha a_4(x_{j+1})\right)r_{j+1} + \left(-\frac{3\alpha a_5(x_{j-1})}{2h} - \frac{\beta a_5(x_j)}{h} + \frac{\alpha a_5(x_{j+1})}{2h} + \alpha a_6(x_{j-1})\right)w_{j-1} + \left(\frac{2\alpha a_5(x_{j-1})}{h} - \frac{2\alpha a_5(x_{j+1})}{h} + 2\beta a_6(x_j)\right)w_j + \left(-\frac{\alpha a_5(x_{j-1})}{2h} + \frac{\beta a_5(x_j)}{h} + \frac{3\alpha a_5(x_{j+1})}{2h} + \alpha a_6(x_{j+1})\right)w_{j+1} + \left(-\frac{3\alpha a_7(x_{j-1})}{2h} - \frac{\beta a_7(x_j)}{h} + \frac{\alpha a_7(x_{j+1})}{2h} + \alpha a_8(x_{j-1})\right)v_{j-1} + \left(\frac{2\alpha a_7(x_{j-1})}{h} - \frac{2\alpha a_7(x_{j+1})}{h} + 2\beta a_8(x_j)\right)v_j + \left(-\frac{\alpha a_7(x_{j-1})}{2h} + \frac{\beta a_7(x_j)}{h} + \frac{3\alpha a_7(x_{j+1})}{2h} + \alpha a_8(x_{j+1})\right)v_{j+1} + \left(-\frac{3\alpha a_9(x_{j-1})}{2h} - \frac{\beta a_9(x_j)}{h} + \frac{\alpha a_9(x_{j+1})}{2h} + \alpha a_{10}(x_{j-1})\right)u_{j-1} +$$

$$\left(\frac{2\alpha a_9(x_{j-1})}{h}-\frac{2\alpha a_9(x_{j+1})}{h}+2\beta a_{10}(x_j)\right)u_j+$$
$$\left(-\frac{\alpha a_9(x_{j-1})}{2h}+\frac{\beta a_9(x_j)}{h}+\frac{3\alpha a_9(x_{j+1})}{2h}+\alpha a_{10}(x_{j+1})\right)u_{j+1}=-\alpha f(x_{j-1})-2\beta f(x_j)-\alpha f(x_{j-1}) \quad (36)$$

Now equations (29-32) could also be written as

$L_j = v_j, \quad L_{j+1} = v_{j+1}, \quad L_{j-1} = v_{j-1},$

$M_j = w_j, \quad M_{j+1} = w_{j+1}, \quad M_{j-1} = w_{j-1},$

$N_j = r_j, \quad N_{j+1} = r_{j+1}, \quad N_{j-1} = r_{j-1},$

$P_j = t_j, \quad P_{j+1} = t_{j+1}, \quad P_{j-1} = t_{j-1}.$

Substituting the above relations in equations (18-21), we had

$$\frac{1}{h^2}(u_{j+1}-2u_j+u_{j-1}) = \alpha v_{j+1}+2\beta v_j+\alpha v_{j-1}, \quad (37)$$

$$\frac{1}{h^2}(v_{j+1}-2v_j+v_{j-1}) = \alpha w_{j+1}+2\beta w_j+\alpha w_{j-1}, \quad (38)$$

$$\frac{1}{h^2}(w_{j+1}-2w_j+w_{j-1}) = \alpha r_{j+1}+2\beta r_j+\alpha r_{j-1}, \quad (39)$$

$$\frac{1}{h^2}(r_{j+1}-2r_j+r_{j-1}) = \alpha t_{j+1}+2\beta t_j+\alpha t_{j-1}. \quad (40)$$

The equations (36-40) associated with the boundary conditions in equations (12-16) form a complete system of the $5(n+1)$ linear equations in the $5(n+1)$ unknowns. This system can be solved by simple numerical algorithms.

