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Original Article

$\ddot{g}_\alpha$-CLOSED SETS IN TOPOLOGY

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Abstract: In this paper, we introduce a new class of sets called $\ddot{g}_\alpha$-closed sets in topological spaces. We prove that this class lies between α-closed sets and gα-closed sets. We discuss some basic properties of $\ddot{g}_\alpha$-closed sets.

Keywords: Topological space, sg-closed set, $\ddot{g}$-closed set, $\ddot{g}_\alpha$-closed set, gp-closed set, gsp-closed set
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1. INTRODUCTION

The concept of generalized closed sets play a significant role in topology. There are many research papers which deals with different types of generalized closed sets. Bhattacharya and Lahiri [3] introduced sg-closed set in topological spaces. Arya and Nour [2] introduced gs-closed sets in topological spaces. Sheik John [16] introduced ω-closed sets in topological spaces. Rajamani and Viswanathan [14] introduced αgs-closed sets in topological spaces. Quite Recently, Ravi and Ganesan [15] introduced $\ddot{g}$-closed sets and proved that they forms a topology. In this paper we introduce a new class of sets, namely $\ddot{g}_\alpha$-closed sets, for topological spaces and study their basic properties.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c or X – A denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1

A subset A of a space (X, τ) is called:

(i) semi-open set [8] if A $\subseteq$ cl(int(A));

(ii) preopen set [11] if A $\subseteq$ int(cl(A));

(iii) α-open set [12] if A $\subseteq$ int(cl(int(A)));

(iv) semi-preopen [1] if A $\subseteq$ cl(int(cl(A))).

The complements of the above mentioned open sets are called their respective closed sets.

The preclosure [13] (resp. semi-closure [5], α-closure [12], semi-pre-closure [1]) of a subset A of X, denoted by pcl(A) (resp. scl(A), αcl(A), spcl(A)) is defined to be the intersection of all preclosed (resp. semi-closed, α-closed, semi-preclosed) sets of (X, τ) containing A. It is known that pcl(A) (resp. scl(A), αcl(A), spcl(A)) is a preclosed (resp. semi-closed, α-closed, semi-preclosed) set. For any subset A of an arbitrarily chosen topological space, the semi-interior [5] (resp. α-interior [12], preinterior [13], semi-pre-interior [1]) of A, denoted by sint(A) (resp. αint(A), pint(A),

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spint(A)), is defined to be the union of all semi-open (resp. α-open, preopen, semi-preopen) sets of (X, τ) contained in A.

Definition 2.2

A subset A of a space (X, τ) is called:

(i) a generalized closed (briefly g-closed) set [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of g-closed set is called g-open set;

(ii) a semi-generalized closed (briefly sg-closed) set [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of sg-closed set is called sg-open set;

(iii) a generalized semi-closed (briefly gs-closed) set [2] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of gs-closed set is called gs-open set;

(iv) an α-generalized closed (briefly αg-closed) set [10] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of αg-closed set is called αg-open set;

(v) a generalized α-closed (briefly gα-closed) set [9] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ). The complement of gα-closed set is called gα-open set;

(vi) a αgs-closed set [14] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of αgs-closed set is called αgs-open set;

(vii) a generalized semi-preclosed (briefly gsp-closed) set [6] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of gsp-closed set is called gsp-open set;

(viii) a generalized preclosed (briefly gp-closed) set [13] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ). The complement of gp-closed set is called gp-open set;

(ix) a ĝ-closed set [17] (= ω-closed [16]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ). The complement of ĝ-closed set is called ĝ-open set;

(x) a $\ddot{g}$-closed set [15] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is sg-open in (X, τ). The complement of $\ddot{g}$-closed set is called $\ddot{g}$-open set.

Remark 2.3

The collection of all $\ddot{g}_\alpha$-closed (resp. $\ddot{g}$-closed, ω-closed, g-closed, gs-closed, gsp-closed, αg-closed, αgs-closed, sg-closed, gα-closed, gp-closed, α-closed, semi-closed) sets is denoted by $\ddot{G}_\alpha$C(X) (resp. $\ddot{G}$C(X), ωC(X), GC(X), GSC(X), GSPC(X), αGC(X), αGSC(X), SGC(X), $G\alpha$C(X), GPC(X), αC(X), SC(X)).

