



# A Characteristic Study of Exponential Distribution Technique in a Flowshop using Taillard Benchmark Problems

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**Abstract:** The main objective of many authors is to find an optimum sequence, which can provide minimum makespan in a Permutation Flow Shop (PFS). In this venture, a new Exponential Distribution Technique (EPDT) is proposed, under the mathematical and computational features. This paper deals with characteristic study of EPDT over the existing algorithms. The taillard benchmark problems are solved for the general comparison. This analysis have been tabulated and graphically represented along with the cumulative performance of it. The solution of this work have shown that Exponential Distribution Technique has better performance in finding an optimal sequence in a permutation flow shop.

**Keywords:** Flowshop, makespan, exponential distribution technique (EPDT), optimal sequence

## 1. INTRODUCTION

In production management, a scheduling problem is defined as work time hypothesis regarding assignment of resources like raw materials, machines, etc. In a complex and dynamic production environment, scheduling is an extremely important issue. Scheduling deals with the allocation of available resources to tasks over time.

Permutation flow shop problem is a special case of the flow shop problem. A possible constraint in the flow shop environment is that the queues for each machine operate according to the FIFO discipline. This implies that the order in which the jobs go through the first machine is the same throughout the system. The most important property of PFS is deciding the job sequence on the first machine because once it is decided, all the jobs follow the same sequence on each machine throughout the sys with  $n$ -jobs, there are  $n!$  solutions that are independent of machine numbers.

The various algorithmic methods have been suggested to obtain optimum makespan. While the

makespan is abridged, the corresponding parameters such as idle time, waiting time, total flow time etc., get distorted. During the last six decades, the flowshop sequencing problem has been the active centre of attention for many researchers.

In 1954, a simple algorithm was given by Johnson [1], for flowshop scheduling problems in the order of 'n' jobs in '2' machines. This work was further developed by Ignall and Scharge [2].

In the 1965's Palmer [3, 4] have been the first to propose heuristic procedures. The first significant work in the development of an efficient heuristic is due to Campell, Dudek and Smith. Their algorithm consists essentially in splitting the given  $m$ -machine problem into a series of an equivalent two-machine flow shop problem and solving by Johnson's rule.

A simple modification and extension of Palmer's heuristic was carried out by Hundal and Rajagopal [5], two sets of indices have been proposed and three sequences were obtained. In 1977, Dannenbring [6] has developed a procedure called 'rapid access', which attempts to combine

the advantages of Palmer's slope index and CDS procedures. Though the procedure by Dannenbring has found to yield a better quality solution than those by Palmer's and CDS methods; it requires much more computational effort.

During 1980s, King and Speeches [7] treated the makespan problem as equivalent to that of minimizing total delay and run-out delay. They have proposed heuristics that aim at matching the two consecutive job time-block profiles by considering these delays. One of the heuristics turns out to be better than the CDS heuristic.

In 1982, Stinson et al [8] has proposed a radically different approach. They treated the makespan oriented problem as a travelling salesman problem and developed a procedure in two steps. The heuristic solution is found to yield a better quality solution than those by Palmer and CDS methods at the cost of increased computational effort.

## 2. STUDY METHODOLOGIES

### 2.1 Gupta Method (GUPTA)

Gupta [9] suggested another heuristic which is similar to Palmer's heuristic using a slope index in a different manner. In this method, the effect of first and last machine is high in predicting slope index. This effect causes the demerits in finding the best sequence.

**Step 1:** Calculate the value of the function associated with job  $i$ ,  $f(i)$ , as follows

$$f(i) = \frac{A}{\text{Min}(t_{ij} + t_{i,j+1})} \text{ for } j=1, 2, \dots, (m-1). \quad (1)$$

Where,  $A = 1$  if  $t_{im} \leq t_{i1}$ ,

$A = -1$  otherwise.

**Step 2:** Arrange  $N$  jobs in ascending order of  $f(i)$  and in a favour of the job with the least sum of process times on all  $M$  machines.

**Step 3:** Calculate the makespan of the predetermined schedule through the recursive relation.

$$T_{ij}^k = \text{Max}[T_{ij}^{k-1}, T_{i,j-1}^k] + t_{ij} \quad (2)$$

Where,  $T_{ij}^k$  is the cumulative processing time up to the  $k^{\text{th}}$  order for the  $i^{\text{th}}$  job through  $j^{\text{th}}$  machine.

### 2.2 Rajendran Heuristic (CR)

Rajendran (CR) [10] has implemented a heuristic for the flowshop scheduling with multiple objectives of optimizing makespan, total flow time and idle time for machines. This improvement heuristics, the first seed is taken from CDS algorithm. The heuristic preference relation is proposed and is used as the basis to restrict the search for possible improvement in the multiple objectives. This method is simple but repeating of steps is needed. This makes the evaluation time of the heuristic as high.