4. CONSTRUCTION OF POLYNOMIAL CUBIC SPLINE SCHEME

To derive polynomial cubic spline approximation for equations (7-11) with boundary conditions in equations (12-16), the interval $[a,b]$ was again divided into n equal subintervals using equally spaced knots: $x_j = a + jh,\ j = 0,1,\dots,n$, where $a = x_0,\ b = x_n,$ and $h = \frac{b-a}{n}$. For each segment $[x_j, x_{j+1}],\ j = 0,1,\dots,n-1$, we have the polynomial cubic spline

$$S_j(x) = a_j + b_j(x-x_j) + c_j(x-x_j)^2 + d_j(x-x_j)^3,\ j = 0,1,\dots,n-1. \quad (41)$$

In a similar way to that for the non-polynomial cubic spline scheme, the polynomial cubic spline scheme was constructed as follows

$$\left(\frac{6}{h^2}-\frac{3a_1(x_{j-1})}{2h}-\frac{2a_1(x_j)}{h}+\frac{a_1(x_{j+1})}{2h}+a_2(x_{j-1})\right)t_{j-1}+\left(-\frac{12}{h^2}+\frac{2a_1(x_{j-1})}{h}-\frac{2a_1(x_{j+1})}{h}+4a_2(x_j)\right)t_j+$$
$$\left(\frac{6}{h^2}-\frac{a_1(x_{j-1})}{2h}+\frac{2a_1(x_j)}{h}+\frac{3a_1(x_{j+1})}{2h}+a_2(x_{j+1})\right)t_{j+1}+$$
$$\left(-\frac{3a_3(x_{j-1})}{2h}-\frac{2a_3(x_j)}{h}+\frac{a_3(x_{j+1})}{2h}+a_4(x_{j-1})\right)r_{j-1}+\left(\frac{2a_3(x_{j-1})}{h}-\frac{2a_3(x_{j+1})}{h}+4a_4(x_j)\right)r_j+$$
$$\left(-\frac{a_3(x_{j-1})}{2h}+\frac{2a_3(x_j)}{h}+\frac{3a_3(x_{j+1})}{2h}+a_4(x_{j+1})\right)r_{j+1}+$$
$$\left(-\frac{3a_5(x_{j-1})}{2h}-\frac{2a_5(x_j)}{h}+\frac{a_5(x_{j+1})}{2h}+a_6(x_{j-1})\right)w_{j-1}+\left(\frac{2a_5(x_{j-1})}{h}-\frac{2a_5(x_{j+1})}{h}+4a_6(x_j)\right)w_j+$$

$$\left(-\frac{a_5(x_{j-1})}{2h}+\frac{2a_5(x_j)}{h}+\frac{3a_5(x_{j+1})}{2h}+a_6(x_{j+1})\right)w_{j+1}$$
$$+\left(-\frac{3a_7(x_{j-1})}{2h}-\frac{2a_7(x_j)}{h}+\frac{a_7(x_{j+1})}{2h}+a_8(x_{j-1})\right)v_{j-1}+\left(\frac{2a_7(x_{j-1})}{h}-\frac{2a_7(x_{j+1})}{h}+4a_8(x_j)\right)v_j+$$
$$\left(-\frac{a_7(x_{j-1})}{2h}+\frac{2a_7(x_j)}{h}+\frac{3a_7(x_{j+1})}{2h}+a_8(x_{j+1})\right)v_{j+1}+$$
$$\left(-\frac{3a_9(x_{j-1})}{2h}-\frac{2a_9(x_j)}{h}+\frac{a_9(x_{j+1})}{2h}+a_{10}(x_{j-1})\right)u_{j-1}+\left(\frac{2a_9(x_{j-1})}{h}-\frac{2a_9(x_{j+1})}{h}+4a_{10}(x_j)\right)u_j+$$
$$\left(-\frac{a_9(x_{j-1})}{2h}+\frac{2a_9(x_j)}{h}+\frac{3a_9(x_{j+1})}{2h}+a_{10}(x_{j+1})\right)u_{j+1}=f(x_{j-1})+4f(x_j)+f(x_{j-1}) \quad (42)$$

$$\frac{6}{h^2}(u_{j+1}-2u_j+u_{j-1})=(v_{j+1}+4v_j+v_{j-1}), \quad (43)$$

$$\frac{6}{h^2}(v_{j+1}-2v_j+v_{j-1})=(w_{j+1}+4w_j+w_{j-1}), \quad (44)$$

$$\frac{6}{h^2}(w_{j+1}-2w_j+w_{j-1})=(r_{j+1}+4r_j+r_{j-1}), \quad (45)$$

$$\frac{6}{h^2}(r_{j+1}-2r_j+r_{j-1})=(t_{j+1}+4t_j+t_{j-1}). \quad (46)$$

The equations (42-46) associated with boundary conditions in equations (12-16) form a complete system of the $5(n+1)$ linear equations in the $5(n+1)$ unknowns. This system can be solved by simple numerical algorithms.

5. RESULTS AND DISCUSSIONS

To observe the computational efficiency of the above developed schemes, we considered the following two test problems.