The collection of all $\ddot{g}_\alpha$-open (resp. $\ddot{g}$-open, ω-open, g-open, gs-open, gsp-open, αg-open, αgs-open, sg-open, gα-open, gp-open, α-open, semi-open) sets is denoted by $\ddot{G}_\alpha$O(X) (resp. $\ddot{G}$O(X), ωO(X), GO(X), GSO(X), GSPO(X), αGO(X), αGSO(X), SGO(X), $G\alpha$O(X), GPO(X), αO(X), SO(X)).

We denote the power set of X by P(X).

Result 2.4

(1) Every semi-closed set is sg-closed [4].

(2) Every $\ddot{g}$-closed set is $\ddot{g}_\alpha$-closed but not conversely [15].

Corollary 2.5 [3]

Let A be a sg-closed set which is also open. Then A $\cap$ F is sg-closed whenever F is semi-closed.

3. $\ddot{g}_\alpha$-CLOSED SETS

We introduce the following definition:

Definition 3.1

A subset A of X is called a $\ddot{g}_\alpha$-closed set if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is sg-open in (X, τ).

Proposition 3.2

Every closed set is $\ddot{g}_\alpha$-closed.

Proof

Let A be a closed set and G be any sg-open set containing A. Since A is closed, we have αcl(A) $\subseteq$ cl(A) = A $\subseteq$ G. Hence A is $\ddot{g}_\alpha$-closed.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {b}, {c}, {b, c}, X}. Here, A = {b} is $\ddot{g}_\alpha$-closed set but not closed.

Proposition 3.4

Every α -closed set is $\ddot{g}_\alpha$-closed.

Proof

Let A be an α -closed set and G be any sg-open set containing A. Since A is α -closed, we have α cl(A) = A ⊆ G. Hence A is $\ddot{g}_\alpha$-closed.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5

Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and α C(X) = {ϕ, {c}, X}. Here, A = {a, c} is $\ddot{g}_\alpha$-closed set but not α -closed.

Proposition 3.6

Every $\ddot{g}_\alpha$-closed set is gα -closed.

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any α -open set containing A. Since any α -open set is semi-open and semi-open set is sg-open, we have α cl(A) ⊆ G. Hence A is gα -closed.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {a}, {b, c}, X} and $G\alpha$ C(X) = P(X). Here, A = {c} is gα -closed set but not $\ddot{g}_\alpha$-closed.

Proposition 3.8

Every $\ddot{g}_\alpha$-closed set is α g-closed.

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any open set containing A. Since any open set is sg-open, we have α cl(A) ⊆ G. Hence A is α g-closed.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {a}, {b, c}, X} and αG C(X) = P(X). Here, A = {c} is α g-closed set but not $\ddot{g}_\alpha$-closed.

Proposition 3.10

Every $\ddot{g}_\alpha$-closed set is gs-closed (sg-closed).

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any open set (semi-open set) containing A. Since any open set (semi-open set) is sg-open, we have scl(A) ⊆ α cl(A) ⊆ G. Hence A is gs-closed (sg-closed).

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}_\alpha$ C(X) = {ϕ, {a}, {b, c}, X} and SG C(X) = GS C(X) = P(X). Here, A = {c} is both sg-closed and gs-closed set but not $\ddot{g}_\alpha$-closed.

Proposition 3.12

Every $\ddot{g}_\alpha$-closed set is αgs -closed.

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any semi-open set containing A. Since any semi-open set is sg-open, we have α cl(A) ⊆ G. Hence A is αgs -closed.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {a}, {b, c}, X} and αGS C(X) = P(X). Here, A = {c} is αgs -closed set but not $\ddot{g}_\alpha$-closed.

Proposition 3.14

Every $\ddot{g}_\alpha$-closed set is gsp-closed.

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any open set containing A. Since any open set is sg-open, we have spcl(A) ⊆ α cl(A) ⊆ G. Hence A is gsp-closed.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}$ C(X) = {ϕ, {a}, {b, c}, X} and GSP C(X) = P(X). Here, A = {c} is gsp-closed set but not $\ddot{g}_\alpha$-closed.

Proposition 3.16

Every $\ddot{g}_\alpha$-closed set is gp-closed.