**Step 1:** The seed sequence is swapped to yield more sequences. In the first step the first and second jobs are interchanged while other jobs are retained in their same position. In the same way as the second step, the first and third jobs are interchanged. Similarly the sequences are generated and the makespan and flow time for the swapped sequences are calculated.

**Step 2:** The seed sequence is assumed as  $S$  and the swapped sequences are taken as  $S'$ .

**Step 3:** Select the best sequence, among the swapped sequences.

### 2.3 Baskar's Pascal's Triangle Method (PTM)

Baskar and Anthony Xavier [11] reported a dummy machine concept along with Pascal's Triangle Method (PTM) and Johnson's algorithm. The advance of PTM had been proposed by Baskar and Anthony Xavier [12] as a current view.

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \\ \cdot & \cdot \\ \cdot & \cdot \\ T_{n-1,n} & T_{n-1,2} \\ T_{n1} & T_{n2} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1m} \\ t_{21} & t_{22} & \dots & t_{2m} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ t_{n-1,1} & t_{n-1,2} & \dots & t_{n-1,m} \\ t_{n1} & t_{n2} & \dots & t_{nm} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} (m-2)C_0 & 0 \\ (m-2)C_1 & (m-2)C_0 \\ \cdot & \cdot \\ \cdot & \cdot \\ (m-2)C_{(m-2)} & (m-2)C_{(m-3)} \\ 0 & (m-2)C_{(m-2)} \end{bmatrix}$$

Pascal's Triangle is the triangular representation of the combinational elements of  $nC_r$ ,

The PFS problem of 'n' jobs to be processed in 'm' machines will be converted to 'n' jobs, two machine problem using this Heuristic and the problem will be solved for minimum Makespan using Johnson's Algorithm which is a proved one for a two machine PFS problem. Even though it is the latest method with better results, the steps involved in this heuristics are long process based and huge to solve.

### 3 PROPOSED HEURISTIC-EXPONENTIAL DISTRIBUTION TECHNIQUE

This algorithm distributes a factor in the processing time of the jobs at each machine from the advancement of classical algorithm. This factor added to the job is evaluated through the exponential equation, which gives a value of the index to the respective job. By sorting the index value in descending an optimal sequence can be obtained.

From the illustration of Palmer, the Palmer's sequence can provide an optimum elapsed time. The advancement of his methodology is proposed as a new heuristic to evaluate the optimal elapsed time. This heuristic provides 'n' values, and these are to be descended and respective jobs to be sequenced.

Using the taillard benchmark problem [13], the newly proposed heuristic is compared with existing algorithms. The processing times vary from 1 to 99 time units and they are generated using a random number generator for different seeds.

**Step 1:** Let 'n' number of jobs to be machined on 'm' machines. It is assumed that all jobs are present for processing at time zero. And one job can run on one machine at a time without changing the machine order.

**Step 2:** The exponential index to be calculated using the exponential equation (4) for n jobs.

$$y_j = \sum_{i=0}^{i=m-1} (2.61 * m - \exp(i)) * T_{m-i} \quad (4)$$

Where,  $Y_j$  = exponential index value for jth job,  
 $m$  = number of machines  
 $T_{(m-i)}$  = process time of job under (m-i)<sup>th</sup> machine

**Step 3:** Sort the exponential index in descending order.

**Step 4:** Based on the sorted order, the jobs to be sequenced.

### 4. ANALYSIS OF EXPONENTIAL DISTRIBUTION TECHNIQUE

The set of results is obtained from the C++ program for this newly proposed heuristic. From Table 1-3, the results are compared and examined to the lower bound values of Taillard's benchmark problems. The results of each set are graphical represented in Fig. 1-3.

The overall percentage nearer to lower bound (LB) is tabulated and graphically represented in Table 4 and Fig. 4 respectively. It shows that the EPDT performs better as compared to existing algorithms and reaches LB about 78.5%.

### 5. CONCLUSIONS

In this research work, a new heuristic was proposed based on the exponential function from mathematical and computational criteria to identify the optimal makespan giving sequence in a flow shop. The attempt made was good for high number jobs or machine; it was proved by the overall % nearer to LB chart. The processing times of jobs on machines are taken from taillard benchmark problems. It noticed that obtained minimum elapsed time for an optimal sequence from the proposed heuristic and it is compared with existing heuristics. The C++ program was generated for comparing the results computationally. The proposed heuristic give a better optimal sequence about 80%, compared to others algorithms.