Test Problem 1

We considered the following tenth-order equation

$$u^{10}(x)=(1-x)\sin x+10\cos x, \quad 0\le x\le 1,$$

with boundary conditions:

$$u(0)=0, \qquad u(1)=0,$$
$$u^2(0)=2, \qquad u^2(1)=2\cos 1,$$
$$u^4(0)=-4, \qquad u^4(1)=-4\cos 1,$$
$$u^6(0)=6, \qquad u^6(1)=6\cos 1,$$
$$u^8(0)=-8, \qquad u^8(1)=-8\cos 1.$$

The analytical solution to the above tenth-order BVP is

$$u(x)=(x-1)\sin x.$$

To quantify the quality of the developed schemes, the first set of experiments was performed to observe the absolute error while comparing the numerical approximations obtained by NPCSM and PCSM with the exact solutions applied to the test problem 1. The associated absolute errors for $h=\frac{1}{15}$ and $h=\frac{1}{20}$ had been showed in Table 1 and Table 2, respectively.

Table 1. Absolute errors at $h = \frac{1}{15}$.

x	Exact	Absolute Error (NPCSM)	Absolute Error (PCSM)
0.2	-0.158938872	4.198E-08	1.299E-04
0.4	-0.233656041	6.206E-08	1.925E-04
0.6	-0.225861866	6.010E-08	1.865E-04
0.8	-0.143471218	3.810E-08	1.182E-04

Table 2. Absolute errors at $h = \frac{1}{20}$.

x	Exact	Absolute Error (NPCSM)	Absolute Error (PCSM)
0.2	-0.158938872	1.328E-08	7.309E-05
0.4	-0.233656041	1.963E-08	1.083E-04
0.6	-0.225861866	1.901E-08	1.050E-04
0.8	-0.143471218	1.205E-08	6.649E-05

Table 1 showed the absolute errors associated with $h = \frac{1}{15}$ and $x = 0.2\ (0.2)\ 0.8$. Here we observed that at spatial displacement $x = 0.4$, for NPCSM and PCSM, the maximum absolute error between the numerical approximation and the exact solution was not more than 6.21×10^{-08} and 1.92×10^{-04}, respectively. Whereas, the best observed numerical accuracy was obtained by the NPCSM at $x = 0.8$ with absolute error approximation 3.81×10^{-08}.

Similar set of experiments for $h = \frac{1}{20}$ and $x = 0.2\ (0.2)\ 0.8$ were performed in Table 2. For all the experiments performed in Table 2, we observed very much the same trend as of the results obtained in Table1. The best observed accuracy was again obtained by the NPCSM with absolute error approximately 1.20×10^{-08}.

Test Problem 2

For the second set of experiments, we considered the following tenth-order equation

$$u^{10}(x) = -(80 + 19x + x^2)e^x, \quad 0 \le x \le 1,$$

with boundary conditions:

$$u(0) = 0, \qquad u(1) = 0,$$
$$u^2(0) = 0, \qquad u^2(1) = -4e,$$
$$u^4(0) = -8, \qquad u^4(1) = -16e,$$
$$u^6(0) = -24, \qquad u^6(1) = -36e,$$
$$u^8(0) = -48, \qquad u^8(1) = -64e.$$

The true solution to the above BVP is given by

$u(x) = x(1 - x)e^x$.

To illustrate the performance of NPCSM and PCSM schemes, we repeated the previous sets of experiments for the test problem 2 as shown in Tables 3-4. The NPCSM again achieved the best observed accuracy compared to PCSM. It has been observed that using NPCSM the best observed accuracy for $h = \frac{1}{15}$ and $h = \frac{1}{20}$ was obtained at $x = 0.2$. The corresponding absolute errors were approximately 2.43×10^{-07} and 7.70×10^{-08} as shown in Table 3 and Table 4, respectively.

Table 3. Absolute errors at $h = \frac{1}{15}$.

x	Exact	Absolute Error(NPCSM)	Absolute Error(PCSM)
0.2	0.195424441	2.433E-07	3.982E-04
0.4	0.358037927	3.986E-07	6.663E-04
0.6	0.437308512	4.428E-07	7.598E-04
0.8	0.356086549	3.328E-07	5.885E-04

Table 4 Absolute errors at $h = \frac{1}{20}$.

x	Exact	Absolute Error (NPCSM)	Absolute Error (PCSM)
0.2	0.195424441	7.698E-08	2.238E-04
0.4	0.358037927	1.261E-07	3.745E-04
0.6	0.437308512	1.401E-07	4.271E-04
0.8	0.356086549	1.053E-07	3.308E-04