Proof

Let A be an $\ddot{g}_\alpha$-closed set and G be any open set containing A. Since any open set is sg-open, we have pcl(A) $\subseteq$ α cl(A) $\subseteq$ G. Hence A is gp-closed.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then $\ddot{G}$ C(X) = {ϕ, {b}, {c}, {b, c}, X} and GP C(X) = {ϕ, {b}, {c}, {a, b}, {a, c}, {b, c}, X}. Here, A = {a, b} is gp-closed set but not $\ddot{g}_\alpha$-closed.

Remark 3.18

The following examples show that $\ddot{g}_\alpha$-closedness is independent of ω-closedness, semi-closedness and g-closedness.

Example 3.19

Let X = {a, b, c} with τ = {ϕ, {a}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {b}, {c}, {b, c}, X} and ωC(X) = {ϕ, {b, c}, X}. Here, A = {c} is $\ddot{g}_\alpha$-closed set but not ω-closed.

Example 3.20

Let X = {a, b, c} with τ = {ϕ, {a}, {b, c}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {a}, {b, c}, X} and ωC(X) = P(X). Here, A = {c} is ω-closed set but not $\ddot{g}_\alpha$-closed.

Example 3.21

Let X = {a, b, c} with τ = {ϕ, {a}, {b}, {a, b}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and S C(X) = {ϕ, {a}, {b}, {c}, {a, c}, {b, c}, X}. Here, A = {b} is semi-closed set but not $\ddot{g}_\alpha$-closed.

Example 3.22

Let X = {a, b, c} with τ = {ϕ, {a, b}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {c}, {a, c}, {b, c}, X} and S C(X) = {ϕ, {c}, X}. Here, A = {b, c} is $\ddot{g}_\alpha$-closed set but not semi-closed.

Example 3.23

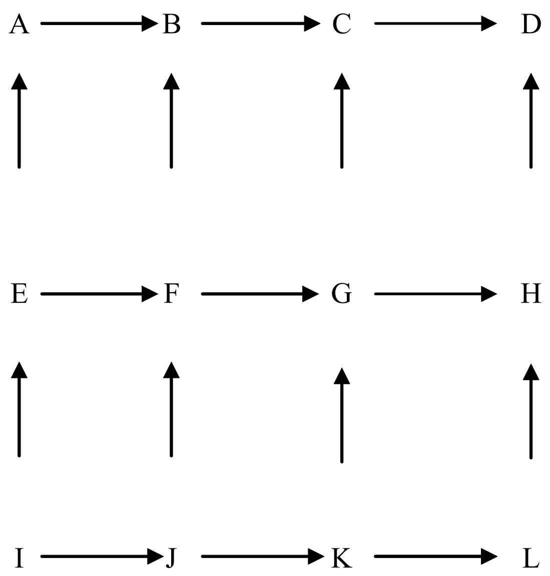
Let X = {a, b, c} with τ = {ϕ, {a}, {a, b}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {b}, {c}, {b, c}, X} and G C(X) = {ϕ, {c}, {a, c}, {b, c}, X}. Here,

(i) A = {b} is $\ddot{g}_\alpha$-closed set but not g closed.

(ii) B= {a, c} is g-closed set but not $\ddot{g}_\alpha$-closed.

Remark 3.24

From the above discussions and known results in [6, 15, 16, 18], we obtain the following diagram, where A → B (resp. A ↮ B) represents A implies B but not conversely (resp. A and B are independent of each other)



Where

A: semi-closed B: sg-closed C: gs-closed D: gsp-closed E: α -closed F: $\ddot{g}_\alpha$-closed G: gα -closed H : αg-closed I: closed J : $\ddot{g}$-closed K: ω-closed L: g-closed.

None of the above implications is reversible as shown in the above examples and in the related papers [6, 15, 16, 18].

4. PROPERTIES OF $\ddot{g}_\alpha$-CLOSED SETS

In this section, we discuss some basic properties of $\ddot{g}_\alpha$-closed sets.

Definition 4.1 [15]

The intersection of all sg-open subsets of (X, τ) containing A is called the sg-kernel of A and denoted by sg-ker(A).

Lemma 4.2

A subset A of (X, τ) is $\ddot{g}_\alpha$-closed if and only if α cl(A) ⊆ sg-ker(A).

Proof

Suppose that A is $\ddot{g}_\alpha$-closed. Then α cl(A) ⊆ U whenever A ⊆ U and U is sg-open. Let x ∈ α cl(A). If x ∉ sg-ker(A), then there is a sg-open set U containing A such that x ∉ U. Since U is a sg-open set containing A, we have x ∉ α cl(A) and this is a contradiction.