**Table 1.** Comparison of heuristic's makespan for 5 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
873654221	1232	1,409	1377	1398	1377
379008056	1290	1424	1468	1481	1360
1866992158	1073	1255	1379	1387	1236
216771124	1268	1485	1548	1438	1564
495070989	1198	1367	1387	1354	1342
402959317	1180	1387	1411	1310	1385
1369363414	1226	1403	1381	1438	1268
2021925980	1170	1395	1404	1339	1504
573109518	1206	1360	1425	1428	1434
88325120	1082	1196	1284	1237	1298

**Table 2.** Comparison of heuristic's makespan for 10 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
587595453	1448	1829	1887	1896	1915
1401007982	1479	2021	2121	2073	1928
873136276	1407	1773	1786	1883	1737
268827376	1308	1678	1628	1703	1727
1634173168	1325	1781	2693	1718	1713
691823909	1290	1813	1835	1757	1618
73807235	1388	1826	1659	1725	1870
1273398721	1363	2031	1878	1821	1928
2065119309	1472	1831	1851	1832	1832
1672900551	1356	2010	1878	1876	2035

**Table 3.** Comparison of heuristic's makespan for 20 machines, 20 jobs.

Taillard Seeds	Lower Bound	GUPTA	CR	PTM	EPDT
479340445	1911	2833	2700	2614	2606
268827376	1711	2564	2600	2608	2516
1958948863	1844	2977	2742	2776	2575
918272953	1810	2603	2550	2628	2561
555010963	1899	2733	2815	2799	2513
2010851491	1875	2707	2518	2588	2697
1519833303	1875	2670	2730	2551	2687
1748670931	1880	2523	2582	2568	2676
1923497586	1840	2583	2615	2615	2553
1829909967	1900	2707	2472	2695	2372

**Table 4.** Overall comparison of heuristics using Taillard benchmark problems.

No. of machines	CR	GUPTA	PTM	EPDT
5	84.80 %	87.20 %	86.38 %	86.86 %
10	73.30 %	74.61 %	75.77 %	75.80 %
20	70.54 %	69.06 %	70.18 %	72.09 %

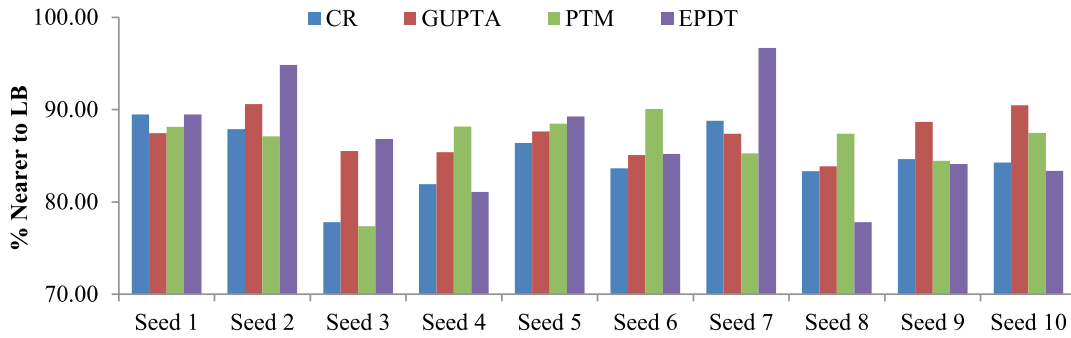


Fig. 1. Comparison of Heuristic's makespan for 5 machines, 20 jobs.a

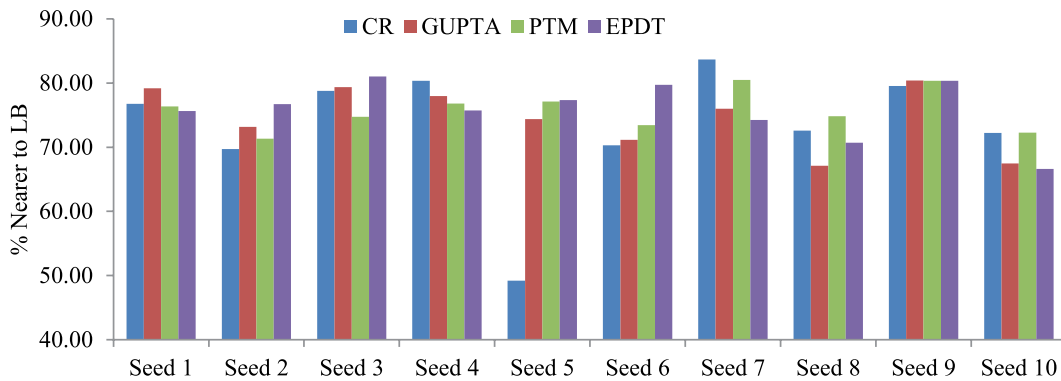


Fig. 2. Comparison of Heuristic's makespan for 10 machines, 20 jobs.