In this research work, the NPCSM and PCSM along with decomposition procedure were used for the spatial derivatives. Siddiqi et al [11] used eleventh degree spline for the numerical approximation of tenth-order linear special case BVPs. The absolute errors were calculated by Shahid S. Siddiqi at different step sizes, $\frac{1}{7}, \frac{1}{14}, \frac{1}{21}$, and $\frac{1}{28}$. The maximum accuracy of approximately 2.13×10^{-8} was obtained at a step size of $\frac{1}{28}$. On the other hand, in this paper we obtained an accuracy of approximately 1.20×10^{-08} was obtained at a step size of $\frac{1}{20}$, using cubic spline method which is much better accuracy with small step size. We further investigate with reduced step size and obtained an accuracy of approximately 3.40×10^{-12} at a step size of $\frac{1}{80}$.

The overall conclusion was that the performance of the developed schemes was remarkably good when implemented on linear tenth-order BVPs and produced encouraging results which were very much close to the exact solution.

6. REFERENCES

1. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability*. Clarendon Press, Oxford, England (1961).
2. Pervaiz, A., Zafar, Z. & M.O. Ahmad. A non-polynomial spline method for solving linear twelfth order boundary-value problems. *Pakistan Academy of Sciences* 51(2): 157-165 (2014).
3. Omotayo, A.T. & O.M. Ogunlaran. A non-polynomial spline method for solving linear fourth order boundary-value problems. *International Journal of the Physical Sciences* 6: 3246-3254 (2011).
4. Usmani, R.A. The use of quartic splines in the numerical solution of the fourth order boundary value problem. *Journal of Computational and Applied Mathematics* 44:187-199 (1992).
5. Twizell, E.H. & A. Boutayeb. Numerical methods for eight, tenth and twelfth order eigenvalue problems arising in thermal instability. *Advances in Computational Mathematics* 2:407-436 (1994).
6. Alberg, J.H. & T. Ito. A collocation method for two-point boundary value problems. *Mathematics of Computation* 29: 761-776 (1975).
7. Siraj-ul-Islam, I.A. Tirmizi, Fazal-i-Haq & M.A. Khan. A non-polynomial spline approach to the solution of sixth-order BVPs. *Applied Mathematics and Computation* 195: 277-284 (2008).
8. Papamichael, N. & A.J. Worsey. A cubic spline method for the solution of linear fourth-order boundary value problems. *Journal of Computational and Applied Mathematics* 7: 187-189 (1981).
9. Siddiqi, S.S. & E.H. Twizell. Spline solution of linear twelfth-order boundary value problems. *Journal of Computational and Applied Mathematics* 78: 371-390 (1997).
10. Siddiqi, S.S. & E.H. Twizell. Spline solution of linear tenth-order boundary value problems. *Journal of Computational and Applied Mathematics* 68: 345-362 (1998).
11. Siddiqi, S.S. & Ghazala Akram. Spline solutions of tenth-order boundary value problems. *Journal of Computational and Applied Mathematics* 185: 115-127 (2007).

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Pakistan Academy of Sciences

Obituary

Prof. Dr. Abdul Ghaffar (1932-2015)

Prof. Dr. Abdul Ghaffar was born in Allahabad, British India on 23rd August, 1932. He obtained his M.Sc. degree from University of Karachi, Pakistan, in 1955 and his Ph.D. from University of Birmingham, UK in 1960.

Prof. Dr. Abdul Ghaffar had a long association with the National Nematological Research Centre (NNRC), University of Karachi, Karachi. He served as Professor and Chairman, Department of Botany (1975-77, 1986-1992); Director, M.A.H. Qadri Biological Research Centre (1990-1992); Director, National Nematological Research Centre (1987-1989); Professor, (Botany) (1979-1992); Associate Professor, (Botany) (1971-1979); and Assistant Professor, (Botany) (1963-1971) at University of Karachi; Assistant Mycologist/ Assistant Plant Pathologist at Department of Plant Protection, Ministry of Agriculture, Government of Pakistan (1950-1963). Prof. Dr. Ghaffar acted as Director of National Nematological Research Centre during 1987-1989, served as Emeritus Editor of "Pakistan Journal of Botany", and Editor-in-Chief of "Pakistan Journal of Nematology" since the inception of this Journal in 1983. Though, Dr. Ghaffar's major contributions concerned fungi of Karachi, he also taught the course of Plant Nematology with immense affection for over 10 years.