Conversely, let α cl(A) ⊆ sg-ker(A). If U is any sg-open set containing A, then α cl(A) ⊆ sg-ker(A) ⊆ U. Therefore, A is $\ddot{g}_\alpha$-closed.

Proposition 4.3

For any subset A of (X, τ), X_2 ∩ α cl(A) ⊆ sg-ker(A), where X_2 = {x ∈ X = $X_1 \cup X_2$: {x} is preopen}.

Proof

Let x ∈ X_2 ∩ α cl(A) and suppose that x ∉ sg-ker(A). Then there is a sg-open set U containing A such that x ∉ U. If F = X – U, then F is sg-closed. Since α cl({x}) ⊆ α cl(A), we have int(α cl({x})) ⊆ A ∪ int(α cl(A)). Again since x ∈ X_2, we have x ∉ X_1 and so int(α cl({x})) = ϕ. Therefore, there has to be some point y ∈ A ∩ int(α cl({x})) and hence y ∈ F ∩ A, a contradiction.

Theorem 4.4

A subset A of (X, τ) is $\ddot{g}_\alpha$-closed if and only if X_1 ∩ α cl(A) ⊆ A, where X_1 = {x ∈ X = $X_1 \cup X_2$: {x} is nowhere dense}.

Proof

Suppose that A is $\ddot{g}_\alpha$-closed. Let x ∈ X_1 ∩ α cl(A). Then x ∈ X_1 and x ∈ α cl(A). Since x ∈ X_1, int(α cl({x})) = ϕ. Therefore, {x} is semi-closed, since int(α cl({x})) ⊆ {x}. Since every semi-closed set is sg-closed [Result 2.4 (1)], {x} is sg-closed. If x ∉ A and if U = X \ {x}, then U is a sg-open set containing A and so α cl(A) ⊆ U, a contradiction.

Conversely, suppose that X_1 ∩ α cl(A) ⊆ A. Then X_1 ∩ α cl(A) ⊆ sg-ker(A), since A ⊆ sg-ker(A). Now α cl(A) = X ∩ α cl(A) = ($X_1 \cup X_2$) ∩ α cl(A) = (X_1 ∩ α cl(A)) ∪ (X_2 ∩ α cl(A)) ⊆ sg-ker(A), since X_1 ∩ α cl(A) ⊆ sg-ker(A) and by Proposition 4.3. Thus, A is $\ddot{g}_\alpha$-closed by Lemma 4.2.

Theorem 4.5

An arbitrary intersection of $\ddot{g}_\alpha$-closed sets is $\ddot{g}_\alpha$-closed.

Proof

Let F = {A_i : i ∈ ∧} be a family of $\ddot{g}_\alpha$-closed sets and let A = $\cap_{i \in \wedge} A_i$. Since A ⊆ A_i for each i, X_1 ∩ α cl(A) ⊆ X_1 ∩ α cl(A_i) for each i.Using Theorem 4.4 for each $\ddot{g}_\alpha$-closed set A_i, we have X_1 ∩ α cl(A_i) ⊆ A_i. Thus, X_1 ∩ α cl(A) ⊆ X_1 ∩ α cl(A_i) ⊆ A_i for each i ∈ ∧. That is, X_1 ∩ α cl(A) ⊆ A and so A is $\ddot{g}_\alpha$-closed by Theorem 4.4.

Corollary 4.6

If A is a $\ddot{g}_\alpha$-closed set and F is a closed set, then A ∩ F is a $\ddot{g}_\alpha$-closed set.

Proof

Since F is closed, it is $\ddot{g}_\alpha$-closed. Therefore by Theorem 4.5, A ∩ F is also a $\ddot{g}_\alpha$-closed set.

Proposition 4.7

If A and B are $\ddot{g}_\alpha$-closed sets in (X, τ), then A ∪ B is $\ddot{g}_\alpha$-closed in (X, τ).

Proof

If A ∪ B ⊆ G and G is sg-open, then A ⊆ G and B ⊆ G. Since A and B are $\ddot{g}_\alpha$-closed, G ⊇ α cl(A) and G ⊇ α cl(B) and hence G ⊇ α cl(A) ∪ α cl(B) = α cl(A ∪ B). Thus A ∪ B is $\ddot{g}_\alpha$-closed set in (X, τ).