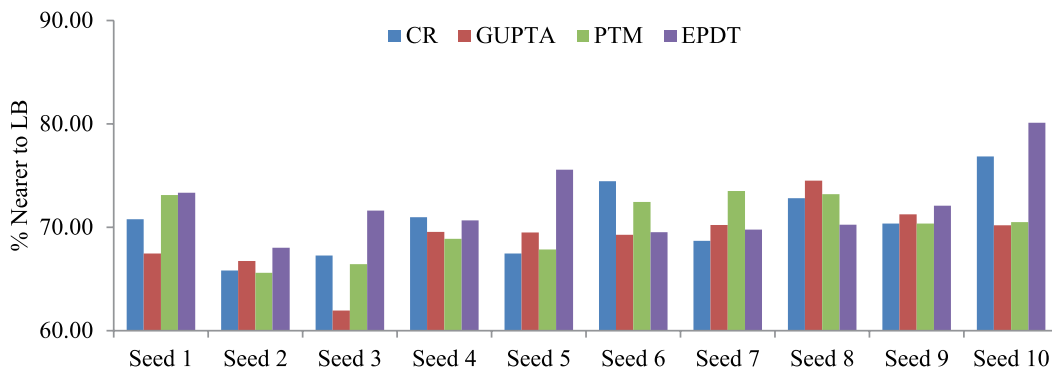


Fig. 3. Comparison of Heuristic's makespan for 20 machines, 20 Jobs

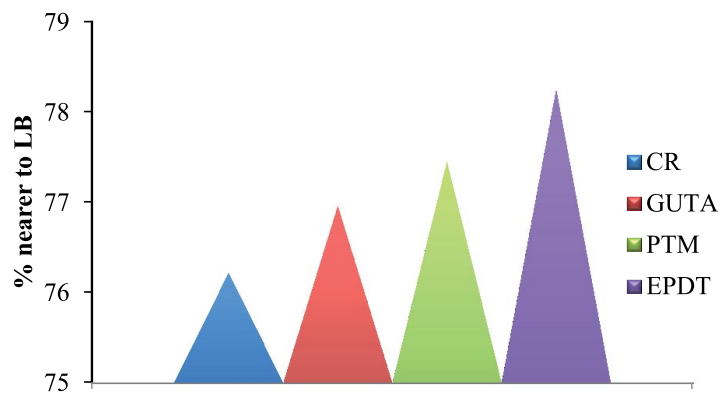


Fig. 4. Overall comparison of Heuristic's using Taillard benchmark problems.

**6. REFERENCES**

1. Johnson, S.M. Optimal two and three machine production scheduling with setup times included. *Naval Research* 1(1): 61–68 (1954).
2. Ignall, E. & L. Schrage. Application of the branch and bound technique to some flow-shop scheduling problems. *Operations Research* 15: 400 – 412 (1965).
3. Palmer, D.S. Sequencing jobs through a multi-stage process in the minimum total time – a quick method of obtaining a near optimum. *Operations Research* 16: 101-107 (1965).
4. Palmer, K. Sequencing rules and due date assignments in a job shop. *Management Science* 30(9): 1093 – 1104 (1984).
5. Hundal, T.S. & J. Rajgopal. An extension of Palmer heuristic for the flow-shop scheduling problem. *International Journal of Production Research* 26: 1119-1124 (1988).
6. Dannenbring, D.G. An evolution of flow-shop sequencing heuristics. *Management Science* 23: 1174-1182 (1977).
7. King, J.R. & A.S. Spachis. Heuristics for flowshop scheduling. *International Journal of Production Research* 18(3): 345-357 (1980).
8. Blackstone, J.H., D.T. Phillips, & G.L. Hogg. A state of art survey of dispatching rules for manufacturing job shop operations. *International Journal of Production Research* 20: 27–45 (1982).
9. Gupta, J.N.D. A functional heuristic algorithm for the flow shop scheduling problem. *Operational Research* 22: 27–39 (1971).
10. Rajendran, C. Theory and methodology heuristics for scheduling in flow shop with multiple objectives. *European Journal of Operational Research* 82: 540–555 (1995).
11. Baskar, A. & M.A. Xavier. A new Heuristic algorithm using Pascal's triangle to determine more than one sequence having optimal/ near optimal make span in flow shop scheduling problems. *International Journal of Computer Application* 39(5): 9-15, (2012).
12. Baskar, A. & M. Anthony Xavier. Optimization of total material processing time in a manufacturing flow shop environment. *Advanced Materials Research* 622-623: 136-141 (2013).
13. Taillard, E. Bench marks for basic scheduling problems. *European Journal of Operational Research* 64(2): 278-285 (1993).