Prof. Abdul Ghaffar's ability as a professor, writer and editor has been the major driving force in the growth of Nematology at early stage in Pakistan.

During his long and active scientific career period over five decades, Dr. Ghaffar made significant contributions in research and academics in Mycology and Nematology.

Prof. Abdul Ghaffar won many prestigious honors and recognitions including National Award of Republic of Brazil "Ordem Nacional do Merito Educado – Classe Comendador", 2000 and "Ordem Nacional do Merito Cientifico – Classe Gra Cruz, 1997; Doctor Honoris Causa, Universidade de Buenos Aires, Argentina, 1997 and Universite Catholique de Louvain, Belgium, 1995; N. van Uden Lecture Award, University of Coimbra, Portugal, 1993; Chemistry Award, Third World Academy of Sciences (TWAS), 1986; and LAFI Award, Pharmaceutical Industries, Brazil, 1978. He was elected Fellow of the Pakistan Academy of Sciences in 1992.

Prof. Dr. Abdul Ghaffar passed away on August 10, 2015 at the age of 83. May Allah bless his soul through His infinitive mercy and place him in Heaven, Aameen!

Dr. Muhammad Ashraf, *FPAS*
Dr. Abdul Rashid, *FPAS*

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Pakistan Academy of Sciences

Obituary

Prof. Dr. M. Ataur Rahman
(1929-2015)

Prof. Dr. Mohammad Ataur Rahman was born in British India on 24 June, 1929. He obtained his M.Sc. (Hons) in Chemistry from Government College, Punjab University Institute of Chemistry, Lahore; Ph.D. in Biochemistry, University College, London University, and D.Sc. (H.C.) from the Open International University for Complementary Medicine, Colombo, Sri Lanka.

Prof. Ataur Rahman spent major period of his career at Jinnah Postgraduate Medical Center, Karachi. He also served as Professor of Clinical Biochemistry at Institute of Biochemistry and Genetic Engineering (KIBGE), University of Karachi, Karachi. Prior to that he served as Assistant Professor of Biochemistry, Jinnah Postgraduate Medical Center, Karachi; Professor and Chairman, Department of Biochemistry; Jinnah Postgraduate Medical Center, Karachi; Instructor, Dow Medical College, Karachi. Principal, School of Medical Technology, Jinnah Postgraduate Medical Center, Karachi; Coordinator and Head of the Basic Medical Sciences Institute, Jinnah Postgraduate Medical Center, Karachi; Chairman, Academic Council, Jinnah Postgraduate Medical Center, Karachi; Professor of Biochemistry, Professor of Biochemistry and Dean of Faculty of Science, Hamdard University, Karachi; Professor of Biochemistry and Dean of Postgraduate Studies, Baqai University, Karachi; and Professor of Biochemistry and Dean of Postgraduate Studies, Ziauddin Medical University, Karachi.

Prof. Ataur Rahman won many prestigious honors and recognitions including President of Pakistan's Award "Pride of Performance" in the field of Medicine and *Hilal-i-Imtiaz* in Medical Sciences by the Government of Pakistan; Gold Medal of Pakistan Academy of Sciences for outstanding contributions in research; Certificate of Achievement of USAID; Japan Society for Promotion of Science Award of Visiting Professor at Kyoto University Medical Center, Kyoto; Royal Society Visiting Scientist Fellowship, London; First Prize for Meritorious Services in the field of research in Medical Sciences, Government of Pakistan; Appreciation Plaque for Contribution to Jinnah Postgraduate Medical Center, Karachi; Dr. Zainul Abedin Commemorative Award of Pakistan Society of Biochemists, Karachi; D.Sc. (HC) from the Open International University, Colombo, Srilanka; and Gold Medal, XIII Star Award, South Asia Publications, Pakistan. He was elected Fellow of the Pakistan Academy of Sciences in 1980.

Prof. Dr. M. Ataur Rahman breathed his last in Karachi on Saturday, 26th September, 2015. May Allah Almighty bless the departed soul with eternal peace. Aameen.

Dr. Abdul Rashid, *FPAS*

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Pakistan Academy of Sciences

Obituary

Dr. Mahbub Ali
(1923–2015)

Dr. Mahbub Ali was born in Gurdaspur, British India on 13th August, 1923. He was elected Fellow of the Pakistan Academy of Sciences in 1988. Dr. Mahbub Ali had the rare distinction of being a Scientist Emeritus of the Department of Agriculture, Government of Punjab and Pakistan Central Cotton Committee. He obtained B.Sc. (Agri.) in 1942 and M.Sc. (Agri.) in 1951 from Punjab Agricultural College & Research Institute, Faisalabad, Pakistan and PhD in 1955 from Agricultural and Mechanical College, Texas, USA. Also, he got Training in Cotton Cultivation from Russia in 1961.