Proposition 4.8

If a set A is $\ddot{g}_\alpha$-closed in (X, τ), then α cl(A) – A contains no nonempty closed set in (X, τ).

Proof

Suppose that A is $\ddot{g}_\alpha$-closed. Let F be a closed subset of α cl(A) – A. Then A ⊆ F^c. But A is $\ddot{g}_\alpha$-closed, therefore α cl(A) ⊆ F^c. Consequently, F ⊆ (α cl(A))^c. We already have

$F \subseteq \alpha$ cl(A). Thus $F \subseteq \alpha$ cl(A) $\cap$ (α cl(A))^c and F is empty.

The converse of Proposition 4.8 need not be true as seen from the following example.

Example 4.9

Let X = {a, b, c, d} with τ = {ϕ, {b, c}, {b, c, d}, {a, b, c}, X}. Then $\ddot{G}_\alpha$C(X) = {ϕ, {a}, {d}, {a, d}, {a, b, d}, {a, c, d}, X}. If A = {a, b, d}, then α cl(A) − A = X − {a, b, d} = {c} does not contain any nonempty closed set. But A is not $\ddot{g}_\alpha$-closed in (X, τ).

Theorem 4.10

A set A is $\ddot{g}_\alpha$-closed if and only if α cl(A) − A contains no nonempty sg-closed set.

Proof

Necessity. Suppose that A is $\ddot{g}_\alpha$-closed. Let S be a sg-closed subset of α cl(A) – A. Then $A \subseteq S^c$. Since A is $\ddot{g}_\alpha$-closed, we have α cl(A) $\subseteq S^c$. Consequently, $S \subseteq$ (α cl(A))^c. Hence, $S \subseteq \alpha$ cl(A) $\cap$ (α cl(A))^c = ϕ. Therefore S is empty.

Sufficiency. Suppose that α cl(A) – A contains no nonempty sg-closed set. Let $A \subseteq G$ and G be closed and sg-open. If α cl(A) $\not\subseteq$ G, then α cl(A) $\cap G^c \neq \phi$. Since α cl(A) is a α -closed set (and hence semi-closed set) and G^c is a sg-closed set and open, α cl(A) $\cap G^c$ is a nonempty sg-closed subset of α cl(A) – A by Corollary 2.5. This is a contradiction. Therefore, α cl(A) $\subseteq$ G and hence A is $\ddot{g}_\alpha$-closed.

Proposition 4.11

If A is $\ddot{g}_\alpha$-closed in (X, τ) and $A \subseteq B \subseteq \alpha$ cl(A), then B is $\ddot{g}_\alpha$-closed in (X, τ).

Proof

Let G be a sg-open set of (X, τ) such that $B \subseteq G$. Then $A \subseteq G$. Since A is an $\ddot{g}_\alpha$-closed set, α cl(A) $\subseteq$ G. Also α cl(B) = α cl(A) $\subseteq$ G. Hence B is also an $\ddot{g}_\alpha$-closed in (X, τ).

Proposition 4.12

Let $A \subseteq Y \subseteq X$ and suppose that A is $\ddot{g}_\alpha$-closed in (X, τ). Then A is $\ddot{g}_\alpha$-closed relative to Y.

Proof

Let $A \subseteq Y \cap G$, where G is sg-open in (X, τ). Then $A \subseteq G$ and hence α cl(A) $\subseteq$ G. This implies that $Y \cap \alpha$ cl(A) $\subseteq Y \cap G$. Thus A is $\ddot{g}_\alpha$-closed relative to Y.

Proposition 4.13

If A is a sg-open and $\ddot{g}_\alpha$-closed in (X, τ), then A is α -closed in (X, τ).

Proof

Since A is sg-open and $\ddot{g}_\alpha$-closed, α cl(A) $\subseteq$ A and hence A is α -closed in (X, τ).

Proposition 4.14

For each $x \in X$, either {x} is sg-closed or {x}^{c} is $\ddot{g}_\alpha$-closed in (X, τ).

Proof

Suppose that {x} is not sg-closed in (X, τ). Then {x}^{c} is not sg-open and the only sg-open set containing {x}^{c} is the space X itself. Therefore α cl({x}^{c}) $\subseteq$ X and so {x}^{c} is $\ddot{g}_\alpha$-closed in (X, τ).

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