During his illustrious career, Dr Mahbub Ali held various prestigious positions, including Managing Director, Punjab Seed Corporation, Govt. of Punjab, 1979-1985; Founding Director, Central Cotton Research Institute, Multan, 1970-1979; Cotton Botanist, Cotton Research Section, Multan, 1958-1970; Research Assistant, Cotton Research Section, Faisalabad, 1955-1958; Agricultural/ Research Assistant, Department of Agriculture, Govt. of Pakistan, 1942-1955; Research Assistant, Plant Physiology, Agronomy, Crop Husbandry, 1942-1945.

In view of his significant contributions in the field of agriculture, Dr Mahbub Ali earned many honors and awards which included civil award of Pride of Performance by Government of Pakistan, 1971; Open Gold Medal by Pakistan Academy of Sciences, 1980; Award of Scientist Emeritus by the Punjab Government, 1985; Award of Scientist Emeritus by the Pakistan Government, 2000; and Shield of Distinction, University of Agriculture, Faisalabad, Pakistan, 1981.

Dr Mahbub Ali was Chairman, Working Group of Agricultural Enquiry Committee, Government of Pakistan, 1975; Pakistan Delegate for Seed Conference in Sri Lanka, February, 1987; Chairman, Cash Corps Committee of National Commission on Agriculture (Pakistan), 1987; Member of USAID Team of Experts, USAID MART Project, 1993; Member, Federal Government's Review Team for PCCC Schemes, 1992-93; Member of several committees on Cotton Research, 1992-2000; and Chairman/Member of several committees of the Govt. of Pakistan, Universities and Colleges, Agriculture Research Review Teams, since 1987.

Dr Mahbub Ali's areas of research included cotton R&D (evolution of new varieties and production technology), quality control, maximizing production and farm income; and seed industry, quality control, production and marketing; Potato seed production and marketing.

I had known Dr Mahbub Ali personally since mid 1980s. Being a founding father in cotton R&D and a mentor to the next generation cotton researchers, he was a legendary scientist in agricultural arena of the country. During the cotton season, one would witness Dr Mahbub Ali almost every day visiting the cotton fields at research stations as well as at farmers' fields. Thus, he remained professionally active till his last days in this world.

Dr Mahbub Ali possessed a pleasing personality. Despite being a hard task master, he was a very kind and humble person, with a smiling face. In the death of Dr Mahbub Ali, Pakistan in general and the Pakistan cotton community in particular have lost an eminent and committed research leader. May the Allah Almighty rest his soul in eternal peace and give fortitude to his family to bear this irreparable loss! Aameen.

Dr. Abdul Rashid, *FPAS*

Proceedings of the Pakistan Academy of Sciences

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ACKNOWLEDGEMENTS (font size 10)**:** In a brief statement, acknowledge financial support and other assistance.

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1. Golding, I. Real time kinetics of gene activity in individual bacteria. *Cell* 123**:** 1025–1036 (2005).
2. Bialek, W. & S. Setayeshgar. Cooperative sensitivity and noise in biochemical signaling. *Physical Review Letters* 100**:** 258–263 (2008).
3. Kay, R.R. & C.R.L. Thompson. Forming patterns in development without morphogen gradients: differentiation and sorting. *Cold Spring Harbor Perspectives in Biology* 1: doi: 10.1101/cshperspect.a001503 (2009).

b. **Books**

4. Luellen, W.R. *Fine-Tuning Your Writing*. Wise Owl Publishing Company, Madison, WI, USA (2001).
5. Alon, U. & D.N. Wegner (Ed.). *An Introduction to Systems Biology: Design Principles of Biological Circuits*. Chapman & Hall/CRC, Boca Raton, FL, USA (2006).

c. **Book Chapters**

6. Sarnthein, M.S. & J.D. Stanford. Basal sauropodomorpha: historical and recent phylogenetic developments. In: *The Northern North Atlantic: A Changing Environment.* Schafer, P.R. & W. Schluter (Ed.), Springer, Berlin, Germany, p. 365–410 (2000).
7. Smolen, J.E. & L.A. Boxer. Functions of Europhiles. In: *Hematology, 4th ed*. Williams, W.J., E. Butler & M.A. Litchman (Ed.), McGraw Hill, New York, USA, p. 103–101 (1991).

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